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Dedicated to the memory
of my parents

QUANTUM MECHANICAL BLACK HOLES

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Abstract

In this paper, Fermions are treated as Kerr-Newman type Black Holes wherein we identify the horizon at the particle's Compton wavelength periphery. A naked singularity is avoided and the singular processes inside the horizon of the Black Hole are identified with Quantum Mechanical effects within the Compton wavelength.
Inertial mass, gravitation, electromagnetism and even QCD interactions emerge from such a description including relative strengths and also other features like the anomalous gyromagnetic ratio, the discreteness of the charge and the reason why the electron's field emerges from Newman's complex transformation in General Relativity.
This model describes the most fundamental stable Fermions viz., the electrons, neutrinos and the quarks.

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1 Introduction

Quantum Mechanics works at distances much greater than the Compton wavelength of elementary particles or roughly $10^{-12}cm$[1, 2]. In the domain of Quantum Field Theory, particles are points, space-time is a continuum and special relativity holds. On the other hand, in Quantum Gravity we attempt to deal with phenomena at distances of the order of the Planck length or $10^{-33}cm$[3]. In this preliminary communication, we consider an alternative view point and deal with distances of the order $<\approx$ the Compton wave length. At this level, Quantum Mechanical phenomena like zitterbewegung and negative energies come in [4]. We show that it is possible to consider electrons, and more generally leptons and also quarks as "Quantum Mechanical Black Holes" (QMBH in what follows), wherein features of Quantum Mechanics and General Relativity are inextricably woven. At the same time, we can trace the origin of inertial mass, gravitation, electromagnetism and even QCD interactions in such a model. Section 2 sets the tone: the origin of inertial mass in purely Quantum Mechanical non-local amplitudes within the Compton wavelength is deduced. Taking the cue from here, it is shown in section 3 that an electron can be identified with a QMBH. This identification leads to the emergence of the gravitational and electromagnetic fields. In section 4, starting from General Relativistic considerations, we deduce

the results of section 3, enlarge their applicability to muons and additionally exhibit the emergence of QCD interactions. There is a discussion of some relevant matters in section 5. Section 6 states the main conclusion.

2 Inertial Mass and Quantum Mechanics

2.1 Preliminary Remarks

We first consider the question of the origin of a particle's inertial mass. Such a question has been considered in the spirit of Mach's principle in the past[5] and even recently with specified forms of macroscopic forces[6]. Even more recently [7], it has been argued from a totally different angle that inertia is a result of the interaction of a particle's constituent partons with the vacuum electromagnetic zero-point field. Yet another view point is adopted here. The origin of inertia is attributed to a particle's properties in ordinary Quantum Mechanics, rather than due to the global result of the universe as a whole or due to the medium. We first deduce both the ordinary and non local Schrodinger equations from simple considerations, invoking Quantum Mechanical axioms only, without using Newtonian mechanics. From these considerations we will show that inertial mass, Newtonian mechanics and even special relativity emerge. An interesting consequence of these considerations is that the energy of inertial mass can be recovered or destroyed under suitable conditions.

2.2 The Schrodinger Equation

In the sequel we will use the following from a larger set of well known postulates of Quantum Mechanics[8, 9]:
i) All physical observables correspond to Hermitian operators - the only measurable values of these observables being the various Eigen values of the corresponding operators.
ii) The state of a physical system is exhaustively characterized by a vector of Hilbert space, the state vector $|\psi>$ upon which the Hermitian operators corresponding to the observables act. The state $|\psi>$ can be specified by an exhaustive state of observables, like the space time coordinates and any other relevant Quantum numbers.

iii) Any state of the system can be expressed as a linear super position of a complete set of Eigen states$|\phi_i>$(of, for example commuting or compatible Hermitian operators representing corresponding physical observables).
iv) The average or expectation value of a physical observable, that is operator H operating on a state vector $|\psi>$ is proportional to $<\psi|H|\psi>$, or if $|\psi>$ is normalized and is expanded in terms of the complete set of orthonormalized Eigen vectors of $H, |\phi_i>$ corresponding to Eigen values $h_i, |\psi>= C_i|\phi_i>$ where $C_i =<\phi_i|\psi>; <H>=<\psi|H|\psi>= \sum|C_i|^2 h_i$. So $C_i \equiv <\phi_i|\psi>$ is the probability amplitude for the state $|\psi>$to be found in the Eigen state $|\phi_i>$. More generally the probability amplitude for a state $|\psi>$ to be found in any state $|\phi>$ is $<\phi|\psi>$.
v) Quantum Mechanically, the momentum is defined by the operator $\frac{h}{i}\frac{d}{dx}$, while the Hamiltonian or energy is assumed to be the operator $i\hbar\frac{d}{dt}$(Later these are identified with the corresponding classical quantities).
We now denote the state of a system at time t by $|\psi(t)>$. Let i be the complete set of base states and $U(t_2, t_1)$ the "time elapse operator" that denotes the passage of time between instants t_1 and t_2, t_2 greater than t_1. We denote by, $C_i(t) \equiv <i|\psi(t)>$, the amplitude for the state $|\psi(t)>$ to be in the state $|i>$, and

$$<i|U|j> \equiv Uij, Uij(t+\Delta t, t) \equiv \delta_{ij} - \frac{i}{\hbar}H_{ij}(t)\Delta t.$$

