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PARTONLIKE BEHAVIOUR IN A PURE FERMIONIC MODEL*

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Abstract: We calculate the electron-positron annihilation cross-section and the electromagnetic quark structure function in a pure fermionic model with composite gluons. We find that the parton model result is not modified.

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In two recent publications [1,2], we showed, with our collaborators, the renormalizability of two pure spinor models in four dimensions, albeit in an unorthodox fashion. Both of these models are asymptotically free ; the coupling constants go to zero as the cut-off, Λ , goes to infinity. In these papers we remarked that a simple $\psi\psi \to \psi\psi$ process goes as $(\ln \Lambda)^{-1}$, if there is any interaction between the spinors. Any production process, for example $\psi\psi \to \psi\psi\bar{\psi}\psi$ goes to zero even faster, with higher powers of $(\ln \Lambda)^{-1}$. It may be of interest to check if this behaviour is typical in any process in which our quarks (the fundamental spinors of our models) take part. Here we calculate the electron-positron annihilation process and the quark-electron structure functions using our model for the strong part of the interaction. There are infinitely many diagrams that in principle give a finite contribution to these processes even when the coupling constant is set to be

$$g^2 = \frac{4\pi^2}{\ln \Lambda} \tag{1}$$

as is required by the renormalizability condition of the model. We want to see if these contributions sum up to give a finite correction to the free parton result.

Since we already fix the coupling constant in the renormalization process, eq. (1) , we can not use it as an expansion parameter. Therefore, we employ a 1/N type expansion[3]. The expansion parameter n = 1/N , is the ratio of the number of colors to the number of flavors. Since experimental results indicate that the number of flavors exceed the number of colors, n , our expansion parameter, is less than one.

Here the calculations are performed in lowest orders in "n" , but we think that the behaviour is characteristic of higher orders too. We find that due to delicate cancellitions between various terms, the parton model result persists since the highest possible degree of divergence of the diagrams is not actually present.

Our starting point is the effective Lagrangian[2],

$$\mathcal{L}_{eff} = \bar{\psi}_{ai}\left((i\not{\partial} - m)\delta_{ij} + \frac{g}{\sqrt{L}}\not{G}_{ij}\right)\psi_{bj}\,\delta_{ab} \tag{2}$$

Here i,j are the color and a,b are the flavor indices. The vector field G^{μ}_{ij} is flavor blind. We have L flavors and N colors. Integration of the spinor fields gives

$$\mathcal{L}_{eff} = L\,\mathrm{Tr}\ln\left[\delta_{ij}(i\not{\partial} - m) + \frac{g}{\sqrt{L}}\not{G}_{ij}\right] \tag{3}$$

which yields for

$$g^2 = \frac{4\pi^2}{\ln \Lambda} \tag{1}$$

a massless vector field propagator for the gluon field G^{ij}_{μ} .

One can couple the electromagnetic interaction to the quark field in the usual way. The functional formalism gives[4]

$$\int e^{iqx}\langle 0|J_\mu(x)J_\nu(0)|0\rangle \propto \Big\{-L^2\,\mathrm{Tr}\,(\Delta^{-1ab}_{ij}\gamma_\nu Q)\,\mathrm{Tr}\,(\Delta^{-1ab}_{ij}\gamma_\mu Q) + iL\,\mathrm{Tr}(\Delta^{-1ab}_{ij}\gamma_\nu Q\,\Delta^{-1ab}_{ji}\gamma_\mu Q)\Big\} \tag{4}$$

Here Q is the electromagnetic charge, and

$$\Delta^{ab}_{ij} = \left[\delta_{ij}(i\not{\partial} - m) + \frac{g}{\sqrt{L}}G_{ij}\right]\delta_{ab} \tag{5}$$

