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Mean field theory of anyon superconductivity

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Abstract

In a mean field scheme, we studied an anyon system in a boson background representation and found that anyon fluid is a superfluid but it has two branch of spectrum, one is a linear spectrum and another has a finite gap which derives from topological term. As we turn on external electromagnetic field, anyon fluid becomes Hall fluid which is incompressible. If we consider the condensation of anyons, anyon fluid goes into a superconducting phase which has two finite gaps. Finally, we find that for different background representations of the anyon system we get different results, in fermion background representation, as we add external electromagnetic field, anyon superfluid directly goes into a superconducting phase, as statistical parameter $\theta' = 1$, this superconducting phase is gapless.

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Anyon superconductivity has attracted considerable interest recently in connection with the cuprate high-T_c superconductivity. In a random- phase-approximation(RPA), one generally thinks that anyon superconductivity is gapless, it has a linear spectrum in the long-wavelength limit[1,2]. In Ref.3, Wen, and Zee develop long distance theories of the Hall fluid and of the anyon fluid, they show that anyon fluid is a superfluid and has not gap, but, if we add a constant external magnetic field B, anyon superfluid will become the Hall fluid which is gapful. This means that the external field maybe makes the system have gaps under some conditions.

The recent experimental data[4,5,6] show the cuprate superconductors are gapful, which motivates us to study anyon superconductivity because anyons can only exist in the two-dimensional materials such as the C_uO_2 planes of the cuprate superconductors. Wen, Wilczek and Zee[7] have studied the antiferromagnetic Heissenberg model on the square lattice with the nearest neighbor and the next nearest neighbor couplings, and indicated the possible existence of the chiral spin state which spontaneously breaks the time inversal invariance under some range of the coupling parameters. They explicitly showed text particles in chiral spin state obey fractional statistics. Therefore, we think anyon superconductivity should be closely related to the cuprate high-T_c superconductivity.

Here we use a mean field method to study anyon superconductivity in a boson background representation of an anyon system. We find that anyon

fluid is a superfluid, but it has two branch of spectrum, one is a linear spectrum and another has a finite gap which derives from topological term. As one turns on external electromagnetic fileds A_μ, anyon fluid becomes an incompressible fluid with the finite gap. If one Bonsiders the condensation of anyons, anyon fluid goes into a superconducting phase which is gapful. Finally, we find that in different background representations of the anyon system we get different results. Therefore, we think that it is inequivalent for an anyon system whether it is in a boson background representation or in a fermion background representation.

We start from the following simple Lagrangian of an anyon system to study its some general properties

$$\mathbf{L} = \phi^*(p_0 + a_0 + A_0)\phi - \frac{1}{2m}\phi^*(\mathbf{p} + \mathbf{a} + \mathbf{A})^2\phi + \mu\phi^*\phi + \frac{\theta}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \quad (1)$$

where ϕ is a charged boson field, a_μ are statistical gauge fields, A_μ external electromagnetic fields, μ is chemical potential. If we change the statistical parameter θ into $\theta \pm 1$, at meanwhile consider ϕ as a charged fermion field, we think these two system are inequivalent, their different will change the whole property of the anyon system(see below).

In order to calculate the current-current correlation function, first we should get the effective Lagrangian of equation (1) by integrating out the boson field. To this end, we make the following approximation that we properly choose chemical potential $\mu = \mu_0 + \mu'$, $-\mu_0 = m$(static energy),

and think the mass m of the boson is very large, we can take the boson as relativistic particle, the Lagrangian in (1) can be written as (here omiting the chemical μ' term)

$$\mathbf{L} = \phi^*[(p_\mu + a_\mu + A_\mu)^2 - m^2]\phi + \frac{\theta}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \quad (2)$$

we consider this approximation is reasonable for large m and is compatible with following results. We take the Lagrangian as the form in (2) so that we can easily integrate out the boson field. After simple calculating and utilizing the large m limit, we get the following effective Lagrangian of the anyon system