We can now deduce, as is well known [10], from the super position of states principle, postulate,(iii) above,

$$C_i(t+\Delta t) = \sum_j [\delta_{ij} - \frac{i}{\hbar}H_{ij}(t)\Delta t]C_j(t)$$

and finally, in the limit,

$$i\hbar\frac{dC_i(t)}{dt} = \sum_j H_{ij}(t)C_j(t) \qquad (1)$$

The matrix $H_{ij}(t)$ will be identified with the Hamiltonian operator according to postulate v. (To facilitate comparison we stick to the notation and development as given in [10]. Before proceeding to derive the Schrodinger equation, we apply equation (1) to the simple case of a two state system

$(i, j = 1, 2$ respectively; (cf. ref.[10]). This will provide a physical understanding of the later work. For a two state system we have

$$i\hbar\frac{dC_1}{dt} = H_{11}C_1 + H_{12}C_2$$

$$i\hbar\frac{dC_2}{dt} = H_{21}C_1 + H_{22}C_2$$

leading to two stationary states of energies $E-A$ and $E+A$, where $E \equiv H_{11} = H_{22}, A = H_{12} = H_{21}$. We can choose our zero of energy such that $E = 2A$. Indeed as has been pointed out [10], when this consideration is applied to the hydrogen molecular ion, the fact that the electron has amplitudes C_1 and C_2 of being with either of the hydrogen atoms, manifests itself as an attractive force which binds the ion together, with an energy of the order of magnitude $A = H_{12}$.

To proceed, we consider in (1), the i to be the space point x_i and we denote by $C(x_n) \equiv C_n$ the probability for the particle to be at this space point. Further let $x_{n+1} - x_n = b$. Then considering only the point x_n and its neighbours $x_{n\pm1}$, the equation (1) goes over into

$$i\hbar\frac{\partial C(x_n)}{\partial t} = EC(x_n) - AC(x_n - b) - AC(x_n + b) \tag{2}$$

In the limit $b \to 0$, with our choice of the arbitrary zero of energy, (2) goes over into

$$i\hbar\frac{\partial C(x)}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2 C(x)}{\partial x^2} \tag{3}$$

where we have now dropped the subscript distinguishing the space point, and $m' = \hbar^2/2Ab^2$.

So far, we have only reproduced the developments of[10]. We now observe that while equation (3) resembles the free Schrodinger equation, as has been pointed out in [10], m' is not really the inertial mass, but an "effective mass" that emerges from the probability amplitude for the particle to be found at a neighbouring point. So (3) is not the Schrodinger equation.

The Schrodinger equation can be obtained from (3) if it can be shown that m' can somehow be replaced by m. This is what we propose to do.

To start with let us suppose that the particle has no mass other than the effective mass m', so that we can treat equation (3) as the Schrodinger type

equation for such a particle which has only amplitude to be at neighbouring points. Let us now suppose that the particle now acquires non zero probability amplitude to be present non locally at other than neighbouring points. We can then no longer work with equations (2) and (3). We will have to use the full equation (1) which explicitly exhibits this possibility. We rewrite equation (1) as

$$i\hbar\frac{dC_i(t)}{dt} = H_{ii}C_i(t) + H_{i,i-1}C_{i-1}(t) + H_{i,i+1}C_{i+1}(t)$$
$$+\sum_j H_{i,i+j}(t)C_j(t), (j = \pm 2, \pm 3,)$$

or as in the transition of equation (2) to equation (3),

$$i\hbar\frac{\partial C(x)}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2 C(x)}{\partial x^2} + \int H(x,x')C(x')dx' \tag{4}$$

where we have replaced H_{ij} by $H(x,x')$ and the points x_i are in the limit taken to be a continuum.
The matrix $H(x,x')$ gives the probability amplitude for the particle at x to be found at x', that is by Postulate iv,

$$H(x,x') =< \psi(x')|\psi(x) > , \tag{5}$$

where as is usual we write $C(x) \equiv \psi(x)$, and denote the state of a particle at the point x by $|\psi(x) >$.
Usually the amplitude $H(x,x')$ is non-zero only for neighbouring points x and x', that is, $H(x,x') = f(x)\delta(x-x')$. But if $H(x,x')$ is not of this form, then there is a non-zero amplitude for the particle to "jump" to an other than neighbouring point. In this case $H(x,x')$ may be described as a non local amplitude. Indeed such non-local amplitudes are implicit in the Dirac equation also and this will be commented on later.
We now give a quick derivation of how the inertial mass emerges from equation (4). The non local Schrodiner equaion (4), given only the effective mass m', can be written, with the help of (5), as,

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2\psi}{\partial x^2} + \int \psi^*(x')\psi(x)\psi(x')U(x')dx', \tag{6}$$

where,
i) $U(x) = 1$ for $|x| < R, R$ arbitrarily large and also $U(x)$ falls off rapidly as $|x| \to \infty; U(x)$ has been introduced merely to ensure the convergence of the integral; and
ii) $H(x, x') =< \psi(x')|\psi(x) >= \psi^*(x')\psi(x)$.
(6) is an integro-differential equation of degree three.
The presence of the, what at first sight may seem troublesome, non-linear and non-local term, viz., the last term on the right side of (6), will be satisfactorily explained in section 2.4.
In (6), in the first approximation $\psi(x)$ can be taken to be the solution of the Schrodinger like equation (3), viz.,

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2\psi}{\partial x^2} \tag{7}$$

