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A modified Fritzsch ansatz with additional first-order perturbation

Dongsheng Du

Institute of High Energy Physics, P.O.Box 918(4), Beijing 100039, China

Zhi-zhong Xing

CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China; and

Institute of High Energy Physics, P.O.Box 918(4), Beijing 100039, China[1]

Abstract

In view of the current experimental indications that the top quark mass m_t is likely to exceed the upper bound predicted by the Fritzsch ansatz for the quark mass matrices, we modify the mass matrices to a new form in which the diagonal elements for both the charm and strange quarks may be nonzero as the additional first-order perturbation. Our ansatz can make the upper bound of m_t *twice* as much as that predicted by the Fritzsch ansatz. The magnitudes of the Kobayashi-Maskawa matrix elements can be restricted very well by the quark masses, and some relations such as $|V_{ub}/V_{cb}|^2 \approx m_u/m_c$, $|V_{td}/V_{ts}|^2 \approx m_d/m_s$, and $|V_{ub}| \approx |V_{ts}|$ can be obtained theoretically in a better accuracy.

[1]Mailing address.

1 Introduction

In the standard electroweak model, the family structure of Yukawa couplings is not constrained by gauge invariance so that the values of the fermion masses and the flavor mixing parameters are completely model-undetermined. In trying to understand this intriguing puzzle, various forms of mass matrices have been proposed to deal with quark mixing and CP violation in the electroweak interactions [1]. Among those suggested ansatze, the most popular and economical one is the Fritzsch ansatz [2] where the masses of the heavy quarks t and b are introduced initially, and the masses of the light quarks u, d, s, and c are generated by weak-interaction mixing. Unfortunately, the phenomenological prediction of the Fritzsch ansatz to the top quark mass m_t [3] is below or at best near to the lower bound of m_t indicated by the current experimental data [4]. This implies that the Fritzsch approach should be either abandoned or modified to fit our better experimental knowledge of the Kobayashi-Maskawa (KM) matrix and the top quark mass. Recently, three different phenomenological approaches have been attempted for the modification of the Fritzsch mass matrices $M_F^{(u)}$ and $M_F^{(d)}$ [5-8]. The first is to treat the Fritzsch ansatz as the tree approximation resulting from a spontaneously broken symmetry of the lagrangian, and to take into account the corresponding radiative corrections so as to make the ansatz compatible with all the present data [5]. Another approach is to introduce two additional nonzero off-diagonal elements into the original Fritzsch matrices $M_F^{(u,d)}$ so that the generalized mass matrices can serve as a full realization of the idea that the lighter quarks in each charge subsector get masses through mixing [6,7]. However, this treatment fails to lead to an appreciable increase in the upper bound of the top quark mass [7]. The third approach, examined by Gupta and Johnson [8], is to introduce an additional nonzero diagonal element for the charm quark c into the Fritzsch matrix $M_F^{(u)}$ based on the fact that the mass of the c quark is much larger than those of the u, d, and s quarks, but comparable with that of the b

quark [9]. They obtain $m_t^{phys} \leq 170$ GeV by comparing their theoretical results for the KM matrix elements with the relevant experimental data [10].

Here, we also follow a phenomenological approach to present a modified form of the Fritzsch mass matrices, in which the diagonal elements for both the charm and strange quarks may be nonzero as the additional first-order perturbation. In such a treatment, the masses of the heavy t and b quarks might still be regarded as the " driving terms " , and the masses of the light u and d quarks as the second-order perturbation which pick up values through mixing of the first and second families. The intermediate masses of the c and s quarks, regarded as the additional first-order perturbative terms, might not gain their values fully via mixing of the second and third families since they are not too small compared with the t and b quarks. Indeed, there is a simple but nontrivial reason which motivates one to improve and develop Gupta and Johnson's work by introdusing an additional nonzero diagonal element for the s quark into $M_F^{(d)}$. On one hand, the masses m_t and m_b set the scales for $M_F^{(u)}$ and $M_F^{(d)}$, respectively. On the other hand, the ratio m_s/m_b is larger than the ratio m_c/m_t in view of the existing lower bound of m_t given by the experimental data [4]. Hence it is more natural and reasonable to introduce both the nonzero diagonal element for the c quark into $M_F^{(u)}$ and that for the s quark into $M_F^{(d)}$ for the modification of the Fritzsch ansatz. Note that this treatment includes the same number of parameters as that with full off-diagonal mixing [6,7], so that it is also worth looking at.

