14

QUANTUM MECHANICAL BLACK HOLES (II)

BSC · CAMCS - 96-09-01

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Technical Report

BSC-CAMCS 96-09-01

September, 1996

Dedicated to the memory
of my parents

QUANTUM MECHANICAL BLACK HOLES (II)

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Abstract

In this paper from four different viewpoints viz. the Bohm-De Broglie hydrodynamical formulation, Dirac's monopole theory, Zitterbewegung considerations within the Compton wavelength and Moller's centre of mass approach to relativistic systems, we arrive at a unified picture which corroborates a recent model of a charged Dirac particle as a Kerr-Newman black hole with a naked singularity shielded by the Zitterbewegung region. These considerations, amongst other things, give a rationale for why monopoles may not exist and why neutrinos are lefthanded.

1 Introduction

Our starting point is the well known hydrodynamical formulation of quantum mechanics[1, 2]. In this formulation we start with the Schrodinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \tag{1}$$

and write

$$\psi = Re^{iS} \tag{2}$$

where R and S are real functions of $\vec{r}$ and t.

Substitution of (2) in (1) and separation of the real and imaginary parts leads to,

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{v}) = 0 \tag{3}$$

$$\frac{1}{\hbar}\frac{\partial S}{\partial t} + \frac{1}{2m}(\vec{\nabla}S)^2 + \frac{V}{\hbar^2} - \frac{1}{2m}\frac{\nabla^2 R}{R} = 0 \tag{4}$$

where $\rho = R^2$and$\vec{v} = \frac{\hbar}{m}\vec{\nabla}S$

and $Q \equiv -\frac{\hbar^2}{2m}(\nabla^2 R/R)$.

Identifying (3) and (4) with the equation of fluid flow, it is well known that we get an alternative formulation of quantum mechanics, but with a hidden variable viz. the definite value of position. This is the Bohm-De Broglie theory. However the Bohm potential Q can be non local and this is one of the important reasons why the formulation has not found favour.

It is well known that equation (2) is also the starting point of Dirac's theory of magnetic monopoles[3] which will be elaborated in section 3.

We will now examine both the hydrodynamical approach and the monopole theory from a slightly different angle in the light of some recent work and come to a harmonious view point.

2 The Hydrodynamical Formulation

We begin with stationary solutions of the above formulation, viz. equation (4), in the absence of external fields. In this case it is known that the non local quantum potential

$$Q = \text{constant} = E,$$

the energy of the system. Further the velocity field is solenoidal,

$$\vec{\nabla}.\vec{v} = 0 \tag{5}$$

Given (5) it is known there is a circulation which is given by (cf. ref.[1]).

$$\Gamma = \int_c \vec{v}.\vec{dr} = \oint ds = \frac{2\pi\hbar n}{m}, n = 1, 2, ... \tag{6}$$

Let us now consider the ultra relativistic case, $|\vec{v}| = c$ for all particles of the fluid for reasons which will become clear below. This time we get from (6),

$$m\Gamma = \int_c m\vec{v}.\vec{dr} = \frac{n\hbar}{2}, n = 1, 2, ... \tag{7}$$

where, an integration over all elements ρ, is implied. Here n is the number of nodes (or, in three dimensions, the end points of nodal lines). We can immediately identify (7) with the quantum mechanical spin $\frac{n}{2}$. Interestingly there are $2 \times \frac{n}{2} + 1 = n + 1$ multiply connected regions, both in hydro-dynamics and in the theory of spin.

It is also worth noting that in (7), if the radius of the vortex is taken to be l, then l turns out to be the Compton wavelength, which thus appears as a fundamental length. This will be commented upon later.

The physical picture is as follows: A particle can be pictured as a fluid vortex which is steadily circulating along a ring (or in three dimensions, a spherical shell) with radius equal to the Compton wavelength and with velocity equal to that of light. Its total energy is given by, as seen above

$$Q = E \tag{8}$$

and its angular momentum, which in quantum theory is quantized is given by (7).

