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NUMBER OF FAMILIES IN A PURE FERMIONIC MODEL

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Abstract

We show that, in the framework of a purely fermionic model, the smallest gauge group which has acceptable fermion representations is SU(16). The simplest embedding of SU(5) into SU(16) implies a maximum number of three chiral families of fermions.

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Composite models in which the basic fields are only fermions[1] have always been one of the main interest areas of physics. More recently in a series of papers, massless scalar fields[2] and gauge fields[3,4] have been constructed in terms of spinor fields starting from a Lagrangian with a dimensionless spinor self-coupling. This coupling, since it is dimensionless, is necessarily nonpolynomial but by introducing two auxiliary fields it can be put into a polynomial form. Moreover, in the path integral formulation, the integration of the auxillary fields yields the original Lagrangian. The application of perturbation theory to the polynomial Lagrangian shows that one of the auxiliary fields propagates as a gauge boson provided that the dependence of the bare coupling e on the cutoff Λ is given by[4]

$$\frac{e^2}{4\pi} = \frac{48\pi}{(8C_f + C_a)\ln\Lambda^2} . \qquad (1)$$

Here C_a is the index of the adjoint representation of the gauge group and C_f is the index of the representation of the gauge group to which chiral left handed fermions belong. For Dirac fermions C_f is to be replaced by $2C_f$. In general the index C for any representation of the gauge group is defined by

$$\mathrm{tr}\, T^a T^b = C\delta^{ab} \qquad (2)$$

Here T^a are the generators in the particular representation. If the theory is to be consistent, the dependence of the bare

coupling on the cutoff has to be identified with the similar expression obtained in standard gauge theories, i.e.

$$\frac{e^2}{4\pi} = \frac{12\,\pi}{(11C_a - 2C_f)\,\ln \Lambda^2} \tag{3}$$

Hence equating the r.h.s. of equations (1) and (3) gives

$$\frac{C_f}{C_a} = \frac{43}{16} \tag{4}$$

This then is the condition which has to be satisfied if the gauge bosons are composites of fermions. Of course for chiral fermions the condition that the fermion representation should be anomaly free must also be satisfied.

In this paper we investigate the groups and their particular representations satisfying equation (4) and the anomaly free condition. It turns out that the acceptable group representations are rather large. To get a handle on the phenomenology we use the strategy adopted in supersymmetric grand unified theories[5]. Namely, when the gauge group is broken to one of the standard grand unifying groups such as SU(5)[6] or SO(10)[7] the number of standard fermion families in the left handed multiplets $(\bar{5} + 10)$ of SU(5) or 16 of SO(10) must at least be three. All other multiplets should either be real themselves or paired with their complex conjugate multiplets into a reducible real multiplet. These real multiplets can be given SU(5) or SO(10) invariant masses and the assumption[5] is that their masses are of the order of the grand unification scale $\sim 10^{14}$ GeV. The remaining

multiplets would then be massless at the grand unification scale and are to be identified with the " light" families of quarks and leptons observed today.

The smallest groups of the unitary, orthogonal or symplectic type with representations satisfying equation (4) are respectively SU(8) , SO(18) and Sp(7) . In all cases fermion representation is reducible. It turns out that no exceptional group has representations satisfying equation (4). Of these groups, the symplectic type is not acceptable since symplectic groups only have real representations and the number of chiral families obtained is zero. SO(18) has complex representations but when this group is broken to SU(5) or SO(10) the complex structure disappears and each multiplet appears together with its complex conjugate multiplet. Hence this group also yields zero number of chiral families. It turns out that of all the representations of SU(8) which satisfy equation (4) none of them can be made anomaly free hence this group is also ruled out. Here, note that since the index of a representation is the same as the complex conjugate representation, equation (4) can only tell us the total number of irreducible multiplets and antimultiplets in a given representation. Since the anomaly of the multiplet and an antimultiplet differ by a sign it is the anomaly free condition which can decide how the total number is divided among multiplets and antimultiplets. Disregarding this ambiguity SU(8) has 67 different (reducible) representations which satisfy equation (4) . However none of these can be made anomaly free by dividing the total number of multiplets and antimultiplets in all possible ways.