Of course, it will be nice to justify the Fritzsch ansatz and its generalized forms by dynamical principles. Some attempts have been made for this purpose in the previous literatures [1]. Instead of trying to construct a model giving rise to the modified Fritzsch mass matrices in a natural way, here we are going to explore the consequences of our ansatz on the flavor mixing and the top quark mass. A remarkable result is that our ansatz can make the upper bound of m_t twice as much as that predicted by the original Fritzsch ansatz. We find that the magnitudes of the KM matrix elements can be restricted very well by the quark masses, and the interesting relations $|V_{ub}/V_{cb}|^2 \approx m_u/m_c$, $|V_{td}/V_{ts}|^2 \approx m_d/m_s$, and $|V_{cb}| \approx |V_{ts}|$ can be obtained theoretically in a better accuracy. Therefore, we expect that our generalized Fritzsch ansatz might provide some clues to the origin of the fermion mass matrices and be helpful in constructing models of the Higgs-boson couplings of quarks, which are currently of much experimental and theoretical interest.

2 The modified Fritzsch ansatz and the KM matrix

Our interest is focused on the following form of the modified Fritzsch mass matrices with three families of quarks:

$$M = \begin{pmatrix} 0 & xe^{i\alpha} & 0 \\ xe^{-i\alpha} & w & ye^{i\beta} \\ 0 & ye^{-i\beta} & z \end{pmatrix}, \tag{1}$$

where the real parameters x, y, z, w and the phases α, β differ for $M^{(u)}$ and $M^{(d)}$ matrices. In Eq.(1),we have required that the diagonal elements corresponding to the light u and d quarks are vanishingly small. Because the masses of the heavy t and b quarks set the scales of $M^{(u)}$ and $M^{(d)}$, respectively, we can require further that the diagonal elements corresponding to t and b are approximately equal to the observed values of their masses:

$$z^{(u)} \approx m_t, \qquad z^{(d)} \approx m_b. \tag{2}$$

It should be noted that the mass matrix M can yield $z^{(u)} \approx m_t$ and $z^{(d)} \approx m_b$ naturally only if $x \ll y \ll z$ and $w \ll z$. Based on the idea that the intermediate masses of the c and s quarks might not pick up values fully through mixing of the second and third families, we can treat $w^{(u)}$ and $w^{(d)}$ as the first-order perturbative variables as $y^{(u)}$ and $y^{(d)}$ in the corresponding $M^{(u)}$ and $M^{(d)}$ matrices, and restrict their values in the following ranges:

$$0 \leq w^{(u)} \leq m_c, \qquad 0 \leq w^{(d)} \leq m_s. \tag{3}$$

It can be seen later on that the nonzero w has little effect on the scale term z and the second-order perturbative term x in the mass matrix M, but affects the first-order perturbative term y.

In order to evaluate the KM matrix, one diagonalizes the quark mass matrix M through the following unitary transformation:

$$M = UM_{diag}U^{+} \tag{4}$$

with

$$M_{diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & -m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{5}$$

Here m_1, m_2 and m_3 correspond to m_u, m_c, m_t for $M^{(u)}_{diag}$, and m_d, m_s, m_b for $M^{(d)}_{diag}$, respectively. In view of our knowledge of the quark masses [11] , we retain only the leading powers of $m_1/m_2, m_1/m_3, m_2/m_3$, and w/m_3, and obtain

$$\begin{aligned} x &= \left(\frac{m_1 m_2 m_3}{m_1 - m_2 + m_3 - w}\right)^{1/2} \approx (m_1 m_2)^{1/2}, \\ y &= \left(-x^2 + zw + m_1 m_2 + m_2 m_3 - m_1 m_3\right)^{1/2} \approx [(m_2 + w)m_3]^{1/2}, \\ z &= m_1 - m_2 + m_3 - w \approx m_3. \end{aligned} \tag{6}$$