$$\mathbf{L}_{\text{eff.}}[a, A] = -\frac{1}{24\pi m}f_{\mu\nu}(a + A)f_{\mu\nu}(a + A) + \frac{\theta}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \quad (3)$$

where $f_{\mu\nu}(a + A) = \partial_\mu(a_\nu + A_\nu) - \partial_\nu(a_\mu + A_\mu)$. After integrating out boson field, the statistical gauge fields become dynamical variables. One easily see that as $m \to \infty$, then $\phi = 0$. the Lagrangian in (2) and (3) are same.

If we turn off external electromagnetic field, the generating functional of the anyon system reads

$$\mathbf{Z}[J] = \int Da_\mu e^{i\int d^3x[\mathbf{L}_{\text{eff.}}[A=0]+a_\mu J_\mu]} \quad (4)$$

where J_μ is a parameter. By simple calculating and choosing $k_\mu = (\omega, k, 0)$, we get the following correlation function

$$\begin{aligned} < a_1(k)a_1(-k) > &= -i\frac{1}{4a}\cdot\frac{1}{\omega^2-k^2} \\ < a_2(k)a_2(-k) > &= -i\frac{8\pi^2 a}{[6\pi^2a^2(\omega^2-k^2)+\theta^2} \end{aligned} \quad (5)$$

where $a = \frac{1}{12\pi m}$. They give two branch spectrum, one is a linear spectrum and another has a finite gap which derives from the topological term in Lagrangian. Therefore, anyon fluid is a superfluid, this is agreed with the result in Ref.3.

As external electromagnetic field is applied to this anyon system, the generating functional reads

$$\begin{aligned} \mathbf{Z}[A] &= \int Da_\mu e^{i\int d^3x \mathbf{L}_{\text{eff}}[a,A]} \\ &= e^{i\mathbf{S}_{\text{eff}}[A]} \cdot \int Da_\mu e^{-i\int d^3x[a_0\Delta_{00}a_0 + a_i G_{ij} a_j]} \end{aligned} \tag{6}$$

$$\begin{aligned} \mathbf{S}_{\text{eff}}[A] = -\int d^3x[&-\tfrac{1}{4}A_0\tilde{F}_{0i}G^{-1}_{ij}F_{0j}A_0 \\ &-\tfrac{1}{2}A_0F_{0i}G^{-1}_{ij}E_{jl}A_l + A_i[\tfrac{1}{2}k_{il} \\ &-\tfrac{1}{4}(k_{0i}\,\Delta^{-1}_{00}\,k_{0l} - \tilde{E}_{ij}G^{-1}_{jk}E_{kl})]A_l] \end{aligned} \tag{7}$$

where $G_{ij} = \Delta_{ij} - \frac{1}{4}\tilde{\Delta}_{0i}\,\Delta^{-1}_{00}\,\Delta_{oj}$, $F_{0i} = -\frac{\theta}{2\pi}\epsilon_{ij}\partial_j$, $E_{ij} = k_{ij} - \frac{1}{2}\tilde{\Delta}_{0i}\,\Delta^{-1}_{00}\,k_{0j}$, and

$$\begin{aligned} \Delta_{00} = a\partial_i^2, \Delta_{0i} &= 2a\partial_0\partial_i + \tfrac{\theta}{2\pi}\epsilon_{ij}\partial_j \\ \Delta_{ij} &= a(\partial_i\partial_j - \bar{\Box}\delta_{ij}) + \tfrac{\theta}{4\pi}\epsilon_{ij}\partial_0 \\ k_{0i} = 2a\partial_0\partial_i, k_{ij} &= 2a(\partial_i\partial_j - \bar{\Box}\delta_{ij}) \end{aligned} \tag{8}$$

where the "$\sim$" over the operator indicates its conjugation. Utilizing equations (6-8), we can easily get the current-current correlation function

$$< j_\mu(x)j_\nu(y) >= (-i)^2\frac{1}{\mathbf{Z}[A]}\frac{\delta^2\mathbf{Z}[A]}{\delta A_\mu(x)\delta A_\nu(y)} \tag{9}$$

