Fermilab Library
0 1160 0022112 2

BIHEP-TH-92-57

IHEP-TH-92-57

The Color Screening Effects in Charmonium*

Dong Yubing, Yu Youwen and Wang Yingcai

Institute of High Energy Physics, Academia Sinica, Beijing, China

Beijing, Inst. High Energy Phys.

Abstract

In this work, by taking the confining potential with color screening effect to be a confining quark-antiquark potential, its effects on heavy quarkonia $c\bar{c}$ and $b\bar{b}$ are studied. In comparison with usual linearly confining potential this one may give some different predictions for orbital angular momentum of high-lying energy levels of $\psi(c\bar{c})$ and improve the widths for leptonic decay mode.

*Work supported by the National Natural Science Foundation of China, and through funds LWTZ-1298 provided by the Academia Sinica

Conventional potential model calculations for heavy quarkonia (the $c\bar{c}$ and $b\bar{b}$ systems) have been performed mainly on the basis of linearly confining potential. In 1986 E. Laermann et al.[1] in their lattice gauge calculations, by taking into account the contribution from first-order loop diagram of fermion they found that at large distances the quark- antiquark potential appreciably deviates downwards from the linear rise. This result indicates that when distances between valence quarks are large the interaction between quark sea and valence quark is clearly enhanced, and the color confinement for valence quarks is reduced, this is known as color screening effect.

Laermann et al. gave a phenomenological form containing an exponential function $V_\mu(1-e^{-\mu_0 r})$ to describe this kind of confining potential with the color screening effect. One can expand it in powers of r, up to the second order this expansion is expressed as $ar - br^2 + ...$ which has a linear behaviour when r is very small, and it tends to a constant as r is large enough. In fact, at present, the confining potential is still less well understood theoretically and there no its accurate form has been given. Therefore, it could be viewed as a useful attempt if one uses a phenomenological potential with the color screening effect to study heavy meson spectrum. It is clear that confining potential including the color screening effect may not be limited to the form quoted above. For example, in Ref.[2] we proposed such type confining potential in a form of the usual error function $V_\mu erf(\mu_0 r)$. At small distances its expansion can be expressed as a combination of first-order and third-order terms of r, namely $a'r - b'r^3 + ...$; with increasing r it approaches a constant. In this work we have calculated energy spectra for heavy quarkonia $c\bar{c}$ and $b\bar{b}$ by using both phenomenological forms of confining potentials above mentioned. It is found that the qualitative features obtained in these two cases are essentially identical. In calculation for vector coupling quark-antiquark (quark) interaction, some authors[3] have taken into account condensations of quark-antiquark pairs and gluons. In their interaction forms, in addition to a term proportional to 1/r, there were also the first-and third-

order terms of r and these two terms were opposite in sign. From here we have got an inspiration which allows us to have the potential form with error function be a preferable choice, that is, we now use an error function form to describe the confining potential with color screening effect. Theoretical values which will be displayed below are just the results calculated by employing this kind of potential.

It should be pointed out especially that the fact the confining potential tends to a constant at large distances does not mean that at that time a quark would be able to escape from a confined color-singlet cluster and hence the color might become visible. Its physical picture could be understood in such a way that a pair of valence quark and antiquark confined in a color-singlet cluster $(q_1\bar{q}_2)_{00}$ interact with quark q_3 and antiquark $\bar{q}_4$ from quark sea, and then they form two new color-singlet clusters, i.e., a decay process occurs there $(q_1\bar{q}_2)_{00} \rightarrow (q_1\bar{q}_4)_{00}(q_3\bar{q}_2)_{00}$. As a result, at this very moment the distance between q_1 and $\bar{q}_2$ might become larger than it was before and therefore they would not interact with each other anymore. As first step of our study, for simplicity such decay process will not be taken into consideration in this work. Replacing linearly confining potential generally accepted in potential model calculations by a phenomenological confining potential with color screening effect we solve Schrodinger equation satisfied by the quark-antiquark relative motion for the purpose of studying its effects on heavy quarkonium structure.

In order to explore main features of color screening effects on some physical properties we don't consider any other corrections here for the time being, and therefore the Hamiltonian of the quark-antiquark system $(q_1\bar{q}_2)_{00}$ can be approximately written as

$$H = B_0 + m_1 + m_2 + H_0 + H_I \tag{1}$$

$$H_0 = -\frac{1}{2\mu}\nabla_r^2 - \frac{4\alpha_s}{3r} + \frac{32\pi\alpha_s}{9m_1m_2}\delta^3(r)\vec{S}_1\cdot\vec{S}_2 + V^{conf} \tag{2}$$

$$H_I = \frac{4\alpha_s}{3m_1m_2r^3}(3\vec{S}_1\cdot\vec{r}\vec{S}_2\cdot\vec{r} - \vec{S}_1\cdot\vec{S}_2) + \frac{1}{2m_1m_2r}(\frac{4\alpha_s}{r^2} - \frac{d}{dr}V^{conf})\vec{L}\cdot\vec{S} \tag{3}$$