One can easily see that in order L·N, only the diagram given in Fig.(1) contributes. This is the well known parton model result. To order L·N·(N/L) , diagrams in Fig. 2 and to order $L\cdot N\cdot(N^2/L^2)$ the diagrams in Fig. 3 contribute. These consist of only a subset of the diagrams that contribute to the similar calculation in QCD up to fourth order[5,6,7]. The rest of the diagrams either do not exist for our model, or they are down by factors of 1/N*. We have to cut these diagrams in all possible ways and look for the parts that are finite after multiplying

by the appropriate factors of "g", i.e. with g^2 and g^4 for diagrams in Fig. (2) and Fig. (3) respectively. At this point to evaluate the diagrams one can simply use the Dine and Sapirstein[5] results. They tabulate the contribution of each diagram with their divergences. For our purposes, only the highestdivergences, $1/\varepsilon^3$ for diagrams in Fig. 3,count. Cutting the diagram reduces its divergence at least by one order. If one only cuts such that the resulting divergence is of order $1/\varepsilon^2$, then one can cut Fig. 3a in three, Fig. 3b in four, Fig. 3c in two, Fig. 3d in three, Fig. 3e in three, Fig. 3f and Fig. 3g in two ways. Using the values of Table I of ref. 5 we see that the contributions addup to zero. The diagrams in Fig. 2 also add up to zero. This can be seen by using the same table of Ref. 5 and adding the contributions of the diagrams labelled by K and L in Fig. 1 b of Dine and Sapirstein. As a result we find that there are no corrections to the parton model result that are finite as the cut-off Λ goes to infinity in tis order. One can easily see that all our arguments also hold for the model described in ref. 1 , the scalar counterpart of the model discussed here, with exactly the same result.

We now turn to the calculation of the electron-quark structure function. To this effect one has to differentiate the partition function

$$\Gamma = \frac{1}{Z} \int [\,]\, e^{i S_{eff}} \tag{6}$$

with respect to quark and electromagnetic current sources twice. Here S_{eff} is the action of our effective lagrangian, eq. (2). Then we have to amputate the external legs and take the absorptive part by using the Cutkosky cutting rule[8].

One can write

$$\left.\frac{\partial^4 \Gamma}{\partial G_\mu(x)\, \partial G_\nu(u)\, \partial \eta(z)\, \partial \bar{\eta}(y)}\right|_{B=0} = \frac{1}{Z} \int [\,]\, e^{i S_{eff}}\, E \tag{7}$$

B is the generic notation for all the sources, G_μ is the electromagnetic current, $\bar{\eta}$ is the quark source. E is given by

$$\begin{aligned} E = &- L^2\, \mathrm{Tr}_u (\Delta^{-1}\gamma_\nu Q)\, \mathrm{Tr}_x (\Delta^{-1}\gamma_\mu Q)\; {}_y(\Delta^{-1})_z \\ &+ iL\, \mathrm{Tr}_x (\Delta^{-1}\gamma_\nu Q \Delta^{-1}\gamma_\mu Q)\; {}_y\Delta^{-1}_z + iL\, (\mathrm{Tr}_x \Delta^{-1}\gamma_\mu Q)\; {}_y(\Delta^{-1}\gamma_\nu Q \Delta^{-1})_z \\ &+ iL\, (\mathrm{Tr}_u (\Delta^{-1}\gamma_\nu Q)\; {}_y(\Delta^{-1}\gamma_\mu Q \Delta^{-1})_z + 2\; {}_y(\Delta^{-1}\gamma_\nu Q \Delta^{-1}\gamma_\mu Q \Delta^{-1})_z \end{aligned} \tag{8}$$

Since the $\gamma \to 2G$ process is ruled out due to charge conjugation and $\gamma \to 3G$ process is down by $(\ln \Lambda)^{-1}$ one only gets contributions from

$$E' = {}_y(\Delta^{-1}\gamma_\nu Q \Delta^{-1}\gamma_\mu Q \Delta^{-1})_z \tag{9}$$

