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LARGE-SCALE STRUCTURE WITH HOT DARK MATTER AND COSMIC STRINGS

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ABSTRACT

A cosmological model in which the energy density perturbations are seeded by cosmic strings and in which the dark matter is hot (e.g. neutrinos with a mass $25h_{50}^2\,eV$) generates a significant amount of structure on scales larger than the scale of clusters. The growth of velocity perturbations produced by long strings moving at relativistic speeds is studied and the conditions under which these perturbations form nonlinear structures are determined. If these conditions are satisfied, then the model predicts overdensities of galaxies in sheets with a thickness of about $2\,Mpc$ and length and width of the order $40\,h_{50}^{-2}\,Mpc$.

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1. INTRODUCTION

Recently, galaxy and cluster formation has been studied[1–5] in a model with cosmic strings as the source of energy density perturbations and with hot dark matter. The results were interesting: stationary string loops seed galaxies with flat halo rotation curves and a mass function with slope -1.5 (again neglecting loop motion). In Refs. 1 - 5 only accretion onto cosmic string loops was considered. However, the network of cosmic strings consists not only of a distribution of loops. At any time t there is also a network of infinite strings[6–8]. The mean curvature radius of an infinite string is about t, and the mean separation between long strings is also of the order of t. The strings are approximately straight on scales smaller than t, and hence there is no local gravitational force. It is well known, however, that long strings moving with relativistic speeds form velocity perturbations (wakes) behind them.[9,10]

The effects of wakes in a model with cosmic strings and cold dark matter have been studied in Refs. 11 - 13. Wakes give rise to planar overdensities. It was shown[12] that the most prominent and numerous sheets have a length, width and mean separation comparable to the comoving horizon at the time t_{eq} of equal matter and radiation, i.e. $40\,h_{50}^{-2}\,Mpc$. These predictions can be compared with the results from the recent Center for Astrophysics redshift survey[14] and from other investigations of large-scale structure.[15] However, with cold dark matter most of the mass in the universe accretes onto small loops rather than wakes, and hence the regions between the wakes will not have a low density of galaxies, in conflict with observations.

In this letter we summarize a detailed analysis[16] of accretion onto wakes in a cosmic string model with hot dark matter. Neutrino free streaming suppresses the growth of density perturbations about small loops. Hence, the fraction of mass which goes nonlinear about loops is much smaller than with cold dark matter, and the relative importance of wakes is much bigger. We show here that, provided the mass per unit length μ of the string obeys $G\mu > f \times 10^{-6}\,h_{50}^{-2}$ (where

f is a constant whose value we calculate using a simple model and which turns out to be 0.5, using a velocity v_s of the string whose mean square is[7,8] about 1/2), the perturbations about wakes go nonlinear and form sheets of thickness $\sim 2\,Mpc$ and mass comparable to the mass of cold dark matter cosmic string wakes. In fact, the total energy density in wakes is approximately equal to the energy density in loops. Hence, the large-scale structure in the cosmic string model with hot dark matter could be dominated by wakes.

We shall follow theoretical prejudice and assume that the universe is flat, i.e. $\Omega = 1$. However, the standard nucleosynthesis scenario[17] constrains the fraction of energy in baryons Ω_B to be smaller than $1 : \Omega_B \leq 0.15$. Hence, the remainder of matter must be nonbaryonic. It is called "hot" if the dark particles have large thermal velocities at t_{eq}, and "cold" if they do not. Free streaming of hot dark matter particles prevents the growth of perturbations on small scales at early times. For this reason, models with linear adiabatic density perturbations are hard to reconcile with observations, since primordial fluctuations on scales smaller than $10^{15} M_\odot$ are erased just before t_{eq}[18,19]. Hence, galaxies can only form by fragmentation of larger structures.

However, with nonadiabatic seeds like cosmic strings as the source of energy density perturbations, the situation is quite different. The seeds survive neutrino free streaming. The growth of perturbations on small scales is delayed but not prevented. Neutrinos have thus been resurrected as a viable dark matter candidate.

