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THE SYMMETRIES OF $Z_n \times Z_n$ BELAVIN MODEL

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The symmetries of $Z_n \times Z_n$ Belavin model [1]

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Abstract

It is shown that there exist some new symmetries in $Z_n \times Z_n$ Belavin model. These symmetric properties can be used to construct the new exactly solvable statistical model with nontrivial boundary terms.

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1. Introduction

The quantum Yang-Baxter equation(QYBE) was discovered by Yang [1] and Baxter [2]. The solutions of QYBE have been classified as rational, trigonometric and elliptic according to their dependence of spectral parameters [3,4]. A host of investigation revealed that QYBE has played an important role in quantum field theories [5-7] and integrable statistical models [8-11]. Much attention was recently paid to the systems on a finite interval with independent boundary conditions on each end proposed by Sklyanian [12], Mezincescu and Nepomechie [13]. For a given trigonometric solution of QYBE based on the classical Lie algebra except $A_n (n > 1)$, it has been shown that the system with special boundary condition has the quantum group symmetry [12,13]. The trigonometric solution of the QYBE is a limit case of elliptic case. Minimal models in conformal field theory are closely related with the critical states of the elliptic RSOS [8]. The trigonometric limit of the Boltzmann weights for the elliptic case are identical with the fusion and braiding matrices in the minimal conformal field theory, which are the Racah coefficients of quantum group [5-7]. The Belavin's Z_n symmetric solution of QYBE is related with $A_{n-1}^{(1)}$ algebra, which may be used to study the systems with $U_q(A_{n-1})$ symmetry. This interesting relation motivates us to study the Belavin Z_n symmetric model with nontrivial boundary condition.

The symmetry of the solution of QYBE plays a key role in procedure of studying the systems on finite interval with independent boundary condition on each end by means of the method proposed by Sklyanian [12] and developed by Mezincescu and Nepomechie [13]. The Z_n elliptic solution of QYBE associated with

the Belavin $Z_n \times Z_n$ symmetric model, which is the generalization of the Baxter's eight-vertex model. Some symmetries of the model was revealed in ref.[14]. The structure constants, both for the classical and for the quantum algebras, and fusion representation in this model were discussed [15]. Hou et al [16] had shown that there exists the quantum symmetric algebra in the Z_n elliptic solution of QYBE under the trigonometric limit. The trigonometric solution of QYBE based on the algebra $A_n(n > 1)$ does not satisfy the restrictive conditions suggested in [12] and [13].

The purpose of this paper is to show some new symmetries and useful properties of the Z_n elliptic solution of QYBE, which can be used to investigate the Belavin $Z_n \times Z_n$ symmetric model with nontrivial boundary condition.

2. Belavin $Z_n \times Z_n$ symmetric model.

The Belavin $Z_n \times Z_n$ symmetric model is the elliptic function solution of QYBE. There exist two equivalent forms of its expression [10,14]

$$\begin{aligned} R(u) &= \exp\{-i\pi u\} \cdot \sum_{\alpha \in Z_n^2} W_\alpha(u) I_\alpha \otimes I_\alpha^h \\ &= \sum_{ij,i'j'} S(u)_{ij}^{i'j'} E_{ii'} \otimes E_{jj'}, \end{aligned} \tag{1}$$

where, superscript h stands for the Hermitian conjugation. E_{ij} and $I_\alpha = h^{\alpha_1} g^{\alpha_2}$ are $n \times n$ matrices with the matrix elements

$$(E_{ij})_{kl} = \delta_{ki}\delta_{jl}, \tag{2}$$

$$(h)_{ij} = \delta_{i(\mathrm{mod.}n)}^{j+1}, \qquad g_{ik} = \omega^{i}\delta_{ij}, \tag{3}$$

ω is equal to $\exp(\frac{i2\pi}{n})$ and $\alpha = (\alpha_1, \alpha_2), \alpha_i = 0, 1, \cdots, n-1$. The coefficient $S(u)_{ij}^{kl}$ in Eq.(1) is called the Boltzmann weight and can be parametrised in terms of