We now compare this result with the results of[4]. There the Schrodinger equation was derived from first principles, allowing non local transitions, to get (considering the one dimensional case for simplicity):

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \int \psi^*(x')\psi(x)\psi(x')U(x')dx' \tag{9}$$

The non local term in the integral in the right side of (9) was then identified with the inertial mass term with the observations that,

i) Such non locality, and apparently superluminal velocities are allowed within the Compton wavelength[5], and,
ii)This non local term is an energy term as in the case of an analogous situation of the hydrogen molecular ion.

Finally the Schrodinger like equation,

$$\imath\hbar\frac{\partial\psi}{\partial t} = [-\frac{\hbar^2}{2\bar{m}}\frac{\partial^2}{dx^2} + m_o c^2]\psi \tag{10}$$

was obtained.

The presence of the extra non local term $m_o c^2$ in (10) is easily explained [6, 7, 8]: The Schrodinger equation is really the limiting case of the Dirac equation in which process an inessential phase factor is dropped. Another way of looking at this is that the constant potential $m_o c^2$ does not affect the dynamics. That is the reason why the Schrodinger equation is not Galilean invariant, as a non relativistic theory should be, and infact exhibits the Sagnac effect, which a strictly Galilean invariant theory should not[9].

The convergence of the above formulation and the Bohm hydrodynamical formulation is evident once we restrict ourselves to the Compton wavelength and luminal velocities. The particle is now a relativisitc fluid vortex circulating along a ring of radius equal to the Compton wavelength. The Q given by (8) is the energy of this system or particle and corresponds to the inertial mass term given by the integral in (9) or equivalently the constant potential term in (10).

3 Monopoles

It is well known that there is a close connection between the hydrodynamic theory and Dirac's theory of monopoles[3]. The starting point in this latter case is precisely the decomposition of the wave function (2), but the focus is on the phase function S which need not be integral: exactly as in the case of the vortex above, there can be nodal singularities. Infact the S in this theory defines the function $\vec{K}$, exactly as it does the momentum vector of section 2. But this time $(\vec{K}, K_0)$ is identified with the electromagnetic

potential and an integral like (7) then consists of, in addition to the term $\frac{n\hbar}{2}$ the electromagnetic flux, again n being the number of nodal lines with end points inside the vortex or region of integration. Thus the well known equation of the magnetic monopole viz. $\mu = \frac{1}{2}n\hbar\frac{c}{e}$, on identifying

$$\vec{K} \equiv \frac{e}{\hbar c}\vec{H} \tag{11}$$

($\vec{H}$ being the magnetic field) with the momentum of section 2 gives back equation (7) for quantized spin.

4 Zitterbewegung and the Compton Wavelength

In the usual theory of the Dirac equation[10], it is well known that the eigen values of the velocity operator $c\vec{\alpha}$are$\pm c$, the velocity of light while the position operator is non Hermitian: It consists of a real part which is the usual position and a rapidly oscillating (or Zitterbewegung) imaginary part,

$$x \equiv (c^2 p_1 H^{-1} t) + \frac{i}{2} c\hbar(\alpha_1 - c p_1 H^{-1}) H^{-1} \tag{12}$$

Both these puzzling facts are reconciled by the fact that our measurements are really averaged over the space intervals of the order of $\hbar/mc$, the Compton wavelength and time intervals of the order $\hbar/mc^2$. In this case the imaginary part in (12) disappears (cf. ref.[10]). Hermiticity and Physics begins after such an averaging necessitated by our gross measurements.

One could say that (12) applies in a non local region bounded by the Compton wavelength as we saw in section 2. Within the region, we have to contend with unphysical phenomena like superluminal velocities and negative energies and in general non Hermitian operators. Outside the Compton wavelength, that is on averaging over space time intervals of this order, we are back in usual Physics.