These considerations lead us to SU(16) as the smallest group that has representations which satisfy equation (4), are anomaly free, and will give chiral fermions when broken down to SO(10) or SU(5) . It is amusing to note that SU(16) has been advocated as a grand unifying group based on entirely different considerations[8]. It turns out that there are 39 representations of SU(16) which satisfy equation (4) and which are anomaly free. 11 of these 39 representations are real. They will yield no chiral fermions at the grand unification scale and thus predict zero families . Thus we are left with 28 representations listed in table 1. We denote the vector representation by V , the two index antisymmetric representation by A , the two index symmetric representation by S and the adjoint (regular) representation by R . The indices and the anomalies of these representations are listed in table 2. For SU(16) $C_a = 32$, hence equation (4) implies that C_f has to be 86 . Using table 2 , this and the anomaly free condition imply that the entries in table 1 satisfy

$$\begin{aligned} V + \bar{V} + 14(A + \bar{A}) + 18(S + \bar{S}) + 32\,R &= 86 \\ V - \bar{V} + 12(A - \bar{A}) + 20(S - \bar{S}) &= 0\ . \end{aligned} \qquad (5)$$

Since SU(16) is rather a large group compared to SU(5) and SO(10) , there are several ways of embedding these groups into SU(16). We now turn to discuss these embeddings and the resulting implications for the number of fermion families.

a. $SU(16) \longrightarrow SU(5)\times[U(1)]^{11} \longrightarrow SU(5)$

This is the simplest embedding of SU(5) into SU(16) . The intermediate stage is shown just as a bookkeeping device to show how SU(5) is embedded into SU(16) . This embedding would result if for example the symmetry breaking occured in eleven steps of $SU(n) \longrightarrow SU(n-1)$. At each stage a vector goes into a vector plus a singlet, an antisymmetric tensor goes into an antisymmetric tensor plus a vector, a symmetric tensor goes into a symmetric tensor plus a vector and a singlet. Hence, disregarding singlets, one has

$$\begin{aligned} SU(16) &\longrightarrow SU(5) \\ V &\longrightarrow V \\ A &\longrightarrow A + 11\,V \\ S &\longrightarrow S + 11\,V \end{aligned} \qquad (6)$$

Since no light fermions belonging to the symmetric representation of SU(5) are observed one must choose solutions satisfying $S = \bar{S}$ in table 1. These are the solutions 1-9 . Number of chiral families $(\bar{V} + A) = (\bar{5} + 10)$ of SU(5) which are left after all real representations and pairs of complex conjugate representations are eliminated is just given by the number $A - \bar{A}$. Hence there is one family for each solution 1-5 , two families for each solution 6-8 and three families for solution 9. Since there are already three families observed in nature we are uniquely led to solution 9 as the only acceptable one.

b. Here we would like to consider the breaking $SU(16) \longrightarrow SO(10)$ such that the sixteen dimensional vector representation of $SU(16)$ goes into the sixteen dimensional spinor representation of $SO(10)$. Under this kind of breaking

$$\begin{aligned} SU(16) &\longrightarrow SO(10) \\ V &\longrightarrow 16 \\ A &\longrightarrow 120 \\ S &\longrightarrow 10 + 126 \end{aligned} \tag{7}$$

Here the only complex representations of $SO(10)$ are the 16 and the 126. Since there are no observed fermion families which belong to the 126, again we must seek solutions with $S = \bar{S}$ in table 1. An inspection of the table shows that there are twelve families for solutions 1-5, twenty-four families for solutions 6-8 and thirty-six families for solution 9. The number of families implied for this kind of symmetry breaking is rather large and perhaps unacceptable. We would like to note that this kind of symmetry breaking is impossible in the Higgs scheme since there is no nontrivial representation of $SU(16)$ which yields a singlet of $SO(10)$.

c. $SU(16) \longrightarrow SU(5) \times SU(5) \times SU(5) \longrightarrow SU_D(5)$

Here, the final $SU(5)$ is the diagonal subgroup of the three $SU(5)$ factors in the previous stage. Again disregarding singlets, the representations break in the following pattern

$$\begin{aligned} SU(16) &\longrightarrow SU(5) \times SU(5) \times SU(5) && \longrightarrow SU_D(5) \\ V &\longrightarrow (V,1,1)+\text{permutations} && \longrightarrow 3V \\ A &\longrightarrow (A,1,1)+(1,V,V)+(V,1,1)+\text{perm.} && \longrightarrow 3S+6A+3V \\ S &\longrightarrow (S,1,1)+(1,V,V)+(V,1,1)+\text{perm.} && \longrightarrow 6S+3A+3V \end{aligned} \tag{8}$$

Hence to cancel S in the $SU(5)$ stage we must pick solutions satisfying $A - \bar{A} + 2(S - \bar{S}) = 0$ in table 1. These are the solutions 10-13 and each of them imply 9 families of $(\bar{5} + 10)$ of $SU(5)$.