Jacobi theta function

$$\begin{aligned} S(u)_{ij}^{kl} &= S(u)_{i+p,j+p}^{k+p,l+p} \\ &= \delta_{i+j,k+l}^{(\mathrm{mod}\, n)} \cdot n \cdot \exp\{-i\pi u\} \frac{\theta\begin{bmatrix} \frac{l-k}{k} + \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}(u+w, n\tau)}{\theta\begin{bmatrix} \frac{i-k}{n} + \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}(w, n\tau) \cdot \theta\begin{bmatrix} \frac{l-i}{n} + \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}(u, n\tau)} \\ &\times \frac{\prod_{\lambda=0}^{n-1} \theta\begin{bmatrix} \frac{\lambda}{n} + \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}(u, n\tau)}{\prod_{\lambda=1}^{n-1} \theta\begin{bmatrix} \frac{\lambda}{n} + \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}(0, n\tau)}. \end{aligned} \tag{4}$$

In another expression form, $W_\alpha(u)$ in (1) reads

$$W_\alpha(u) = \frac{\theta\begin{bmatrix} \frac{\alpha_1}{n} + \frac{1}{2} \\ \frac{\alpha_2}{n} + \frac{1}{2} \end{bmatrix}(u + \frac{w}{n}, \tau)}{\theta\begin{bmatrix} \frac{\alpha_1}{n} + \frac{1}{2} \\ \frac{\alpha_2}{n} + \frac{1}{2} \end{bmatrix}(\frac{w}{n}, \tau)}. \tag{5}$$

The definition of Jacobi theta function is given by

$$\theta\begin{bmatrix} a \\ b \end{bmatrix}(u, \tau) = \sum_{m \in Z} \exp\{i\pi\tau(m+a)^2 + i2\pi(m+a)(u+b)\}. \tag{6}$$

R matrix (1) satisfies the QYBE

$$R_{12}(u-v)R_{13}(u)R_{23}(v) = R_{23}(v)R_{13}(u)R_{12}(u-v). \tag{7}$$

Introducing an operator

$$L(u) = \sum_{\alpha \in Z_n^2} W_\alpha(u) I_\alpha S_\alpha, \tag{8}$$

we can rewrite the QYBE as

$$R_{12}(u-v)L^1(u)L^2(v) = L^2(v)L^1(u)R_{12}(u-v). \tag{9}$$

The relation (9) is equivalent to

$$\sum_{\gamma\in Z_n^2} \omega^{(\beta_1-\gamma_1)(\gamma_2-\alpha_2)} W_{\alpha\beta\gamma}(u,v) S_{\alpha+\beta-\gamma} S_\gamma = 0, \tag{10}$$

here

$$W_{\alpha\beta\gamma}(u,v) = W_{\gamma-\alpha}(u-v) W_{\alpha+\beta-\gamma}(u) W_\gamma(v) - W_{\beta-\gamma}(u-v) W_\gamma(u) W_{\alpha+\beta-\gamma}(v). \tag{11}$$

In order to simplify the equation (10), one can introduce the formal solution of $W_{\alpha\beta\gamma}$

$$\begin{aligned} W_{\alpha\beta\gamma}(u,v) &= C_{\alpha\beta\gamma}(u,v)\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(u,\tau)\theta\begin{bmatrix}\frac{\alpha_1}{n}+\frac{1}{2}\\ \frac{\alpha_2}{n}+\frac{1}{2}\end{bmatrix}(u+v+\tfrac{2w}{n},\tau) \\ &\quad \cdot\theta\begin{bmatrix}\frac{\beta_1}{n}+\frac{1}{2}\\ \frac{\beta_2}{n}+\frac{1}{2}\end{bmatrix}(v,\tau). \end{aligned} \tag{12}$$

It can be shown that the constant $C_{\alpha\beta\gamma}(u,v)$ is independent of the spectral parameters u and v [15]. Hence, Eq.(10) can be rewritten as the following algebraic relation

$$\sum_\gamma C_{\alpha\beta\gamma} S_{\alpha+\beta-\gamma} S_\gamma = 0. \qquad \text{for } \alpha,\beta \in Z_n^2 \tag{13}$$

Since Eq.(13) can be reduced to the Sklyanin algebra when $n = 2$, it is regarded as the generalization of the Sklyanin algebra.