In the following analysis of large-scale structure with cosmic strings and neutrinos, we use units in which $c = k_B = \hbar = 1$. We use a Friedmann–Robertson–Walker metric with scale factor $a(t)$. H is the Hubble expansion parameter which we write as $H = h_{50}\, 50\, km\, s^{-1}\, Mpc^{-1}$. t_0 denotes the present time, and $z(t)$ is the redshift at time t.

2. ACCRETION ONTO WAKES

As mentioned in the introduction, moving long strings produce velocity perturbations in their wake. These perturbations originate because of the unusual gravitational field of a long straight string. There is no local gravitational force but the spatial section perpendicular to the string is conical.[20] Represented on a plane, the spatial section is flat with a missing wedge of angle $8\pi G\mu$ (Figure 1a).

A long straight string moving with relativistic speed v_s in its perpendicular plane gives rise to a velocity perturbation $\Delta v = 4\pi G\mu v_s \gamma_s$ in its wake (γ_s is the usual Lorentz factor). Note that Δv is independent of the height h from the center of the wake up to a distance comparable to the curvature radius R of the long string. At that point, the approximate form of the metric breaks down. The length of the wake is approximately R, the width $v_s R$ (Figure 1b).

According to the cosmic string simulations, at any time t there will be about 10 long string segments with a typical curvature radius t passing through each horizon volume. While the universe is radiation dominated (i.e. for $t < t_{eq}$), the velocity perturbations will be damped. However, for $t \geq t_{eq}$ they can grow and turn into density perturbations. Since the energy density perturbations in the matter dominated phase grow as $a(t)$, those created at t_{eq} will have had the longest time to grow and will be the most prominent today. They will appear today as planar density enhancements of length t^c_{eq} and width $v_s t^c_{eq}$, where t^c_{eq} is the comoving horizon at t_{eq}.

$$t^c_{eq} = t_{eq} z_{eq} \simeq 40\, h_{50}^{-2}\, Mpc \tag{2.1}$$

In order to calculate the growth of an initial velocity perturbation in the wake of a moving long straight cosmic string beginning at time $t_i \geq t_{eq}$ we shall use a simple model based on the Zel'dovich approximation.[21,22] We consider a particle with unperturbed comoving distance q to the center of the wake (Figure 2).

Since within one Hubble expansion time, a density perturbation with $\delta\rho/\rho = 1$ inside the wake will develop, we will replace the velocity perturbation by a planar density perturbation with surface density σ at the center of the wake ($h = 0$). As a consequence of the extra mass, a comoving position change ψ will develop. The physical height of the particle is:

$$h(q,t) = a(t)\,(q - \psi(q,t)) \tag{2.2}$$

In the Newtonian approximation, h obeys Newton's equation

$$\ddot{h} = -\nabla_h \Phi \quad , \tag{2.3}$$

where the Newtonian potential Φ satisfies Poisson's equation

$$\nabla_h^2 \Phi = 4\pi G\,(\rho + \sigma\delta(h)) \tag{2.4}$$

where we have chosen coordinates with the center of the wake at $h = 0$. After linearizing in ψ, we obtain the following differential equation

$$\ddot{\psi} + 2\frac{\dot{a}}{a}\dot{\psi} + 3\frac{\ddot{a}}{a}\psi = 4\pi G a^{-1}\sigma\theta(z) \quad . \tag{2.5}$$

With initial conditions $\psi(t_s) = \dot{\psi}(t_s) = 0$ at some time $t_s \geq t_i$, the solution is

$$\psi(t) = \frac{18\pi G}{5}\sigma(t_s)\left(\frac{t_s}{t_0}\right)^{2/3}\left(\frac{t}{t_0}\right)^{2/3} t_0^2 \quad . \tag{2.6}$$

The mean surface density of the wake at time t_s is

$$\sigma(t_s) = 4\pi G\mu t_i\left(\frac{t_s}{t_i}\right)^{2/3}\rho(t_s)v_s\gamma_s \quad . \tag{2.7}$$