Thus the matrix U can be approximately given by

$$U = \begin{pmatrix} 1 & -\left(\frac{m_1}{m_2}\right)^{1/2} & \left[\frac{m_1 m_2 (m_2+w)}{m_3^3}\right]^{1/2} \\ \left(\frac{m_1}{m_2}\right)^{1/2} e^{-i\alpha} & e^{-i\alpha} & \left(\frac{m_2+w}{m_3}\right)^{1/2} e^{-i\alpha} \\ -\left[\frac{m_1(m_2+w)}{m_2 m_3}\right]^{1/2} e^{-i(\alpha+\beta)} & -\left(\frac{m_2+w}{m_3}\right)^{1/2} e^{-i(\alpha+\beta)} & e^{-i(\alpha+\beta)} \end{pmatrix}. \tag{7}$$

The flavor mixing matrix

$$V_{KM} \equiv U^{(u)+}U^{(d)} \tag{8}$$

is then obtained as follows

$$V_{KM} = \begin{pmatrix} 1 & -B + Ae^{i\phi_1} & Ae^{i\phi_1}(\xi D - \zeta C e^{i\phi_2}) \\ -A + Be^{i\phi_1} & e^{i\phi_1} & e^{i\phi_1}(\xi D - \zeta C e^{i\phi_2}) \\ Be^{i\phi_1}(\zeta C - \xi D e^{i\phi_2}) & e^{i\phi_1}(\zeta C - \xi D e^{i\phi_2}) & e^{i(\phi_1+\phi_2)} \end{pmatrix}. \tag{9}$$

where

$$A=\left(\frac{m_u}{m_c}\right)^{1/2},\quad B=\left(\frac{m_d}{m_s}\right)^{1/2},\quad C=\left(\frac{m_c}{m_t}\right)^{1/2},\quad D=\left(\frac{m_s}{m_b}\right)^{1/2},$$
$$\zeta=\left(\frac{m_c+w^u}{m_c}\right)^{1/2},\quad \xi=\left(\frac{m_s+w^d}{m_s}\right)^{1/2},\tag{10}$$

and

$$\phi_1=\alpha^u-\alpha^d,\quad \phi_2=\beta^u-\beta^d.\tag{11}$$

In obtaining Eq.(9),we have kept only the leading term of every matrix element [12]. To transform V_{KM} in Eq.(9) into a simpler form, we make the following successive rotations of the quark fields:

$$\begin{aligned}&(1)\quad c\longrightarrow ce^{i\phi_1},\quad t\longrightarrow te^{i(\phi_1+\phi_2)},\\&(2)\quad u\longrightarrow ue^{i(\phi_1-\phi)},\quad d\longrightarrow de^{i(\phi_1-\phi)},\\&(3)\quad t\longrightarrow te^{i\theta},\quad b\longrightarrow be^{i\theta},\end{aligned}\tag{12}$$

where

$$\begin{aligned}\phi&=\arctan\left[\frac{\sin\phi_1}{\cos\phi_1-\frac{A}{B}}\right],\\ \theta&=\arctan\left[\frac{\sin\phi_2}{-\cos\phi_2+\frac{\xi D}{\zeta C}}\right].\end{aligned}\tag{13}$$

Accordingly, we obtain

$$V_{KM}=\begin{pmatrix}1-\frac{1}{2}\eta^2 & \eta & A\rho e^{i\phi}\\ -\eta & 1-\frac{1}{2}\eta^2 & \rho\\ -B\rho e^{i(\phi_1-\phi)} & -\rho & 1\end{pmatrix},\tag{14}$$

where

$$\begin{aligned}\eta&=(A^2+B^2-2AB\cos\phi_1)^{1/2},\\ \rho&=(\zeta^2C^2+\xi^2D^2-2\zeta\xi CD\cos\phi_2)^{1/2},\end{aligned}\tag{15}$$

and we have added the next-to-leading terms in the diagonal elements V_{ud} and V_{cs} by using the unitary condition.