3. The symmetries and the properties of the Belavin $Z_n \times Z_n$ symmetric model.

Richey and Tracy [14] had discussed the symmetries based on the invariances of the model. These symmetries are not enough to construct the exactly solvable statistical model with nontrivial boundary conditions. We must investigate new symmetries and some useful properties of R matrix of the model.

By directly calculating, one can show the relations $R(0)A \otimes BR(0) = n^2(B \otimes A)$ and $R^2(0) = n^2$, in which A and B are two arbitrary $n \times n$ matrices. From these relstions, we have the following consequence:

Proposition 1. The value of R(u) at u=0 is in proportion to the permutation operator

$$R(0) = nP \tag{14}$$

Proposition 2.

$$P_{12}R_{12}(u)P_{12} = R_{12}^{h_1h_2}(u) \tag{15}$$

where, h_i represents the hermitian conjugation of the i-th vector space and $P_{12}(a \otimes b) = b \otimes a$.

This property of the R matrix is obvious. It means that the Z_n elliptic solution of QYBE possess the PT invariance

Proposition 3. The matrix R(u) has the unitary property

$$R_{12}(u)R_{12}^{h_1h_2}(-u) = N(u,\tau)\mathrm{id} \tag{16}$$

here, id stands for the identical operator and

$$N(u,\tau) = n^2\frac{\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(u+w,\tau)\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(-u+w,\tau)}{\theta^2\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(w,\tau)}. \tag{17}$$

Proof. According to the definition of R(u), the left hand side of Eq.(16) is read

as

$$
\begin{aligned}
R_{12}(u)R_{12}^{h_1h_2}(-u) &= \sum_{\alpha,\beta} W_\alpha(u)W_\beta(-u)I_\alpha I_\beta^h \otimes I_\alpha^h I_\beta \\
&= \sum_{\alpha,\beta} W_\alpha(u)W_\beta(-u)I_{\alpha-\beta} \otimes I_{\alpha-\beta}^h \omega^{\alpha_1\beta_2-\alpha_2\beta_1} \\
&= \sum_{\alpha\in\mathcal{Z}_n^2}\left[\sum_{\beta\in\mathcal{Z}_n^2} W_{\alpha+\beta}(u)W_\beta(-u)\omega^{\alpha_1\beta_2-\alpha_2\beta_1}\right] I_\alpha \otimes I_\alpha^h.
\end{aligned}
\tag{18}
$$

Because of Z_n symmetry it is sufficient to consider S_{0a+b}^{ab} defined by (4), for $a, b \in \mathcal{Z}_n$, or $S^{a,b}$ as we shall henceforth abbreviate. From (1) and (2), we have

$$
W_{(a,b)}(u,\tau) = \frac{1}{n}\sum_{c\in\mathcal{Z}_n} S^{-a,c}\omega^{bc}. \tag{19}
$$

Thus, (18) can be rewritten as

$$
R_{12}(u)R_{12}^{h_1h_2}(-u) = \frac{1}{n}\sum_{\gamma_1,\gamma_2\in\mathcal{Z}_n^2}\omega^{c\gamma_2}\{\sum_{\beta\in\mathcal{Z}_n^2} S^{-(\gamma-c)-b,b}(u,w,\tau)\cdot S^{c-b,-b-\gamma_2}(-u,w,\tau)\}. \tag{20}
$$

Let

$$
\varphi_{\sigma\sigma'}(u) = \sum_{\beta\in\mathcal{Z}_n^2} S^{-(\sigma'-\sigma)-b,b}(u,w,\tau)\cdot S^{\sigma-b,-b-\sigma'}(-u,w,\tau). \tag{21}
$$

Now, we want to determine the explicit expression of $\varphi_{\sigma\sigma'}(u)$, which can be obtained by exploring the properties of its zeros. By means of the transformation properties of the theta function, one can show that

$$
\begin{aligned}
S^{a,b}(u+\xi_1\tau+\xi_2,w,\tau) &= exp\{-i2\pi\xi_1^2\tau - i2\pi\xi_1(u+\tfrac{w}{n}+\tfrac{\tau}{2}+\tfrac{1}{2})\} \\
&\quad\cdot\omega^{-a\xi_2}S^{a,b+\xi_1}(u,w,\tau)
\end{aligned}
\tag{22}
$$

$$
S^{a,b}(u+\xi_1\tau+\xi_2,w,\tau) = exp\{-i2\pi\frac{\xi_1}{n}u\}\omega^{b\xi_2}S^{a-\xi_1,b}(u,w,\tau) \tag{23}
$$