In phase space (choosing $k_\mu = (\omega, k, 0)$), it is simplified into

$$\begin{aligned} < j_0(k)j_0(-k) > &= -i\tfrac{2a\theta^2k^2}{16\pi^2a^2(\omega^2-k^2)+\theta^2} \\ < j_1(k)j_1(-k) > &= 0 + O(\omega^2, k^2) \\ < j_2(k)j_2(-k) > &= i\tfrac{2a\theta^2(\omega^2-k^2)}{16\pi^2a^2(\omega^2-k^2)+\theta^2} + O(\omega^2, k^2) \\ < j_1(k)j_2(-k) > &= 0 \\ < j_2(k)j_1(-k) > &= -\tfrac{\theta}{2\pi}\omega + O(\omega^2, k^2) \end{aligned} \tag{10}$$

We see that in existing external electromagnetic field, anyon fluid has only one spectrum with a finite gap which derives from the topological term, then anyon fluid becomes the Hall fluid which is incompressible. It is very natural that a anyon system will describe the quantum Hall effect as external electromagnetic field is applied. But the anyon system described by the Lagrangian in (3) cannot be in a superconducting phase, the current-current correlation functions do not give the Meissner term in (10), it is only in a Hall phase. In order to gain anyon superconductivity, we must consider the boson condensation. This can be easily done by writing the boson field as $\phi = \sqrt{\rho} + \phi'$ in equation (1) and omiting the linear terms of ϕ' and redefining statistical gauge fields $\mathbf{a} = \mathbf{a^0} + \mathbf{a}'$, $\frac{\theta}{2\pi}\epsilon_{ij}\partial_i a^0_j = -\rho$, now the Lagrangian is (omiting the prime of a_μ)

$$\begin{aligned} \mathbf{L}_{\text{eff}}[a, A] = &-\tfrac{1}{24\pi m}f_{\mu\nu}(a+A)f_{\mu\nu}(a+A) \\ &-\tfrac{\rho}{2m}(a_i + A_i)^2 + \tfrac{\theta}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \end{aligned} \tag{11}$$

Utilizing the same method as above, we can get the following current-current correlation functions(choosing $k_\mu = (\omega, k, 0)$)

$$\begin{aligned} < j_0(k)j_0(-k) > &= -i\tfrac{2am\theta^2k^2}{16\pi^2ma^2(\omega^2-k^2)+m\theta^2+8\pi^2a\rho} \\ < j_1(k)j_1(-k) > &= -i\tfrac{4a\rho(\omega^2-k^2)}{4am(\omega^2-k^2)+\rho} + O(\omega^2, k^2) \\ < j_2(k)j_2(-k) > &= i[(\tfrac{\rho}{m} - \tfrac{8\pi^2a\rho^2}{8\pi^2am\rho+m^2\theta^2}) \\ +(\tfrac{8\pi^2a^2\rho}{8\pi^2a\rho+m\theta^2} &- 2a)\tfrac{8\pi^2a\rho(\omega^2-k^2)}{16\pi^2ma^2(\omega^2-k^2)+m\theta^2+8\pi^2a\rho} + O(\omega^2, k^2) \\ < j_1(k)j_2(-k) > &= 0 + O(\omega^2, k^2) \\ < j_2(k)j_1(-k) > &= -\tfrac{\theta}{2\pi}\omega + O(\omega^2, k^2) \end{aligned} \tag{12}$$

The first term in $< j_2(k)j_2(-k) >$ is the Meissner term which indicates the anyon system being in a superconducting phase, then we see that the

Meissner term depends upon the statistical parameter θ, if $\theta = 0$, it will disappear. This anyon superconducting state has two finite gaps $\Delta_1^2 = \frac{\rho}{2am}$, $\Delta_2^2 = \frac{\rho}{2am} + \frac{\theta^2}{16\pi^2 a^2}$. If we consider the high order corrections, these two gaps will have a small change, but they do not disappear. Therefore, we think the anyon superconductivity is gapful, and so is the high-T_c superconductivity.