In lowest order we get the diagram given by Fig. 4 . One order higher one gets those in Fig. 5 . A simple calculation shows that Fig. 5a and Fig. 5b give the negative of the result of Fig. 5c. Since in highest order Fig. 5c can be cut twice, whereas the other two diagrams can be cut only once, we get zero for this order. We can give examples of the sort of diagrams one gets for higher orders. For instance to N^2/L^2 order, those in Fig. 6 cancel those in Fig. 7, and diagrams of Fig. 8 cancel those in Fig. 9. One can find pairs for all the possible diagrams at this order. So the cancellation is complete at this order. The end result is exactly that of the parton model. The dynamical part of the structure function is $\delta\,(1-x)$, where $x = -q^2/2kq$, k quark, q off-shell photon momentum. There is not even a multiplicative renormalization of the usual factor multiplying this result. This is in contrast to the behaviour in two dimensional models, where the parton model result is modified, and a QCD like expression is obtained[4].

We have found that in these two models[1,2] there are no corrections to the parton model result for these two processes, in the orders we have calculated. We would like to note that this result is by no means tritrivial. Although the individual Feynman diagrams we have considered are the same as in QCD, our handling of the perturbation series is very different. From a pure mathematical point of view our result has to do

with the cancellation of the highest divergences. If these did not cancel among themselves in QCD, they would be cancelled by two loop counterterms. So their cancellation in QCD is an irrelevant point, whereas in our model it leads to the parton model result. An interesting question is whether this behaviour persists in higher orders and we believe that this is the case. A possible way of introducing corrections to the parton model result is keeping the cut-off Λ finite. This would result in a QCD-like result.

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Footnote

* Actually one can show that the contributions due to these diagrams cancel, so our end result do not change.

FIGURE CAPTIONS

Figure 1- Diagram contributing to the electron-positron annihilation process in order $L^{\cdot}N$. Straight lines show the fermion, dotted lines the gluon and wiggly lines the photon propagator.

Figure 2- The same process in order $L^{\cdot}N$ (L/N)

Figure 3- The same process in order $L^{\cdot}N$ (L^2/N^2)

Figure 4- The diagram contributing to the electromagnetic quark structure function in lowest order

Figure 5- The same process in order L/N

Figure 6- Diagrams contributing to the electromagnetic quark structure function in order N^3/L^3.

Figure 7, 8, 9 are the same as Figure 6.