Combining (22) and (23) with (21), we get the transformation relations of $\varphi_{\sigma\sigma'}(u)$

$$
\varphi_{\sigma\sigma'}(u+1) = \omega^{\sigma'}\varphi_{\sigma\sigma'}(u) \tag{24}
$$

$$
\varphi_{\sigma\sigma'}(u+n\tau) = exp\{-i2\pi(n^2\tau+2nu\}\varphi_{\sigma\sigma'}(u) \tag{25}
$$

It is an elementary complex analysis argument that if f is entire, not identically zero, and satisfies

$$
f(z+\tau) = exp\{-2\pi i(A_1+A_2z)\}f(z),
$$

$$
f(z+1) = exp\{-2\pi iB\}f(z).
$$

Then necessarily A_2 is a positive integer, and f has zeros in Λ_τ with

$$
\sum \text{zero} = \frac{1}{2}A_2 + B\tau - A_1 \qquad (\text{mod } \Lambda_\tau),
$$

where, $\Lambda_\tau = \{\xi_1\tau+\xi_2|\xi_1,\xi_2\in\mathcal{Z}\}$ the lattice generated by 1 and τ. We apply this to $\varphi_{\sigma\sigma'}(u)$ to conclude that there are $2n$ zeros in $\Lambda_{n\tau}$ with sum

$$
\sum \text{zero} = nu - n^2\tau - \sigma'\tau \qquad (\text{mod } \Lambda_{n\tau}). \tag{26}
$$

In order to determine $\varphi_{\sigma\sigma'}(u)$, we must find the locations of the 2n zeros. It is from the $S^{a,b}(0,w,\tau)=0$ for $b\neq 0$ and the relation (22) that

$$
S^{a,b}(k\tau,w,\tau) \sim S^{a,b+k}(0) = 0 \qquad for\ k \neq -b(n). \tag{27}
$$

This means that $\varphi_{\sigma\sigma'}(k\tau)$ $(k=0,\cdots,n-1)$ is identically equal to zero unless $\sigma'=0$. By using of (25), one can write $\varphi_{\sigma\sigma'}(u)$ at $u=-w+k\tau$ as

$$
\begin{aligned}
\varphi_{\sigma\sigma'}(-w+k\tau) &= exp\{-i2\pi k^2\tau - i2\pi(k\tau-w)\} \\
&\quad\cdot\sum_b S^{\sigma-\sigma'-b,b+k}(-w,w,\tau)S^{\sigma-b,-b-\sigma'-k}(w,w,\tau)
\end{aligned}
\tag{28}
$$

Recall the $S^{a,b}(u)$ is zero at $u=(a-b)\tau-w$. Thus the first factor in each term in the sum of (28) is zero if $b+k=\sigma-\sigma'-b$. The b satisfies this condition, which

is read as

$$b = \begin{cases} \frac{1}{2}(\sigma - k - \sigma')(mod\ n), & for\ \sigma - \sigma' - k = even,\ and\ n\ =\ odd\ or\ even \\ \frac{1}{2}[\sigma - \sigma' - k(mod\ n)], & for\ \sigma - \sigma' - k = even,\ and\ n = even \\ \frac{1}{2}(n + \sigma - k - \sigma'), & for\ \sigma - \sigma' - k = odd,\ and\ n\ =\ odd, \\ no\ solutions, & for\ \sigma - \sigma' - k = odd,\ and\ n\ =\ even. \end{cases} \tag{29}$$