In effect, we linearize (6), so that we get,

$$i\hbar\frac{\partial\psi}{\partial t} = [-\frac{\hbar^2}{2m'}\frac{\partial^2}{\partial x^2} + m_0]\psi \tag{8}$$

where,

$$m_0 = \int \psi^*(x')\psi(x')U(x')dx'.$$

In operator language, (8) becomes,

$$\bar{H} = \frac{p^2}{2m'} + m_0 \tag{9}$$

where by Postulate v, $\bar{H}$ is the Hamiltonian operator, p the momentum operator and where, what can now be anticipated as a rest mass like term m_0, appears for a particle assumed not to have any rest mass in the absence of the non-local amplitude term in (6). Also we have replaced the Hamiltonian matrix H by $\bar{H}$ to stress that, to start with, in (4) and (6), the particle has no inertial mass. To facilitate comparison with the usual theory, we next multiply both sides of (9) by the constant $\frac{m'}{m}$, where,

$$m = (m_0 m')^{\frac{1}{2}}/c,$$

c being the velocity of light. (The reason for the appearance of the velocity of light, c can be seen below (cf.equation (11)) and the constant could be absorbed into the state vector, whose direction is all that matters (cf. Postulate

ii). We then get,

$$\hat{H} = \frac{p^2}{2m} + mc^2 \tag{10}$$

The physical meaning of (10) is now clear. In an expansion of the classical relativistic expression for energy,

$$E = (p^2c^2 + m^2c^4)^{1/2}$$

as is well known, if we keep terms up to the order $(p/mc)^2$, we get,

$$E = \frac{p^2}{2m} + mc^2 \tag{11}$$

Anticipating for convenience (though it is not necessary), the discussion after (12), regarding the identification of Quantum Mechanical Operators and their Eigen values with corresponding classical quantities, we can now easily identify m in (10) with the rest mass on comparing this equation with (11). (Interestingly it is not accidental that equation (10) corresponds to the approximation (11) as will be seen in section 2.4). If further, we denote

$$H = \hat{H} - mc^2,$$

where H can be easily identified with the usual kinetic energy operator (or energy operator in non-relativistic theory, remembering that we are considering a free particle only), (10) becomes

$$H = \frac{p^2}{2m} \tag{12}$$

In a strictly non relativistic context, where the rest energy of the particle is not considered, the Hamiltonian is given by (12); otherwise, it is given approximately by (10). Remembering that as yet H and p are operators and that (11) has been introduced only to facilitate an identification with classical quantities, we get from (12), by using Pustulates i, ii and v, the Schrodinger equation,

$$\imath\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \tag{13}$$

All these considerations can be generalized in a simple way to three dimensions [11], but as there is no new physical insight, the details are not given and we continue to work in one-dimension.

2.3 NEWTONIAN MECHANICS AND INERTIAL MASS

The usual derivation of the Schrodinger equation (13) is rather different. First, the Newtonian relation,

$$E = p^2/2m, \tag{14}$$

is invoked, where E and p are the classical energy and momentum of a free particle. The scalars E and p are then identified with the corresponding Quantum Mechanical operators. Thus Newtonian Mechanics is an input. The equation (13) however has been deduced only from Quantum Mechanical Postulates, without invoking Newtonian Mechanics.
If we now invoke Postulates i and ii in equation (12), we get (14), where E and p are now Eigen values of the operators $i\hbar\frac{d}{dt}$ and $-i\hbar\frac{d}{dx}$ which we have defined to be the energy and momentum operators in Postulate v. It is these Eigen values which show up as classical quantities. In other words, the Newtonian relation (14) is a consequence of Quantum Mechanical Postulates. Once we view equation (14) as the Eigen value - and classical - version of the Quantum Mechanical operator equation (12), it is easy to deduce the classical Laws of Motion from (14), using now, classical concepts. For example, (14) gives,

$$\Delta E = \frac{p}{m}\Delta p,$$

while

$$\Delta E \equiv F.\Delta x = \frac{F}{m}p\Delta t,$$

F being the (classical) Force, so that we get,

$$\Delta p = F\Delta t,$$

from which Newton's first and second Laws follow.
This is also the reason why we can compare the Quantum Mechanical equation (10) with the classical equation (11) and identify 'm' in (10) with the rest mass of the particle. The physical origin of the rest mass in (10) is also clear: In the two state Hydrogen molecular ion case considered earlier, it was the amplitude for the single electron to be with one Hydrogen atom or the

other, which manifested itself as a binding energy. In our case, the amplitude of a particle to be at x or x', viz., the second term on the right side of (6), manifests itself as an (attractive) energy, the mass energy of the particle, or what may be called self energy (or self interaction on energy). This is the origin of a particle's inertial mass.

2.4 THE ORIGIN OF SPECIAL RELATIVITY AND DISCUSSION

In section 2.2 it was mentioned that the appearance of the rest mass term in (10) corresponding to the approximation (11) of the relativistic energy momentum relation, was not accidental. Infact it is well known [4] that the Schrodinger equation can also be obtained as the non-relativisitic limit of the Dirac equation. However, in this transition, the phase factor $\phi \equiv \exp[-\imath mc^2 t/\hbar]$, of the original relativistic wave function is dropped. If this phase factor is taken into account in the Schrodinger equation (that is if we continue to work in terms of the original wave function), it can be easily verified that we get equation (10). Conversely, if in the equation with Hamiltonian (10), the wave function is transformed by the factor ϕ, we get back the usual Schrodinger equation (13).

This is to be expected because the energy operator $\imath\hbar\frac{d}{dt}$ operating on ϕ gives the mc^2 term. To the extent that the Quantum Mechanical phase or the rest mass which corresponds to a constant potential in the Schrodinger equation, is inessential, this is of no consequence.