Let us for simplicity consider the free particle Dirac equation. The solutions are of the type,

$$\psi = \psi_A + \psi_S \tag{13}$$

where

$$\psi_A = e^{\frac{i}{\hbar}Et}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \text{ or } e^{\frac{i}{\hbar}Et}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \text{ and}$$

$$\psi_S = e^{-\frac{i}{\hbar}Et}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \text{ or } e^{-\frac{i}{\hbar}Et}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \tag{14}$$

denote respectively the negative energy and positive energy solutions. From (13) the probability of finding the particle in a small volume about a given point is given by

$$|\psi_A + \psi_S|^2 = |\psi_A|^2 + |\psi_S|^2 + (\psi_A\psi_S^* + \psi_S\psi_A^*) \tag{15}$$

Equations (14) and (15) show that the negative energy and positive energy solutions form a coherent Hilbert space and so the possibility of transition to negative energy states exists. This difficulty however is overcome by the Hole theory which uses the Pauli exclusion principle.

However the last term on the right side of (15) is like the Zitterbewegung term. When we remember that we really have to consider averages over space time intervals of the order of $\hbar/mc$ and $\hbar/mc^2$, this term disappears and effectively the negative energy solutions and positive energy solutions stand decoupled in what is now the physical universe.

A more precise way of looking at this is[11] that as is well known, for the homogeneous Lorentz group, $\frac{p_0}{|p_0|}$ commutes with all operators and yet it is not a multiple of the identity as one would expect according to Schur's lemma: The operator has the eigen values ± 1 corresponding to positive and negative energy solutions. This is a super selection principle pointing to the two incoherent Hilbert spaces or universes [12] now represented by states ψ_A and ψ_S which have been decoupled owing to the averaging over the Compton wavelength space- time intervals. But all this refers to energies such that our length scale is greater than the Compton wavelength. As we reach energies

corresponding to the Compton wavelength scale, negative energy solutions show up as anti particles. Thus the super selection principle which comes into play on averaging over Compton wavelength scales dispenses with the Pauli exclusion principle.

5 A Classical Viewpoint

From a classical point of view, we could say that if in the Lorentz transformation,

$$x = \gamma(x' - vt), \gamma = (1 - v^2/c^2)^{-1/2} \tag{16}$$

$v > c$ is allowed, then the coordinates become imaginary, this being true within the Compton wavelength as in (12), in the sense that non locality is allowed there. So (12) can be understood as representing a coordinate which is imaginary within the Compton wavelength but becomes the usual position coordinate outside, that is after averaging over these intervals. One way of interpreting (12) would be that from our physical point of view using (16) there is a region where v is $> c$, consisting of virtual or superluminal ghost particles bounded by a region, a sphere of radius equal to the Compton wavelength consisting of massless particlets (to distinguish them from partons-, instantons and the like, or to make a clean break, "Ganeshas") with velocity of light. Only on averaging over this vortex like sphere or region, do we come to the domain of conventional physics and the usual particles moving with sub luminal velocities. It may be remarked that the De Broglie-Bohm picture of a particle is that of an average over an ensemble (cf. ref.[1]) but the above picture is different: It is an averaging over a physically inaccessible region.

Indeed it is known that for a collection of relativistic particles, the various mass centres form a two-dimensional disc perpendicular to the angular momentum vector $\vec{L}$ and with radius (ref.[13])

$$r = \frac{L}{mc} \tag{17}$$

Further if the system has positive energies, then it must have an extension greater than r, while at distances of the order of r we begin to encounter negative energies.

If we consider the system to be a particle of spin or angular momentum $\frac{\hbar}{2}$, then equatión (17) gives, $r = \frac{\hbar}{2mc}$. That is we get back the Compton wavelength.

On the other hand it is known that (cf. ref.[14]), if a Dirac particle is represented by a Gausssian packet, then we begin to encounter negative energies precisely at the same Compton wavelength as above. Thus a particle can indeed be treated as a vortex or a spherical shell of relativistic sub constituents or particlets (or Ganeshas).

6 General Relativity

Taking the cue from the above considerations let us treat a particle as a relativistic fluid[7]. Our starting point is the linearized theory[15]

$$g_{\mu\nu} = n_{\mu\nu} + h_{\mu\nu}, h_{\mu\nu} = \int \frac{4T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' \tag{18}$$

(A bar on T has been dropped.)