The $b's$ values of the remaining nonzero terms in the sum are expressed by $\{b \in \mathcal{Z}_n | 2b \neq \sigma - \sigma' - k\} = \bigcup_i^x (b_i, -\sigma' - b_i + \sigma)$, where $x = \left[\frac{n}{2}\right]$ if n is odd or $2b = \sigma - \sigma' + k$ has no solutions or $x = \left[\frac{n}{2}\right] - 1$ if $2b = \sigma - \sigma' - k$. Thus the sum in (28) can be rewritten

$$\begin{aligned} \sum_b S^{\sigma-\sigma'-b,b+k}(-w,w,\tau) S^{\sigma-b,-b-\sigma'-k}(w,w,\tau) \\ = \sum_i S^{\sigma-\sigma'-b_i,b_i+k}(-w,w,\tau) S^{\sigma-b_i,-b_i-\sigma'-k}(w,w,\tau) \\ + \sum_i S^{k+b_i,\sigma-\sigma'-b_i}(-w,w,\tau) S^{k+\sigma'+b_i,b_i-\sigma}(w,w,\tau). \end{aligned} \tag{30}$$

Noting the fact that

$$S^{a,b}(-w,w,\tau) = -S^{b,a}(-w,w,\tau)$$

and

$$S^{a,b}(w,w,\tau) = -S^{-b,-a}(w,w,\tau),$$

we obtain that

$$\varphi_{\sigma\sigma'}(-w + k\tau) = 0, \quad k = 0,\ 1, \cdots, n-1. \tag{31}$$

From the equations (27) and (31), we immediately see that $\varphi_{\sigma\sigma'}(u)$ has the 2n zeros with the sum $n(n-1)\tau - nu$ if $\sigma' \neq 0$. Comparing it with (26), we conclude

that

$$\varphi_{\sigma\sigma'}(u) \equiv 0 \qquad k = 0, 1, \cdots, n-1. \tag{32}$$

that is

$$\varphi_{\sigma\sigma'}(u) \equiv \delta_{\sigma',0}\varphi_{\sigma 0}(u) = \delta_{\sigma',0} \sum_{b \in \mathcal{Z}_n} S^{\sigma-b,b}(u,w,\tau) S^{\sigma-b,-b}(-u,w,\tau) \tag{33}$$

In order to calculate $\varphi_{\sigma 0}$, we introduce a function

$$\psi_{\sigma\sigma'}(u) = \varphi_{\sigma 0}(u) - \varphi_{\sigma' 0}(u)$$

Because $\varphi_{\sigma 0}(u)$ and $\varphi_{\sigma'}(u)$ have the same transformation properties, $\psi_{\sigma\sigma'}(u)$ has 2n zeros with $\sum$ zero $= 0$ mod $\Lambda_{n\tau}$. On other hand, at $u = k\tau$, $\psi_{\sigma\sigma'}(u)$ is equal to zero. By using of (31), one can see that $\psi_{\sigma\sigma'}(u)$ has $2n$ zeros with the sum $-nu$ mod $\Lambda_{n\tau}$. This results in that $\psi_{\sigma\sigma'}(u)$ is independent of σ. From the definition of $\varphi_{\sigma 0}(u)$, it is obvious that $\varphi_{\sigma 0}$ has the following transformation properties

$$\begin{aligned} \varphi_{\sigma 0}(u+1) &= \varphi_{\sigma 0}(u), \\ \varphi_{\sigma 0}(u+2\tau) &= exp\{-i2\pi(4\tau + 4u)\}\varphi_{\sigma 0}(u). \end{aligned} \tag{34}$$

Hence, there are four zeros of $\varphi_{\sigma 0}(u)$ with $\sum$ zero $= 0$ mod $\Lambda_{2\tau}$. Two zeros are determined by Eq.(31), which are $-w$ and $-w + \tau$. By using of the properties of $S^{a,b}(u,w,\tau)$, one can show that $u = w + \tau$ is the zero of $\varphi_{\sigma 0}(u)$. The condition $\sum$ zero $= 0$ results in that the rest zero is w. It is now straightforward to prove that $N(u,w,\tau)$ has the same transformation properties and zero set as $\varphi_{\sigma 0}(u)$. By applying the elementary arguments of the entire function, we know that the difference between $\varphi_{\sigma 0}(u)$ and $N(u,w,\tau)$ is a scalar factor, which does not depend on the spectral parameter u. The factor is obtained by comparing $\varphi_{\sigma 0}(0)$ with $N(0,w,\tau)$. The result is 1. Now, (16) follows immediately.