Solution (6) describes how the expansion of $h(t)$ is gradually slowed down by the gravitational attraction of the wake. The Zel'dovich approximation can be

used until the shell turns around, i.e. $\dot{h}(t) = 0$. At any time t, the comoving coordinate q for which $\dot{h}(q,t) = 0$ determines the thickness of the nonlinear planar overdensity formed by the wake. We denote this value of q by $q_{n\ell}(t)$. From (2), (6) and (7) it follows that

$$q_{n\ell}(t) = 2\psi_0\left(\frac{t}{t_s}\right)^{2/3} \tag{2.8}$$

with

$$\psi_0 = \frac{12\pi}{5}G\mu v_s\gamma_s z(t_i)^{-1/2}t_0 \quad . \tag{2.9}$$

With cold dark matter, perturbations on all scales q would start to grow at t_{eq}, i.e. $t_s(q) = t_{eq}$. With hot dark matter, the growth of the perturbation on a scale q begins when q first exceeds the neutrino free streaming length $\lambda_J^c(t)$ (the superscript c stands for comoving coordinates). For $t \geq t_{eq}$

$$\lambda_J^c(t) = v(t)ta(t)^{-1} = v_{eq}t_{eq}z_{eq}^{1/2}z(t)^{1/2} \equiv \lambda_0 z(t)^{1/2} \tag{2.10}$$

where $z_{eq} = z(t_{eq})$ and v_{eq} is the mean neutrino velocity at t_{eq}. We take $v_{eq} = 0.05$.[1] Hence, for $q < q_{max} = v_{eq}t_{eq}z_{eq}^{1/2}z(t_i)^{1/2}$

$$z(t_s) = \left(\frac{q}{\lambda_0}\right)^2 \quad . \tag{2.11}$$

Obviously, for $q > q_{max}, z(t_s) = z(t_i)$.

This recipe for including the effects of free streaming is rather naive. It neglects the nontrivial velocity distribution of the neutrinos. Furthermore, using the Zel'dovich approximation in the way given by (2-4) neglects the facts that the initial perturbation is a velocity perturbation rather than a density perturbation and that the initial perturbation is made up of neutrinos which can themselves free stream. All these deficiencies are corrected in a more detailed analysis[16] in

which we use the Gilbert equation[19] to study the growth of inhomogeneities in the phase space distribution of neutrinos starting with a pure velocity perturbation. The results of this more detailed analysis[16] are in agreement with the results derived here.

Combining (11) and (8), we find that for $q > q_{max}$ the redshift z_{eq} at which the sheet turns around is

$$z_q = 2\psi_o z(t_i) q^{-1} \tag{2.12}$$

whereas for $q < q_{max}$,

$$z_q = 2\psi_o \lambda_o^{-2} q \tag{2.13}$$

(see Figure 3). This implies that for hot dark matter there is no slow growth of the nonlinear structure seeded by the wake (like for cold dark matter). Instead, at a redshift

$$z_{max} = z_{q_{max}} = 2\psi_o \lambda_o^{-1} z(t_i)^{1/2} \tag{2.14}$$

the entire region $q < q_{max}$ goes nonlinear. Since $\psi_o \sim z(t_i)^{-1/2}$ and $q_{max} \sim z(t_i)^{1/2}$, it follows that (as was already pointed out in Ref. 12) nonlinear structures form at the same time for all wakes created after t_{eq}. (When taking into account the free streaming of the neutrinos which make up the initial perturbation we find that wakes from larger t_i form later [16].) However, the thickness of the structure scales like $z(t_i)^{1/2}$. Hence, the thickest wakes are those laid down at t_{eq}.

It is important to point out that nonlinear structures about wakes form only if $z_{max} > 1$. This gives a lower bound on $G\mu$

$$G\mu > \frac{5}{24\pi} v_{eq} v_s^{-1} \gamma_s^{-1} z_{eq}^{-1} \sim 5 v_s^{-1} \gamma_s^{-1} 10^{-7} h_{50}{}^{-2} \quad . \tag{2.15}$$

According to numerical simulations,[6-8] $v_s \gamma_s \sim 1$. Hence the bound is $G\mu > 5 \times 10^{-7} h_{50}{}^{-2}$ which is consistent with the present upper bounds from microwave

anisotropy measurements [24]. The more detailed analysis of Ref. 16 gives a condition on $G\mu$ which agrees with the above to within a factor of 3.