In (18), velocities comparable to the velocity of light c are allowed and also, the stresses T^{jk} and momentum densities T^{oj} can be comparable to the energy momentum density T^{00}. As in ref.15, we can easily deduce that, when $|\frac{\vec{x}'}{r}| << 1$, where $r \equiv |\vec{x}|$, and in a frame with origin at the centre of mass and at rest with respect to the particle,

$$Gm = \int T^{00} d^3x \tag{19}$$

$$S_k = \int \epsilon_{klm} x^l T^{mo} d^3x \tag{20}$$

where m is the mass (or approximate mass because of the linear approximation), and S_k is the angular momentum. We next observe that,

$$T^{\mu\nu} = \rho u^\mu u^\nu \tag{21}$$

If we now work in the Compton wavelength region we have, while $u^o = 1$,

$$|u^i| = c \tag{22}$$

(This is the Quantum Mechanical input) Substitution of (21) and (22) in (20) gives on using the Mean Value Theorem,

$$S_k = c < x^l > \int \rho d^3x$$

As $< x^l > \sim \frac{\hbar}{2mc}$, using (19), we get, $S_k \approx \frac{\hbar}{2}$, as required for a spin half particle. Infact as pointed out in ref.[7] this relation becomes exact if we treat the particle as effectively a rotating shell distribution of radius $\hbar/2mc$, keeping in mind the fact that the interior region is in any case unphysical as seen in section 4, and is described by complex space-time coordinates. The gravitational potential can similarly be obtained from (18) and (19) (cf.ref.[15]).

$$\Phi = -\frac{1}{2}(g^{00} - n^{00}) = -\frac{Gm}{r} + 0(\frac{1}{r^3}) \tag{23}$$

Infact (cf.ref.[7]) the particle can be treated as a Kerr-Newman black hole and as is well known we get back the correct electromagnetic field of a Dirac electron including the anamolous gyromagnetic ratio $g = 2$ (Though there is a naked singularity, as explained in [7], this is shielded by the Compton wavelength region.)

Thus a particle could be treated as a relativistic vortex, that is a vortex where the velocity of circulation equals that of light or a spherical shell, whose constituents are again rotating with the velocity of light or as a black hole described by the Kerr-Newman metric for a spin $\frac{1}{2}$ particle.

The fact that we get the gravitational potential $\frac{m}{r}$ in equation (23) again confirms that mass comes from the Compton wavelength region.

7 Discussion

1. The equation (6) emerges on using the fact that S is defined only up to a multiple of 2π, whence we get equation (7) giving quantized spin. As pointed out from equation (7) the Compton wavelength emerges. On the other hand equation (17) shows that given the spin $\frac{\hbar}{2}$, we get the Compton wavelength. It is also to be noted that equation (20) gives the spin $\frac{\hbar}{2}$ if we

use the Compton wavelength. The Compton wavelength itself appears in quantum mechanics due to the Heisenberg uncertainity principle. So it appears that the quantum mechanical quantized spin and Compton wavelength can be obtained from classical considerations like relativistic vortices. The same conclusion is drawn from a slightly different viewpoint in [7]. In any case the remarkable universality of the Compton wavelengh was pointed out by Wigner[16] - the above considerations show why it emerges in a natural way.

2. The fact that the spin of the particle is directly connected to the number of end points of the nodal lines, as seen in Section 2 appears to indicate that Fermions are primary and that Bosons can be treated as bound states of Fermions. As pointed out in[7], quarks also could be treated as Quantum Mechanical Black Holes in the foregoing sense, and as it is known pions are indeed treated as bound states of a quark and an anti quark. (Indeed from considerations of the symmetry between leptonic and hadronic currents, leptons and hadrons appear to be the same[17].)

3. The fact that the magnetic field which arises in the monopole formulation as given by equation (11) and the quantized spin angular momentum which arises in the hydrodynamical formulation as given by equation (7) appear to be one and the same is remarkable. This is caused by the fact that the vector K and the momentum vector as given in the two formulations are really one and the same, as pointed out in Section 3. Indeed the Coriolis and other effects of rotating frames[18] bear a strong resemblence to the magnetic effects. As pointed out in[7], electromagnetism and gravitation can be unified in a general relativistic version of quantum mechanics as symbolised by the complete description of the electron in terms of the Kerr-Newman metric. This has been indicated in Section 6. Thus in this picture monopoles disappear. Indeed they have not been found todate and Dirac himself expressed his conviction that they do not exist[19].