However, there are subtler issues in this context which are worth noting. Firstly, as is well known, the Schrodinger equation is not quite invariant under the Galilean group, as one would expect a strictly non-relativistic equation to be [12]. For, Galilean invariance would imply that there would be no Quantum Mechanical Sagnac effect - a purely relativistic effect due to the interference of different phases. However the Quantum Mechanical Sagnac effect has been observed [13].

There has been much discussion about all this recently [12, 14]. As has been pointed out, under Galilean transformations, the phase of the Schrodinger wave function is not invariant and this is what leads to the Sagnac effect. But if the phase factor ϕ above, which is dropped in the passage from the Dirac to the Schrodinger equation is retained, then under a Lorentz transformation,

to the order $(v/c)^2$, there is phase invariance also; and the relativistic Sagnac effect is satisfactorily explained. It has been correctly concluded in references [12] and [14], that the original de Broglie theory, on which Schrodinger based his work, was relativisitic.

To sum up, in the usual theory, the Schrodinger equation can also be obtained as the limit of the Dirac equation together with the suppression of the phase factor ϕ, which amounts to dropping the rest energy term in (11). It is this latter step which gives a non-Galilean character to the Schrodinger equation, viz., the appearance of the Sagnac effect.

The second point is that in the usual theory, in the above discussion, we start with the relativistic energy momentum relation, then deduce the Dirac equation and finally take the non-relativistic limit to get the Schrodinger equation. However, in section 2.2 we have deduced the equation (10) and subsequently the Schrodinger equation (13), from an independant, purely Postulative Quantum Mechanical view point without the inputs of Newtonian or relativistic mechanics. So equation (10) shows the origin of not just Newtonian Mechanics, but special relativity also, in Quantum Mechanics. This is not surprising, because the very concept of rest energy or energy of mass is relativistic. Infact, for a particle at rest, the operator equation (10) and its Eigen value classical version would give (as $p = 0$), the well known relation,

$$E = mc^2$$

However, we have not deduced Einstein's mass-energy relation - the constant c was introduced only to facilitate comparison: We have not Quantum Mechanically proved that c is the velocity of light. Infact what we have deduced is,

$$E \alpha m$$

This has been elaborated in an earlier communication[11] and will be justified in section 5.

We finally comment on the non-local term in (6) which is also non-linear. This gives rise to the inertial mass of a particle. Non- locality implies superluminal velocities and the breakdown of causality. Classically this is impossible. But in Quantum Mechanics, owing to the Uncertainity Principle, this is allowed within the Compton wavelength $\hbar/mc$ of a particle[15]. So there is no contradiction if the non-local amplitude is integrated within this region of the particle's Compton wavelength. That is, the inertial mass is a

result of non-local, non-linear processes within the Compton wavelength of the particle.
One could argue that the usual Dirac equation also has a non-local character: the operator $c\vec{\alpha}.\vec{p} + \beta mc^2$ is equivalent to and replaces the non-local square-root operator, $(-\hbar^2\nabla^2 + m^2c^4)^{1/2}$. Here also, the non-local effects in the form of negative energies are encountered - again within the Compton wavelength region (cf. ref.[4]).
In the light of the preceding considerations, we can derive the Schrodinger equation from an alternative angle: It appears that the "point" particle is really spread over the non-locality region $\sim \bar{b} = \frac{\hbar}{mc}$, the Compton wavelength. Further, the energy of the particle i.e., the energy tied up within this region viz., 2A of Section 2.2 is the inertial mass energy mc^2. We could now, as in Section 2.2 (equation (3)), speak of the amplitude for the particle at x to be found (locally) at a neighbouring point $x + b$, except that in the limit, $b \to \bar{b}$ (and not 0). The effective mass m' in equation (3) is then given by,

$$m' = \frac{\hbar}{2A\bar{b}^2} = m,$$

that is the mass itself.
So, equation (3) can be interpreted as the Schrodinger equation.
It is worth re-emphasizing that it is the force of binding of non-local positions within the Compton wavelength, rather like the Hydrogen molecular ion binding, that manifests itself as inertial mass.
According to this reasoning, if a particle is subjected to a field powerful enough to penetrate the Compton wavelength region, in contrast to usual relatively low energy fields which merely accelerate the particle as a whole, then it can be disruptively destroyed and the mass energy released (cf. also ref.[16]).

3 Particles as Black Holes

The fact that the mass generating non-local amplitudes are confined to a region of width $\sim \frac{\hbar}{mc}$ suggests that the particle could be a black hole, because in this case also, there is a width, the horizon, inside which such unphysical phenomena appear. The possibility that a particle could be a Schwarzchild black hole has been examined earlier by Markov and others[17] and leads to

a high particle mass of $10^{-5}gm$.
However, our approach in what follows is different. Let us consider a charged Dirac (spin half) particle. If we treat this as a spinning black hole, there is an immediate problem: The horizon of the Kerr-Newman black hole[18] becomes in this case, complex,

$$r_+ = \frac{GM}{c^2} + ib, b \equiv (\frac{G^2Q^2}{c^8} + a^2 - \frac{G^2M^2}{c^4})^{1/2} \tag{15}$$

where G is the gravitational constant, M the mass and $a \equiv L/Mc, L$ being the angular momentum. That is, we have a naked singularity apparently contradicting the cosmic censorship conjecture. However, in the Quantum Mechanical domain, (15) can be seen to be meaningful. (cf-Appendix) Infact, the position coordinate for a Dirac particle is given by [9]

$$x = (c^2 p_1 H^{-1} t + a_1) + \frac{i}{2} c\hbar(\alpha_1 - cp_1 H^{-1})H^{-1}, \tag{16}$$