Proposition 4. The R matrix satisfies the crossing unitary symmetry

$$\left[R_{12}^{h_1}(u)R_{12}^{h_2}(-u-nw)\right]^{h_1} = M(u,w,\tau)\mathrm{id}, \tag{35}$$

where,

$$M(u,w,\tau) = n^2\exp\{i\pi nw\}\frac{\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(u,\tau)\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(-u-nw,\tau)}{\theta^2\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(w,\tau)}. \tag{36}$$

The proof of proposition 4 is very similar to that of proposition 3. We shall omit some analogous to above but rather long calculation.

Proof. Recalling the definition and some algebra properties of the R matrix, we can write the left hand side of (35) as

$$r.h.s.\ \text{of}\ (35) = \sum_{\gamma}\sum_{\beta} W_{\beta+\gamma}(u)W_{\beta}(-u-nw)I_{\gamma}\otimes I_{\gamma}^{h}. \tag{37}$$

Substituting (19) into (37), we find that

$$\begin{aligned} F_{\gamma}(u) &= \sum_{\beta} W_{\beta+\gamma}(u)W_{\beta}(-u-nw) \\ &= \frac{1}{n}\cdot\sum_{\beta_1,a} S^{-\beta_1-\gamma_1,a}(u)S^{-\beta_1,-a}(-u-nw)\omega^{-a\gamma_2}. \end{aligned} \tag{38}$$

The following conclusion can be obtained by applying the some formulas in Ref.[14] and performing the same procedure in the proof of proposition 3, which is

$$F_{\gamma}(u) = M(u,w,\tau)\delta_{\gamma_1,0}\delta_{\gamma_2,0}. \tag{39}$$

Inserting (39) into (37), we get (35).

A direct corollary of proposition 4 is read as

$$R_{12}^{h_1}(u)R_{12}^{h_2}(-u-nw) = M(u,w,\tau)\mathrm{id}. \tag{40}$$

4. Remarks.

Sklyanin's method [12] and its generalization can be used not only to solve the exactly solvable models on a finite interval with independent boundary conditions on each end, but also to construct the statistical systems with the quantum group symmetry. It plays an important role in Sklyanin's formalism that the solution R of QYBE satisfies the symmetric properties. Sklyanin assumes

$$\begin{aligned} &P_{12}R_{12}(u)P_{12} = R_{12}(u), \\ &R_{12}^{t_1}(u) = R_{12}^{t_2}(u), \\ &R_{12}(u)R_{12}(-u) = \rho(u)\mathrm{id}, \\ &R_{12}^{t_1}(u)R_{12}^{t_2}(u-2\eta) = \tilde{\rho}(u)\mathrm{id}, \end{aligned} \tag{41}$$

where t_i denotes transposition in the ith vector space. The $\rho(u)$ and $\tilde{\rho}(u)$ are some scalar functions. Mezincescu and Nepomechie [13] extend Sklyanin's formalism to the case of a 'non-symmetric' R matrix, which satisfies the less restrictive conditions

$$\begin{aligned} &P_{12}R_{12}(u)P_{12} = R_{12}^{t_1t_2}(u), \\ &R_{12}(u) = \overset{1}{V}\, R_{12}^{t_2}(-u-\eta)\, \overset{1}{V}^{-1}, \\ &R_{12}(u)R_{12}^{t_1t_2}(-u) = \rho(u)\mathrm{id}, \\ &R_{12}^{t_1}(u)\, \overset{1}{M}\, R_{12}^{t_2}(-u-2\eta)\, \overset{1}{M}^{-1} = \tilde{\rho}(u+\eta)\mathrm{id}, \end{aligned} \tag{42}$$

here $\overset{1}{V} = V\otimes 1$ and the forth relation can be derived from the second and third relations. Comparing the propositions shown by us with (41) and (42), we can see that the Z_n symmetric elliptic solution of QYBE does not satisfy (41) and (42). Hence, the symmetries (15), (16) and (35) can be regarded as the starting point to construct the exactly solvable Belavin's model with nontrivial boundary terms to solve it by means of the quantum inverse scattering method. It is emphasized

that for the case of the trigonometric limit the Hamiltonian of this system should have the quantum group symmetry of $U_q(A_{n-1})$. The detail is reported elsewhere [17].

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