4. The double valuedness that arises from a nodal singularity on the one hand and half integral spin on the other finds an immediate echo in the Kerr-Newman metric. This can be seen as follows.

In natural units the metric is given by (cf. ref.[15]).

$$ds^2 = -\frac{\Delta}{\rho^2}[dt' - a\sin^2\theta d\phi]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\phi - adt]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2,$$

where, a is the Compton wavelength and,

$$\Delta = r^2 - 2mr + a^2 + m^2 + e^2, \rho^2 \equiv r^2 + a^2\cos^2\theta$$

At $r = a$ and $\theta = \pi/2$, $\Delta = 2a^2$ as both e and $m << a$, and $\rho^2 = a^2$.

If further, we take $a\frac{d\phi}{dt} = \lambda$, we get,

$$ds^2 = (2\lambda^2 - 1)dt^2 + \frac{1}{2}dr^2$$

The choice $\lambda = \frac{1}{2}$ leads to,

$$ds^2 = -\frac{1}{2}dt^2 + \frac{1}{2}dr^2,$$

which is Mirkowski like, except for the scale factor $\frac{1}{\sqrt{2}}$. In the foregoing model, $a\frac{d\phi}{dt}$ = velocity of light = 1. The choice $\lambda = \frac{1}{2}$ can be understood as follows: If the azimuthal angle measured by an observer at rest far away, is ϕ', then we get back the velocity of light at $r = a$ for this observer, if $\phi' = 2\phi$, which is precisely spinorial behaviour.

In other words, special relativity for the spin $\frac{1}{2}$ electron can be seen to emerge from the Kerr-Newman metric.

5. Treating the particle as a vortex as in Section 2, arguments for a monopole in Section 3 then show that there would be the Bohm-Ahranov like effect[20] at the Compton wavelength scale.

6. It is interesting to note that the above model of a particle could explain the left handedness of the neutrino in the light of Sections 4 and 5. In the case of the neutrino, as the mass is vanishingly small, the Compton wavelength tends to infinity or turns out to be very large. On the other hand we encounter the negative energy solutions within this region. That is

we encounter negative energy neutrinoes only. The equation for a negative energy neutrino is (cf. ref.[11]).

$$(-p_o)v(p) = +\vec{\sigma}.\vec{p}v(p)$$

This is the equation for a left handed neutrino in the physical world of positive energy solutions.

7. It was pointed out in ref.[7] that a complex displacement,

$$x^\mu \rightarrow x^\mu + \imath a^\mu \tag{24}$$

leads to the electromagnetic field of an electron. Considering the time component, for example, we have, for the wave function ψ,

$$\psi(t) \rightarrow \psi(t + \imath a^\circ) = \frac{a^\circ}{\hbar}[\imath\hbar\frac{\partial}{\partial t} + \frac{\hbar}{a^\circ}]\psi(t)$$

So, as $\imath\hbar\frac{\partial}{\partial t} \equiv p^o$, the usual fourth component of the energy-momentum operator, we can identify the electrostatic potential, ϕ, from an equation like

$$e\phi = \frac{\hbar}{a^\circ} \tag{25}$$

Infact, we could identify K^μ of section 3 and the momentum vector p^μ from section 2 with $\frac{1}{a^\mu}$. If further a^i is taken to be of the order of the Compton wavelength, $\frac{h}{mc}$ and similarly a^o to be of the order of $\frac{\hbar}{mc^2}$ in the light of equations (12) and (16), we get,

$$|\vec{p}||\vec{a}| \sim mc\frac{\hbar}{mc} = \hbar,$$

which can also be obtained from the Heisenberg uncertainity principle.

It is interesting to note that quite independantly, the space components of (24) give the other components of the electromagnetic vector potential, and infact as is known, given the charge (or equation (25)), the above displacement leads us to the Kerr-Newman metric[21].

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