where a_1 is an arbitrary constant and $c\alpha_1$ is the velocity operator with eigen values $\pm c$. The real part in (16) is the usual position while the imaginary part arises from zitterbewegung. Interestingly, in both (15) and (16), the imaginary part is of the order of $\frac{\hbar}{mc}$, the Compton wave length, and leads to an immediate identification of these two equations. We must remember that our physical measurements are gross - they are really measurements averaged over a width of the order $\frac{\hbar}{mc}$(cf.ref. [1]). Similarly, time measurements are imprecise to the tune $\sim \frac{\hbar}{mc^2}$ (cf. ref.[9]): Very precise measurements if possible, would imply that all Dirac particles would have the velocity of light, or in the Quatum Field Theory atleast of Fermions, would lead to divergences. (This is closely related to the non-hermiticity of position operators in relativistic theory as can be seen from equation (16) itself [19].) Physics begins after an averaging over the above unphysical space-time intervals. In the process as is known (cf. ref.[19]), the imaginary or non-hermitian part of the position operator in (16) disappears. That is in the case of the QMBH (Quantum Mechanical Black Hole), obtained by identifying (15) and (16), the naked singularity is shielded by a Quantum Mechanical censor.
To continue, we first naively treat a Dirac particle as a Kerr-Newman black hole of mass m, charge e and spin $\frac{\hbar}{2}$. The gravitational and electromagnetic fields at a distance are given by (cf. ref.[5]),

$$\Phi(r) = -\frac{Gm}{r} + 0(\frac{1}{r^3})E_{\hat{r}} = \frac{e}{r^2} + 0(\frac{1}{r^3}), E_{\hat{\theta}} = 0(\frac{1}{r^4}), E_{\hat{\phi}} = 0,$$
$$B_{\hat{r}} = \frac{2ea}{r^3}\cos\theta + 0(\frac{1}{r^4}), B_{\hat{\theta}} = \frac{ea\sin\theta}{r^3} + 0(\frac{1}{r^4}), B_{\hat{\phi}} = 0, \tag{17}$$

exactly as required. Infact, as is well known, (17) also exhibits the electron's anomalous gyromagnetic ratio $g = 2$.

We next examine more closely, the identification of a Dirac particle with a Kerr-Newman black hole. We reverse the arguments after equation (16) which lead from the complex or non-Hermitian coordinate operators to Hermitian ones: We consider the displacement,

$$x^\mu \rightarrow x^\mu + \imath a^\mu \tag{18}$$

and first consider the temporal part, $t \rightarrow t + \imath a^0$, where $a^0 \approx \frac{\hbar}{2mc^2}$, as before. That is, we probe into the QMBH or the zitterbewegung region inside the Compton wavelength. Remembering that $|a| << |$, we have, for the wave function,

$$\psi(t) \rightarrow \psi(t + \imath a^0) = \frac{a^0}{\hbar}[\imath\hbar\frac{\partial}{\partial t} + \frac{\hbar}{a^0}]\psi(t)$$

As $\imath\hbar\frac{\partial}{\partial t} \equiv p^0$, the usual fourth component of the energy momentum operator, we identify, by comparison with the well known electromagnetism - momentum coupling, $p^0 - e\phi$, the usual electrostatic charge as,

$$e \equiv \frac{\hbar}{a^0} = 2mc^2 \tag{19}$$

In the case of the electron, we can verify that the equality (19) is satisfied: the charge is seen to be related to the mass and the velocity of light.

It may be noted in passing that in the usual displacement operator theory ([9])the operators like $\frac{d}{dx}$ or $\frac{d}{dt}$ are indeterminate to the extent of a purely imaginary additive constant which is adjusted against the hermicity of the operators concerned. We next consider the spatial part of (18), viz.,

$$\vec{x} \rightarrow \vec{x} + \imath\vec{a}, \text{where}|\vec{a}| = \frac{\hbar}{2mc},$$

given the fact that the particle is now seen to have the charge e (and mass m). As is well known [20], this leads in General Relativity from the static Kerr metric to the Kerr-Newman metric where the gravitational and electromagnetic field of the particle is given by (17), including the anomalous factor

$g = 2$. In General Relativity, the complex transformation (18) and the subsequent emergence of the Kerr-Newman metric has no clear explanation. But in the Quantum Mechanical context we can see the rationale: the origin of (18) lies in the QMBH. We started with a massless particle. Then we saw the emergence of mass and also the origin of gravitation and electromagnetism in the processes inside the Compton wavelength - the nonlocal QMBH region. There is another way to see the emergence of electromagnetism. It is well known that for the Dirac four spinor $\begin{pmatrix} \theta \\ \chi \end{pmatrix}$, where θ denotes the positive energy two spinor and χ the negative energy two spinor, at and within the Compton wave length, it is χ that dominates. Further, under reflections, while $\theta \rightarrow \theta, \chi$ behaves like a psuedo-spinor[4]

$$\chi \rightarrow -\chi$$

Hence the operator $\frac{\partial}{\partial x^\mu}$ acting on χ, a density of weight $N = 1$, has the following behaviour[21],

$$\frac{\partial \chi}{\partial x^\mu} \rightarrow \frac{1}{\hbar}[\hbar \frac{\partial}{\partial x^\mu} - NA^\mu]\chi \tag{20}$$

where,

$$A^\mu = \hbar \Gamma^{\mu\sigma}_{\sigma} = \hbar \frac{\partial}{\partial x^\mu} \log(\sqrt{|g|}) \equiv \nabla^\mu \Omega \tag{21}$$