The thickness q of the nonlinear structure created by a wake formed at t_i follows immediately from (8) and (11).

$$q = v_{eq} t_{eq} z_{eq} \frac{G\mu}{(G\mu)_c} \left(\frac{z(t_i)}{z_{eq}} \right)^{1/2} \tag{2.16}$$

where $(G\mu)_c$ is the critical value of $G\mu$ given by (15). For $G\mu = (G\mu)_c, t_i = t_{eq}$ and $h_{50} = 1$ the value is $q \simeq 2\,Mpc$.

3. CONSEQUENCES

In order to assess the relative importance of wakes we shall compare the fraction of matter which goes nonlinear in wakes and about loops. The energy in wakes per unit volume is given by

$$\rho_w(t_o) = \int_{t_i}^{t_o} dt_i n_w(t_i) M(t_i) \tag{3.1}$$

where $n_w(t_i)$ is the comoving number density of long strings at time t_i

$$n_w(t_i) = \nu_w t_i^{-2} t_o^{-2} \tag{3.2}$$

(ν_w is a constant), and $M(t_i)$ is the nonlinear mass in wakes produced by these string segments

$$M(t_i) = q_{n\ell}(t_o) v_s t_i^2 z(t_i)^2 \rho(t_o) \quad . \tag{3.3}$$

From (2.8) and (2.9) we obtain

$$\rho_w(t_o) = \frac{6}{5} \nu_w v_s^2 \mu z_{eq} t_o^{-2} \quad . \tag{3.4}$$

We can determine the energy in loops per unit volume $\rho_\ell(t)$ using the same approximations used in Section 2. We treat the accretion onto loops using the

Zel'dovich approximation and the naive model of (2.11) to take into account for free streaming. For a loop of radius R and mass $m = \beta\mu R$, the comoving scale of turnaround is given by

$$q_{n\ell}^3(t_o) = \frac{9}{5}Gm\left(\frac{t_o}{t_s}\right)^{2/3} t_o^2 \quad . \tag{3.5}$$

For loops with radius $R > R_J$, where R_J is determined by $q_{n\ell}(t_o) = q_{max}, t_s = t_i$ and the non-linear mass is the same as in the cold dark matter model. For $R < R_J$, however, free streaming decreases the total nonlinear mass bound to the loop. R_J is given by

$$R_J = \frac{5}{9}\beta^{-1}(G\mu)^{-1}v_{eq}^3 z_{eq}^{-1} t_{eq} \tag{3.6}$$

For $R < R_J$, we may use (2.11) to obtain

$$q_{n\ell}(t_o) = \frac{9}{5}\beta G\mu R\left(\frac{t_o}{\lambda_o}\right)^2 \quad . \tag{3.7}$$

The total mass of the structure gravitationally bound to the loop is

$$M(R) = \frac{4\pi}{3}q_{n\ell}^3(t_o)\rho(t_o) \tag{3.8}$$

where $q_{n\ell}$ is either given by (3.5) for $R > R_J$ or by (3.7) for $R < R_J$. $\rho_\ell(t_o)$ can be obtained using the known distribution $n_\ell(R)$ of loops

$$n_\ell(R) = \nu_\ell R^{-5/2} t_{eq}^{1/2} t_o^{-2} \quad (R < t_{eq}) \tag{3.9}$$

(ν_ℓ is a constant) and $M(R)$:

$$\rho_\ell(t_o) = \int_0^{t_o} dR n(R) M(R) \tag{3.10}$$

The integral is dominated by loops with $R \simeq R_J$. Since $R_J \ll t_{eq}$ we can use

(3.9) in the evaluation. The result is

$$\rho_\ell(t_o) = \frac{16}{15}\nu_\ell\mu\beta\left(\frac{9}{5}\beta G\mu\right)^{1/2}\left(\frac{z_{eq}}{v_{eq}}\right)^{3/2} t_o^{-2} \quad . \tag{3.11}$$

The ratio of (3.4) and (3.11) gives

$$\frac{\rho_w(t_o)}{\rho_\ell(t_o)} \simeq \frac{\nu_w}{\nu_\ell} v_s^2 v_{eq}^{3/2}\left(\frac{9}{5}\beta G\mu z_{eq}\right)^{-1/2}\beta^{-1} \quad . \tag{3.12}$$