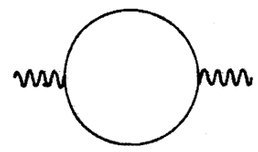


Fig. 1

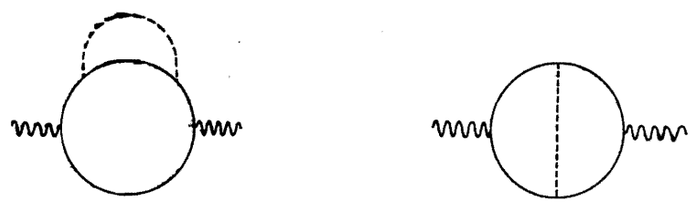


Fig. 2

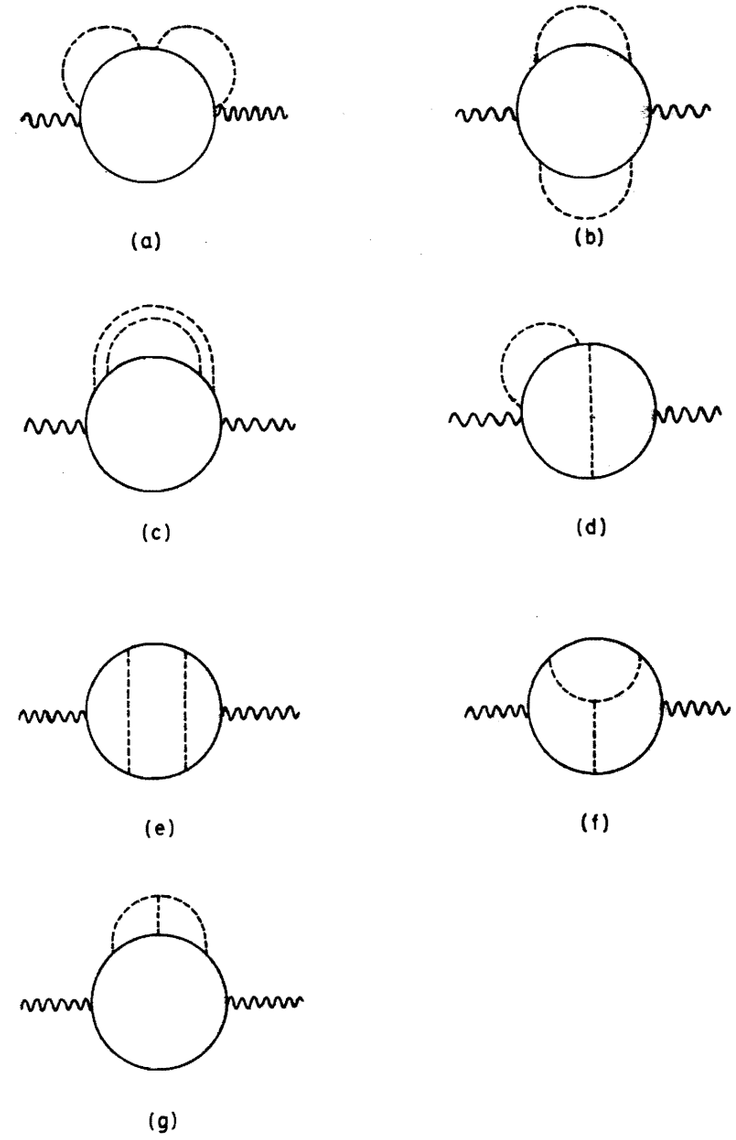


Fig. 3

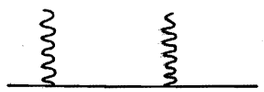
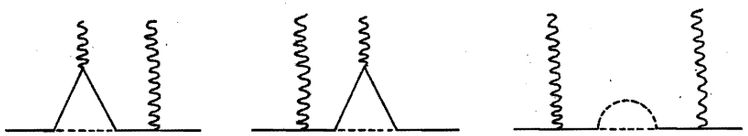


Fig. 4



(a) (b) (c)

Fig. 5

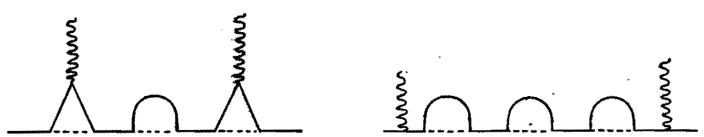


Fig. 6

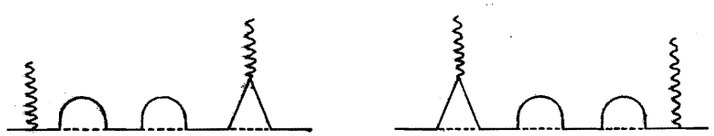


Fig. 7

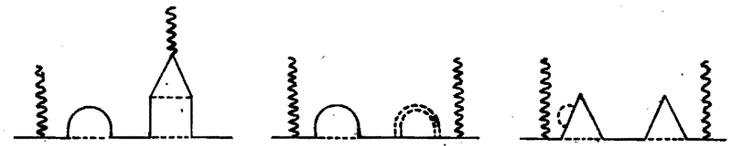
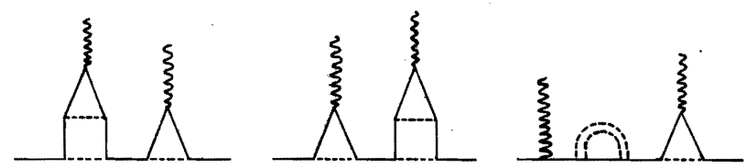


Fig. 8



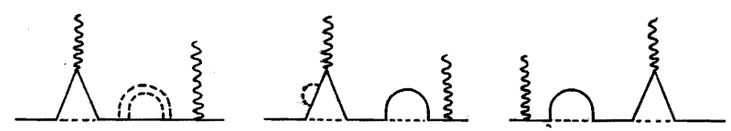


Fig. 9