As before we can identify NA^μ in (20) with the electro- magnetic four potential. That $N = 1$, explains the fact that charge is discrete. It will be shown in the next section that,

$$A^\mu \sim \frac{mc^2}{r} \sim \frac{e}{r} \tag{22}$$

in agreement with (19). That is, electromagnetism is the result of the covariant derivative that arises due to the Quantum Mechanical behaviour of the negative energy components within the Compton wavelength region.
We observe, that in case the mass $m \rightarrow 0$, the considerations of section 2 imply that there are no negative energy components while (19) and (22) show that such a particle has no charge. The massless neutrino fits this description exactly: it has a two component wave function and is chargeless.
There is also the muon which satisfies (22) in the order of magnitude sense. But it is unstable and disintegrates into an electron (or positron) and two

neutrinos anyway.
Thus these considerations describe the stable leptons, viz., electrons and neutrinos, and approximately the remaining unstable lepton.
It is interesting that an application of Maxwell's equations in this Compton wavelength region of "charged matter" leads to meaningful results: In this case the fact that A^μ in (21) is a four gradient poses no problem. We have, using Maxwell's equations (22).

$$\begin{aligned} &\phi \equiv A^0 = \tfrac{\partial \Omega}{\partial t}, \vec{B} = \vec{\nabla} X \vec{A} = \vec{\nabla} X (\vec{\nabla}\Omega) = 0, \\ &\vec{E} = -\tfrac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi = -2\vec{\nabla}\Omega, \vec{E} = -2\vec{\nabla}\phi \end{aligned} \tag{23}$$

Also,

$$\vec{\nabla}.\vec{E} = 2\nabla^2\phi = 4\pi\sigma \tag{24}$$

$$\vec{\nabla}.\vec{B} = 0,$$

while

$$\vec{\nabla} X \vec{B} = 0 = 4\pi s + \frac{\partial \vec{E}}{\partial t} \tag{25}$$

and,

$$\vec{\nabla} X \vec{E} = -\sigma \frac{\partial \vec{B}}{\partial t} = \vec{\nabla} X (2\vec{\nabla}\phi) = 0$$

Equations (23),(24), and (25) show that effectively this is a steady field with potential ϕ that is, we could work as if we have a steady field of potential ϕ except that there is an anomalous doubling of the charge and current. Now, as is well known the usual orbital magnetic momentum is given by [23]

$$\mu = \frac{e}{2mc} p_\phi \tag{26}$$

where p_ϕ is the angular momentum and e is the charge. In our case, e in (26) is effectively replaced by $2e$, so that in the usual units of $e/2mc$, we now have for the Dirac particle, instead of (26),

$$g = \frac{\mu}{p_\phi} = 2$$

This is the anomalous gyromagnetic ratio.

4 A General Relativistic Approach: Origin of QCD Interactions

Thus far it appears that the QMBH description applies to electrons and more generally Leptons. In the light of the preceding considerations, we will now approach the problem from a General Relativistic point of view. This will also reveal the origin of QCD interactions. Our starting point is the linearized theory[5]:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, h_{\mu\nu} = \int \frac{4T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' \tag{27}$$

(A bar on T has been dropped.)
In (27), velocities comparable to the velocity of light c are allowed and also, the stresses T^{jk} and momentum densitities T^{oj} can be comparable to the energy momentum density T^{00}. As in ref.5, we can easily deduce that, when $\frac{|\vec{x}'|}{r} << 1$, where $r \equiv |\vec{x}|$, and in a frame with origin at the centre of mass and at rest with respect to the particle,

$$Gm = \int T^{00} d^3x \tag{28}$$

$$S_k = \int \epsilon_{klm} x^l T^{mo} d^3x \tag{29}$$

where m is the mass (or approximate mass because of the linear approximation), and S_k is the angular momentum. We next observe that,

$$T^{\mu\nu} = \rho u^\mu u^\nu \tag{30}$$

If we now work in the Compton wavelength region of the QMBH, we have, while $u^0 = 1$,

$$|u^i| = c \tag{31}$$

(This is the Quantum Mechanical input)
Substitution of (30) and (31) in (29) gives on using the Mean Value Theorem,

$$S_k = c < x^l > \int \rho d^3x$$

As $< x^l >\sim \frac{\hbar}{2mc}$, using (28), we get, $S_k \approx \frac{\hbar}{2}$, as required for a spin half particle. Infact this relation becomes exact if we treat the QMBH as effectively a rotating shell distribution of radius $\hbar/2mc$, keeping in mind the fact that the interior region is in any case unphysical as seen in section 3, and is described by complex space-time coordinates. The gravitational potential can similarly be obtained from (27) and (28) (cf. ref.[5]),

$$\Phi = -\frac{1}{2}(g^{00} - \eta^{00}) = -\frac{Gm}{r} + 0(\frac{1}{r^3}) \tag{32}$$

We saw in section 3 that the electromagnetic potential is given by,

$$A^\mu = \hbar\Gamma^{\mu\sigma}_{\sigma}$$

Using the expression for the Christoffel symbols, we have,

$$A_\sigma = \frac{1}{2}(\eta^{\mu v}h_{\mu v}), \sigma,$$

so that, from (27),

$$A_0 = 2\int \eta^{\mu v}\frac{\partial}{\partial t}[\frac{T_{\mu v}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}]d^3x'$$

Remembering that $|\vec{x} - \vec{x}'| \approx r$ for the distant region we are considering, we have,

$$A_0 \approx \frac{2}{r}\int \eta^{\mu v}[\frac{\partial}{\partial\tau}T_{\mu v}(\tau, \vec{x}').\frac{d}{dt}(t - |\vec{x} - \vec{x}'|)]d^3x' \approx \frac{2}{r}\int \eta^{\mu v}\frac{d}{d\tau}T_{\mu v}.(1 + c)d^3x',$$

or finally

$$A_0 \approx \frac{2c}{r}\int \eta^{\mu v}\frac{d}{d\tau}T_{\mu v}d^3x' \tag{33}$$

as $c >> 1$, and where we have used the fact that in the Compton wavelength region, $|u_v| = c$.