Unfortunately, some of the constants which appear in (3.12) must be determined by numerical simulations, and the results on ν_ℓ are still very uncertain.[6-8]. We shall take $G\mu = 10^{-6}, \beta = 10, v_s^2 = \frac{1}{2}, v_{eq} = 0.05, \nu_w = 10$ and $\nu_\ell = \nu_{0.01}10^{-2}$. Then the ratio is

$$\frac{\rho_w(t_o)}{\rho_\ell(t_o)} \simeq \frac{1}{4}h_{50}^{-1}\nu_{0.01}^{-1} \quad . \tag{3.13}$$

Independent of the specific values of the constants we may conclude that wakes are much more important with hot dark matter than with cold dark matter. For the values of the cosmic string parameters chosen above wakes are almost as important as loops. The model predicts that a large fraction of the galaxies should be concentrated in sheets of length and width $40\,h_{50}^{-2}\,Mpc$, thickness $\sim 2\,Mpc$ and mean separation $\sim 20h_{50}^{-2}\,Mpc$.

From (3.11) it follows that for the values of the cosmic string parameters used above, the fraction $\Omega_{n\ell}$ of the total matter which has gone nonlinear by the present time is less than one. Thus, the model has the potential to explain why the dynamical measurements of mass in clusters give $\Omega < 1$ in a flat universe.

4. CONCLUSIONS

We have studied the accretion of hot dark matter onto sheets formed in the wakes of long strings moving with relativistic speed. The method used is adaptation of the Zel'dovich approximation to hot dark matter. Although the method is not at all rigorous, it manifests in a clear way most of the physics of the problem and the results derived here agree to within a factor of 3 with those obtained[16] using the Gilbert equation approach.

The cosmic string model with hot dark matter predicts that a substantial fraction of the nonlinear mass in the universe should be in sheets of length width $40\,h_{50}^{-2}\,Mpc$ and thickness $2\,Mpc$. The mean separation of the sheets is also of the order $40\,Mpc$. The above numbers depend slightly on parameters which must be determined from cosmic string simulations. The mean separation of the sheets is reduced by the cube root of the number of long strings which cross a horizon volume, and the length of the wake depends linearly on the ratio of the curvature radius of a typical long string to the horizon length. It is tempting to relate this to the recent observations of large-scale structure.[14,15].

The predictions of this model on the scale of galaxies and clusters are no longer straightforward. In particular, the usual[25] one loop-one object relation breaks down. Predictions for the cluster correlation function[26] have to be reconsidered. On even smaller scales, that of dwarf spheroidals, the model based on cosmic strings and neutrinos could be seriously flawed. The Gunn-Tremaine[27] phase space constraints imply that the dark matter in dwarf spheroidals cannot consist of neutrinos with a mass $25\,h_{50}^{2}\,eV$.[28] However, since it follows from nucleosynthesis constraints[17] that some dark baryons must exist, the halos of dwarf spheroidals could be made up of dark baryons. One of the important issues to tackle in order to resolve the problem of the halo of dwarf spheroidals is to study the fragmentation of the baryonic component of the wakes.

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FIGURE CAPTIONS

*Fig.*1 Space around a long straight string (a) and the induced velocity perturbation created when the string is moving with velocity v_s.

*Fig.*2 Comoving height q above a string wake.

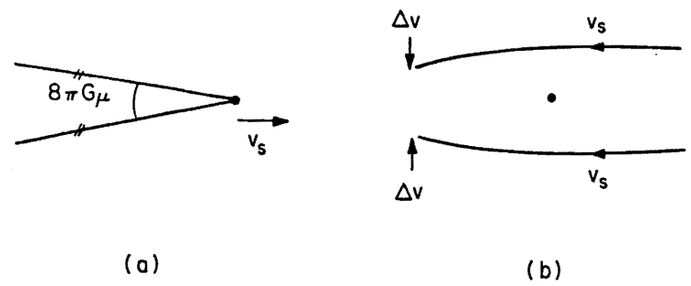

FIG. 1

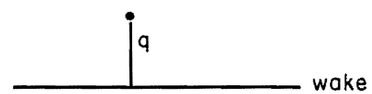

FIG. 2