If we consider as the boson field ϕ in (1) becomes fermion field by changing the statistical parameter θ into $\theta \pm 1$, how do we calculate the effective Lagrangian of the anyon system? We are reminded by the following relation

$$\begin{aligned} &\int d\bar{\psi} d\psi e^{i\int d^3x \bar{\psi}[\gamma^\mu(p_\mu + a_\mu) \pm M]\psi} \\ &= e^{i\int d^3x[\pm\frac{1}{8\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda - \frac{1}{16\pi M} f_{\mu\nu}f_{\mu\nu}]} \end{aligned} \tag{13}$$

which is valid for large M. However, when a nonrelativistic fermion is considered as a relativistic one, we must consider its spin degrees. For large m limit, the Lagrangian in (1) can be approximatly written as (omiting μ')

$$\begin{aligned} \mathbf{L}' = \textstyle\sum_{i=1}^2 \bar{\psi}_i[\gamma^\mu(p_\mu + a_\mu + A_\mu) - m']\psi_i \\ + \frac{\theta+1}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \end{aligned} \tag{14}$$

where the difference of the mass m and m' is only a constant coefficient(see below), which can be seen as a microscopic inequivalence of two representations when we take ψ as a boson field and a fermion field, respectively. This can be easily understood by the equivalence of the boson and fermion in 2+1 dimensions. Its effective Lagrangian reads

$$\begin{aligned} \mathbf{L}'_{\text{eff.}} = -\frac{1}{8\pi m'} f_{\mu\nu}(a + A) f_{\mu\nu}(a + A) \\ + \frac{\theta+1}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda - \frac{1}{4\pi}\epsilon_{\mu\nu\lambda}(a_\mu + A_\mu)\partial_\nu(a_\lambda + A_\lambda) \end{aligned} \tag{15}$$

Then, if we first consider ϕ in (1) as a charged fermion field, using the same method as above, we can get the following effective Lagrangian

$$\begin{aligned} \mathbf{L}'_{\text{eff.}} = -\frac{1}{8\pi m'} f_{\mu\nu}(a + A) f_{\mu\nu}(a + A) \\ + \frac{\theta'}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda - \frac{1}{4\pi}\epsilon_{\mu\nu\lambda}(a_\mu + A_\mu)\partial_\nu(a_\lambda + A_\lambda) \end{aligned} \tag{16}$$

where the relation of the statistical parameters θ' and θ is $\theta' = \theta + 1$. We see that the Lagrangian in (15) and in (16) is same, but they are different from the Lagrangian in (3), this difference induces a different physical picture. In a fermion background representation of the anyon system, we can get the following current-current correlation function with the effective Lagrangian in (16)(choosing $k_\mu = (\omega, k, 0)$)

$$< j_2(0) j_2(0) >= i\frac{m'}{4\pi} \tag{17}$$

which means that anyon superfluid directly goes into superconducting phase when external electromagnetic field is applied. As the statistical parameter $\theta' = 1$, i.e., $\theta = 0$, this superconducting phase is gapless. However, this superconducting phase derives from the topological term $-\frac{1}{2\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda$, i.e., the interaction of external electromagnetic field A_μ with the topological current $J_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda$ in (16). If one only considers the interaction of external electromagnetic field with the topological current in the Lagrangian of an anyon system, one certainly gets the result that anyon superconductivity is gapless. If the statistical parameter $\theta' \neq 1$ in (16), one gets that anyon superconductivity is gapful.

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