It has already been observed that QMBH can be treated as a rotating shell distribution with radius $R \equiv \frac{\hbar}{2mc}$. So we have,

$$|\frac{du_v}{dt}| = |u_v|w \tag{34}$$

where w, the angular velocity is given by,

$$w = \frac{|u_v|}{R} = \frac{2mc^2}{\hbar} \tag{35}$$

We get the same relation in the theory of the Dirac equation, remembering that in (28) and (29) the centre of mass is at rest:

$$i\hbar\frac{d}{dt}(c\alpha_i) = -2mc^2(c\alpha_i),$$

where $c\alpha_i$ is the velocity operator (cf.ref.[9]) Finally, on using (30), (34) and (35) in (33), we get,

$$\frac{e}{r} = A_0 \sim \frac{\hbar c^3}{r}\int \rho w d^3x' \sim (Gmc^3)\frac{mc^2}{r} \tag{36}$$

If we use the value of G, m and c, we get as in section 3, for the electron,

$$e \sim 14 \times 10^{-10} esu$$

in agreement with its value and with (19). In the order of magnitude sense the above holds for the muon also.
Our input in the above has been the value of the Gravitational constant G. On the other hand, if we had used the value of e from (19), this analysis would have yielded an approximate value for G.
Equally important, (36) yields the well known ratio 10^{+40} for the electromagnetic and Gravitational interactions.
So far we have been considering distances far from the particle: $|\vec{x}' - \vec{x}| >> |\vec{x}'|$. This is the approximation invoked in a transition from (27) to equations (28), (29) etc. Let us now see what happens when $|\vec{x}| \sim |\vec{x}'|$. In this case, we have from (27), expanding in a Taylor series about t,

$$\begin{aligned} h_{\mu\nu} &= 4\int \frac{T_{\mu\nu}(t,\vec{x}')}{|\vec{x}-\vec{x}'|}d^3x' + (\text{terms independent of}\vec{x}) + 2 \\ &\int \frac{d^2}{dt^2}T_{\mu\nu}(t,\vec{x}').|\vec{x}-\vec{x}'|d^3x' + 0(|\vec{x}-\vec{x}'|^2) \end{aligned} \tag{37}$$

The first term gives a Coulombic $\frac{\alpha}{r}$ type interaction except that the coefficient α is of much greater magnitude as compared to the gravitational or electromagnetic case, because in this approximation, in an expansion of $(1/|\vec{x}-\vec{x}'|)$, all terms are of comparable order. To proceed further, using (34), we have,

$$\frac{d}{dt}T^{\mu\nu} = \rho u^\nu\frac{du^\mu}{dt} + \rho u^\mu\frac{du^\nu}{dt} = 2\rho u^\mu u^\nu w,$$

so that,

$$\frac{d^2}{dt^2}T^{\mu\nu} = 4\rho u^\mu u^\nu w^2 = 4w^2 T^{\mu\nu}$$

where w is given by (35). Substitution in (37) gives,

$$h_{\mu\nu} \approx -\frac{\beta M}{r} + 8\beta M(\frac{Mc^2}{\hbar})^2.r \tag{38}$$

β being a constant.
This resembles the QCD quark potential[24], with both the Coulombic and confining parts. Taking for M the mass of a typical Cquark $\sim 1.8Gev$ (cf. ref.[24]), the ratio of the coefficients of the r term and the $\frac{1}{r}$ term as obtained from (38) is $\sim \frac{1}{\hbar^2}(Gev)^2$ as in the case of QCD (ref.[24]).
In this picture, how do we accommodate anti-particles, for example positrons? While treating the negative energy spinor as a density in section 3 we had assumed that $N = 1$. Equally well, we could have chosen $N = -1$. This reverses the sign of the charge, all else remaining the same. So with $N = -1$, we get a positron. Similarly for quarks, N can be taken to be fractional. But this apart it must be remembered that whereas for electrons we took the asymptotic expansions of equations like (27), in the case of quarks we had to consider the region near the Compton wavelength itself.
Thus, it appears that the treatment of Leptons and quarks as QMBH leads to meaningful results. On the other hand, these are the most fundamental constituents of matter.

5 Discussion and Comments

1) As has been pointed out, QFT works with point particles and a space-time continuum in a special relativistic context. Divergences appear when we go right upto $r = 0$. However, it appears that for Fermionic Fields atleast, this picture may be valid only for distances greater than the Compton wavelength: The above model forbids such a limiting process for Fermions and sets a cut off. Once we enter the QMBH region, a very high energy phenomenon, space-time in the conventional sense becomes unphysical. The appearance of complex coordinates or non-Hermitian operators is a manifestation of this unphysical feature.
2) In the usual formulation of the Hole Theory, the Dirac sea is filled with

negative energy electrons, and by invoking the Pauli exclusion principle, transitions to negative energy states are forbidden. In the present formulation, in effect, the Dirac sea of negative energy states is squeezed into the QMBH and the Quantum Mechanical censor of section 3 forbids transitions.