d. $SU(16) \longrightarrow SU(8) \times SU(8) \longrightarrow SU_D(8) \longrightarrow SU(5)$ or

$SU(16) \longrightarrow SU(10) \longrightarrow SU(5) \times SU(5) \longrightarrow SU_D(5)$

A detailed analysis shows that these two symmetry breaking patterns imply the same solution and the same number of families. Note that in both cases the final $SU(5)$ is embedded as a diagonal subgroup of $SU(n) \times SU(n)$ which is embedded into $SU(16)$. Here we will only give the argument for the second case.

$$\begin{aligned} SU(16) &\longrightarrow SU(10) \longrightarrow SU(5) \times SU(5) && \longrightarrow SU_D(5) \\ V &\longrightarrow V \longrightarrow (V,1)+(1,V) && \longrightarrow 2V \\ A &\longrightarrow A + 6V \longrightarrow (A,1)+(1,A)+(V,V)+6(V,1)+6(1,V) && \longrightarrow S+3A+12V \\ S &\longrightarrow S + 6V \longrightarrow (S,1)+(1,S)+(V,V)+6(V,1)+6(1,V) && \longrightarrow 3S+A+12V \end{aligned} \tag{9}$$

Hence we must pick solutions satisfying $A - \bar{A} + 3(S - \bar{S}) = 0$ in table 1. There is only one solution satisfying this condition. This is solution 14 and after cancelling resulting representations with complex conjugate representations precisely eight families

of $(\bar{5} + 10)$ of SU(5) are left.

e. One can also pick more exotic embeddings of SU(5) into SU(16). One case is given by $SU(16) \longrightarrow SU(10) \longrightarrow SU(5)$ where in the last stage the symmetry breaking is such that the ten dimensional vector representation of SU(10) goes into the ten dimensional antisymmetric representation of SU(5). Another case is $SU(16) \longrightarrow SU(15) \longrightarrow SU(5)$ where in the last stage the symmetry breaking is such that the fifteen dimensional vector representation of SU(15) goes into the fifteen dimensional symmetric representation of SU(5). An analysis shows that there are no solutions compatible with our assumptions in these cases.

We have found solutions containing 1,2,3,8,9,12,24,36 families. The solutions with eight or more families follow from rather exotic types of symmetry breaking patterns, whereas the simplest pattern implies a unique solution with three families. A fundamental assumption in reaching this conclusion is that the non-chiral fermions obtain masses at the grand unification scale. Although this assumption is reasonable , the model under consideration cannot explain why this is so. However once this assumption is made the conclusion for the number of families is inevitable. Interestingly enough there do not seem to be any solutions which imply any number of families between three and eight. Hence if a fourth family is experimentally discovered then this model implies that there are at least eight families.

Based on our findings we are unable to assert that this

model is perfect with regard to its implications about the number of families. However, the fact that the model contains only fermions as elementary fields and that it has something to say about the number of quark-lepton families is both interesting and appealing.

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	V	$\bar{V}$	A	$\bar{A}$	S	$\bar{S}$	R
1	0	12	2	1	0	0	1
2	2	14	3	2	0	0	0
3	12	24	1	0	1	1	0
4	14	26	1	0	0	0	1
5	30	42	1	0	0	0	0
6	1	25	2	0	0	0	1
7	3	27	3	1	0	0	0
8	17	41	2	0	0	0	0
9	4	40	3	0	0	0	0
10	0	4	2	0	1	2	0
11	2	6	2	0	0	1	1
12	4	8	3	1	0	1	0
13	18	22	2	0	0	1	0
14	5	21	3	0	0	1	0
15	2	6	0	3	2	0	0
16	5	13	0	1	2	1	0
17	7	15	0	1	1	0	1
18	3	19	0	2	2	0	0
19	9	17	1	2	1	0	0
20	6	26	0	0	2	1	0
21	8	28	0	0	1	0	1
22	4	32	0	1	2	0	0
23	10	30	1	1	1	0	0
24	16	28	2	1	0	0	0
25	5	45	0	0	2	0	0
26	11	43	1	0	1	0	0
27	23	31	0	1	1	0	0
28	24	44	0	0	1	0	0

← Table 1
Fermion representations of SU(16) satisfying equation (4) and the anomaly free condition. $V, \bar{V}, A, \bar{A}, S, \bar{S}$ and R denote the number of irreducible representations of each kind occuring in the representation.

	V	A	S	R
C	1	14	18	32
A	1	12	20	0

Table 2
Indices (C) and anomalies (A) of low dimensional representations of SU(16).

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