Clearly, the form of V_{KM} in Eq.(14) is quite similar to that parametrized by Wolfenstein [13]. To some extent, this similarity might imply the reasonableness of the quark mass matrix M given in Eq.(1), and one may be interested in speculating the underlying physics which gives rise to M. In the next section, we will explore some consequences of our modified Fritzsch ansatz by means of Eqs.(1) and (14).

3 Predictions and discussion

Except the top quark mass, there still exist four unknown parameters in the KM matrix V_{KM} (see Eqs.(13-15)): $\phi_1, \phi_2, w^{(u)}$ and $w^{(d)}$, originating from the proposed mass matrices $M^{(u)}$ and $M^{(d)}$. Here we do not want to allow those unknown parameters to vary numerically so as to achieve close agreement between the theoretical results and the experimental determined central values of the magnitudes of the KM matrix elements [8]. Instead of quantitative evaluation, we use our ansatz to restrict the magnitudes of the nine KM matrix elements, to give an upper bound of the top quark mass, and to look at the rephasing-invariant measure of CP violation.

According to our argument that the diagonal element $w^{(u)}$ ($w^{(d)}$) may vary from zero to m_c (m_s) in the modified mass matrix $M^{(u)}$ ($M^{(d)}$), we obtain from Eqs.(3) and (10) that

$$1 \leq \zeta \leq \sqrt{2}, \quad 1 \leq \xi \leq \sqrt{2}. \tag{16}$$

For various possible values of the unknown phases ϕ_1 and ϕ_2, the ranges of η and ρ in Eq.(15) are limited by

$$\begin{aligned} B - A \, \leq \, \eta \, \leq \, B + A \, , \\ D - \sqrt{2}C \, \leq \, \rho \, \leq \, \sqrt{2}(D + C) \, . \end{aligned} \tag{17}$$

In obtaining the lower bound of ρ in Eq.(17), we have taken $\zeta = \sqrt{2}$ and $\xi = 1$, in view of the fact $m_t^{\rm phys} > 89$ GeV and $C < D$ [4,11], to ensure sufficient cancellation between C and D. Using Eqs.(14-17), the magnitudes of the KM matrix elements can be approximately restricted by the quark masses as follows:

$$\begin{aligned}
&1-\frac{m_u}{m_c}-\frac{m_d}{m_s}-2\left(\frac{m_u m_d}{m_c m_s}\right)^{1/2} \leq |V_{ud}| \approx |V_{cs}| \leq 1-\frac{m_u}{m_c}-\frac{m_d}{m_s}+2\left(\frac{m_u m_d}{m_c m_s}\right)^{1/2}, \\
&\left(\frac{m_d}{m_s}\right)^{1/2}-\left(\frac{m_u}{m_c}\right)^{1/2} \leq |V_{us}| \approx |V_{cd}| \leq \left(\frac{m_d}{m_s}\right)^{1/2}+\left(\frac{m_u}{m_c}\right)^{1/2}, \\
&\left(\frac{m_s}{m_b}\right)^{1/2}-\sqrt{2}\left(\frac{m_c}{m_t}\right)^{1/2} \leq |V_{cb}| \approx |V_{ts}| \leq \sqrt{2}\left[\left(\frac{m_s}{m_b}\right)^{1/2}+\left(\frac{m_c}{m_t}\right)^{1/2}\right], \\
&\frac{|V_{ub}|}{|V_{cb}|} \approx \left(\frac{m_u}{m_c}\right)^{1/2}, \\
&\frac{|V_{td}|}{|V_{ts}|} \approx \left(\frac{m_d}{m_s}\right)^{1/2}, \\
&|V_{tb}| \approx 1.
\end{aligned} \tag{18}$$

Three features should be noted in the above expressions.

(i) The magnitudes of V_{us} and V_{cd} (V_{ud} and V_{cs}), which correspond to the flavor mixing between (within) the first and second quark families, are mainly limited by m_u, m_d, m_c and m_s. They are approximately independent of the masses of the third quark family, m_t and m_b.