3) In ordinary Quantum Mechanics, ψ being the wave function, $\psi\psi^*$ is proportional to the *probability* density. On the other hand, we saw in section 2 that the mass density is produced by the non-linear amplitude $\psi\psi^*$ in the Compton wavelength region. More specifically we saw in sections 3 and 4 that it is χ, the negative energy part of the Dirac four spinor (which dominates in this region), that is relevant. That is, ρ being the *material* density,

$$\rho \alpha \chi \chi^* \tag{39}$$

As observed in section 3, for the two component neutrino, $\chi = 0$, and the neutrinos are massless.

It was shown in an earlier communication[25], how Gravitation can emerge from the Schrodinger equation self-consistently. Again, it is the identification of the material density in (39) which gives substance to that result.

4) More general than the radius of the horizon given in (15) is the static limit of the Kerr-Newman Black Hole, wherein 'a' is replaced by acos θ, θ being the usual polar coordinate. However in the QMBH, as we approach the Compton wavelength $\sim a$, we encounter the unphysical zitterbewegung region where θ ceases to have any physical meaning. In other words, the spin is insensitive to θ unlike in the classical case. This is ofcourse well known in Quantum Mechanics.

5) The emergence of electromagnetism from spin should not be too surprising. As is well known, rotating frames and the accompanying drag do produce effects similar to magnetism[26].

6) It may be remarked that we started in section 2 with purely Quantum Mechanical postulates and deduced mechanical effects. We came a full circle in section 4 wherein, from purely classical considerations, we deduced Quantum Mechanical phenomena.

7) It is interesting that from a different angle, using considerations of self-similarity, if we assume that the scale of the Universe is broken at some stage, that is, that there is an ultimate micro-level, then the Compton wavelength again appears as a fundamental length[27].

8) If in the position formula (16), we consider the real part and also a time

interval of $\sim \frac{h}{mc^2}$, we get for $p = mc$,

$$x = \frac{\hbar}{mc}$$

This result can also follow from the Heisenberg Uncertainity Principle.
As in the spirit of the preceding observation, we see the emergence of the Planck constant h in the extreme situation of the maximum velocity and minimum physical space-time intervals. Moreover, h, m and c are inter-related. Taking the cue from here, we pick up the result in section 2 viz.,

$$E \alpha m \quad \text{or} \quad E = my,$$

where y is the constant of proportionality which was identified adhoc earlier with c^2. Using now Heisenberg's Uncertainity relations, and considering the extreme case, we have, firstly,

$$t \sim \frac{\hbar}{my},$$

so that $x = ct \sim \frac{\hbar c}{my}$, where c is the maximum possible velocity. Further, in this case,

$$p = mc \sim \frac{\hbar}{x} = \frac{my}{c},$$

so that $y = c^2$.
This provides a Quantum Mechanical justification for the formula $E = mc^2$, without taking recourse to special relativity. Indeed as pointed out in section 2, the origin of special relativity could be traced to these Quantum Mechanical considerations.

6 Conclusion

It appears that the most fundamental Fermions, viz., the quarks, the electrons and the neutrinos can be thought of as Quantum Mechanical Black Holes. These are Black Holes apparently exhibiting a naked singularity, but which are meaningful in terms of the Quantum Mechanical Compton wavelength and processes near and within this region.
In classical terms, effectively this is a rotating shell distribution with a radius

equalling half the Compton wavelength and where the velocity equals that of light as at a Black Hole horizon.
All this explains the origin of mass, gravitation, electromagnetism and even QCD interactions, their relative magnitudes and also features like the discreteness of charge and the anomalous gyromagnetic ratio of the electron.

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APPENDIX

There is another way in which the result of section 3, viz., the treatment of elementary particles as black holes can be viewed.
What we call the position of the point particle is given by the real part of equation (16) - after accounting for Quantum Mechanical effects in the form of the imaginary part. In this case, we have the Hermitian position operator $\vec{x}$. From here on we are in the domain of point particles and the usual space-time continuum described by the Lorentz metric both in Quantum Theory and QFT. From this point of view, we can interpret (15) as follows.
After accounting for the Quantum Mechanical effects in the form of the imaginary parts of (15) and (16), the radius of the Kerr- Newman black hole, as described by the real part of (15) (and (16)), is given by,

$$r_+ = M,$$

where for convenience, we work in units such that $G = 1, c = 1$. Now, $r_+ \sim 10^{-56} cm$ for the electron and a few orders higher for protons and quarks.
This is well below the Planck length and effectively the particles are points. The Kerr-Newman metric which gives (15) is:

$$ds^2 = -\frac{\Delta}{\rho^2}[dt - a\sin^2\theta d\phi]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\phi - adt]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2,$$

where

$$\Delta \equiv r^2 - 2Mr + a^2 + Q^2, \rho^2 = r^2 + a^2\cos^2\theta$$

Even for distances much smaller than the Compton Wave length a, as above, this goes over to,

$$-dt^2 + a^2\sin^2\theta d\phi^2 + \cos^2\theta dr^2 + a^2\cos^2\theta d\theta^2,$$

where $\theta \neq \pi/2$.
This is broadly similar to the Lorentz metric of special relativity.
The concept of point particles vis a vis Heisenberg's uncertainity principle which denies precise positions is, in any case, one of the paradigms of modern physics (cf. "Super Strings and the search for the Theory of Everything", F.D. Peat, Abacus, London, 1992, for a detailed discussion.)