(ii) The magnitudes of V_{cb} and V_{ts}, which correspond to the flavor mixing between the second and third quark families, are mainly determined by m_c, m_s, m_t and m_b. They are approximately independent of the masses of the first quark family, m_u and m_d.

(iii) The ratios $|V_{ub}/V_{cb}|^2$ and $|V_{td}/V_{ts}|^2$ are approximately equal to m_u/m_c and m_d/m_s, respectively [9,14]. They are almost irrelevant to the corresponding heavy quark masses m_b and m_t.

It is remarkable that Eq.(18) can straightforwardly yield the upper bound of the top quark mass:

$$m_t \leq 2m_c\left[\left(\frac{m_s}{m_b}\right)^{1/2}-|V_{cb}|\right]^{-2}. \tag{19}$$

This result shows that our modified ansatz can make the upper bound of m_t twice as much as that predicted by the original Fritzsch ansatz [3]. The coefficient 2 in Eq.(19) come out under the condition $w^{(u)} = m_c$ and $w^{(d)} = 0$. This means that in our generalized quark mass matrices the maximal value of the upper bound of m_t can be obtained when the diagonal elements corresponding to the light quarks u, d, and s are vanishingly small, while those corresponding to the heavy quarks c, b, and t are approximately equal to the observed values of their masses. This special situation has just been chosen by Gupta and Johnson [8]. Here we get it in a more general and natural way. Using the current values $|V_{cb}| = 0.030 - 0.058$ [10], m_c(1 GeV)=1.35 ± 0.05 GeV and $m_s/m_b = 0.033 \pm 0.011$ [11], and transforming m_t(1 GeV) into $m_t(m_t)$ [15], we obtain the approximate value of the upper bound of the top quark mass as follows:

$$m_t^{phys} \leq 190 \text{ GeV}. \tag{20}$$

Of course, this result increases the previous values of the upper bound of m_t predicted in Refs.[3,6,7].

Finally, let us look at the rephasing-invariant measure of CP violation [16]:

$$J \equiv |\mathrm{Im}(V_{ij} V_{kl} V_{il}^* V_{kj}^*)|. \tag{21}$$

Using our flavor mixing matrix $V_{\rm KM}$ with leading-order elements in Eq.(14), we obtain

$$J \approx AB\rho^2 \sin\phi_1. \tag{22}$$

Note that our CP-violating phase factor ($e^{i\phi_1}$) is associated with the second-order perturbative terms in the quark mass matrices $M^{(u)}$ and $M^{(d)}$. Thus CP violation might be vanishingly small if m_u and m_d tend to be zero or α_u is equal to α_d. A similar result has also been obtained in Ref.[9] with more detailed arguments. As a result, the maximal value of J in Eq.(22) is given by

$$J_{max} \approx 2AB(C+D)^2, \tag{23}$$

which is of order $10^{-4} - 10^{-3}$ for current values of the quark masses [4,11].

4 Summary

We have followed a phenomenological approach to find a modified form of the Fritzsch quark mass matrices to deal with the flavor mixing and CP violation in the electroweak interactions. Based on the idea that the intermediate masses of the c and s quarks might not pick up values fully via mixing of the second and third families, we have introduced two nonzero diagonal elements for the c and s quarks into the original Fritzsch matrices as the additional first-order perturbation. Our ansatz leads to an interesting Wolfenstein pattern of the flavor mixing matrix, and makes the upper bound of the top quark mass twice as much as that predicted by the Fritzsch ansatz. In addition, the magnitudes of the KM matrix elements can be restricted very well by the quark masses, and some relations such as $|V_{ub}/V_{cb}|^2 \approx m_u/m_c$ and $|V_{td}/V_{ts}|^2 \approx m_d/m_s$ are obtained in a better accuracy. We have also looked at the rephasing-invariant measure of CP violation using our mass matrices.

In conclusion, our modified Fritzsch ansatz for the quark mass matrices is in agreement with the present experimental knowledge on the KM matrix and the top quark mass. No doubt, any simple ansatz (containing only a few free parameters), which can account for the observed systematics of fermion masses, is useful in order to find clues of the origin of the fermion mass matrices. We are looking forward to obtaining more precise experimental data to examine our phenomenological model.

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