$$V^{conf} = V_\mu \, erf(\mu_0 r) \tag{4}$$

where B_0 is zero-point energy of the system, m_1 and m_2 are the quark and antiquark masses, respectively, μ is their reduced mass, α_s is the quark-gluon coupling constant, V_μ is the confining strength, μ_0^{-1} is called the color screening constant and r is the relative distance between q and $\bar{q}$. To start with, we solve Schrodinger equation $H_0\psi = E\psi$ exactly and treat H_I as first-order perturbation. In practical solution of this problem, as in Ref.[4] we replace the $\delta^3(r)$ in H_0 by its approximated function

$$f(r) = \frac{1}{4\pi \, r \, r_0^2} e^{-r/r_0} \, , \qquad \lim_{r_0 \to 0} f(r) = \delta^3(r) \tag{5}$$

where r_0 is phenomenologically chosen as

$$r_0 = 0.08 \, m^{-0.6} (fm) \, , \qquad (m \;\; in \;\; GeV); \;\; m = 2\mu \tag{6}$$

In calculations there are five parameters: $\alpha_s, m_q, \mu_0, V_\mu$ and B_0. for a given choice of parameters α_s, m_q, μ_0 and V_μ the B_0 is determined so that the evaluated 1 $^3S_1$ $c\bar{c}$ energy fits the measured J/ψ mass (3097 MeV) and the 1 $^3S_1$ $b\bar{b}$ value fits Υ mass (9460 MeV), respectively; other parameters are adequately adjusted by the requirement that one can get good coincidence of energy levels with the experimental data.

Fig.1 presents a comparison of theoretical calculations for $c\bar{c}$ energy spectrum with the experiment. The solid lines represent experimental data, the dashed lines denote theoretical values for parameter set I while dot-dashed lines are theoretical ones for set II. These two sets of parameters are chosen as

$$\begin{aligned} Set \;\; I \; : \;\; M_c &= 1.3 \, GeV, \;\; \mu_0^{-1} = 2.0 fm, \;\; V_\mu = 1.6 \, GeV, \;\; \alpha_s = 0.39 \; ; \\ Set \;\; II \; : \;\; M_c &= 1.3 \, GeV, \;\; \mu_0^{-1} = 2.0 fm, \;\; V_\mu = 1.6 \, GeV, \;\; \alpha_s = 0.50 \; . \end{aligned} \tag{7}$$

Symbols nL for each energy level denote the nth state with angular momentum of relative motion L. Judging from the locations of energy levels one can see that both

calculated results are in good agreement with experiment, each experimental level can be in correspondence with a theoretical value calculated by using each set of parameters. But there is a difference between sets I and II in expecting the values of nL for very few energy levels.

In comparison with results obtained by using linearly confining potential with some relativistic corrections the calculated results in this work significantly differ from those on the problems of nL identification for high-lying energy level and the widths for leptonic decay mode, but they are close to each other in other aspects. Linearly confining potential can only give a result similar to that obtained in this work by using parameter set II. In contrast, our confining potential with color screening effect via different choices of parameters V_μ, μ_0^{-1} and α_s could yield two distinct nL identifications for some individual energy levels. Hereafter we shall concentrate our discussions on the differences between our results and those of linearly confining potential, namely the problem of orbital momentum L identification for $J^{PC} = 1^{--}$ energy levels of $c\bar{c}$ and the widths of its leptonic decays. It should be noted that we have also made calculations corresponding to another choice of quark mass $M_c = 1.8GeV$, but we found that the physical results are quite similar. Therefore, in this paper only results for $m_c = 1.3GeV$ will be reported.

Table 1 presents comparison between theory and experiment for $J^{PC} = 1^{--}$ energy levels of charmonium. Six experimental energy levels are listed in the first column, among them only two lowest ones have been experimentally identified as states with orbital angular momentum S, but others are still uncertain. Results calculated on basis of linearly confining potential[5] are given in second and third columns. One can see that the locations of energy levels are reproduced well, but in all cases (even with various corrections) the level $\psi(4160)$ has been interpreted as 2D-state and level $\psi(4415)$ as 4S-state. It seems that this is the consequence of employing linearly confining potential. Things will be different if this type potential is replaced by a

confining potential with color screening effect in the form of error function. Results given by the latter potential with two different choices of parameters (set I and II) are plotted in the 4th and 5th columns, respectively. From the results evaluated with our parameter set I we notice that due to color screening effects this type confining potential becomes flat gradually as r increases, and hence may reduce the energies of high-lying states. For instance, by a proper choice of parameters one may get five S-wave solutions below 4.41 GeV. Although calculated energy level of state $\psi(2D)$ lies around 4.07 GeV, yet from point of being consistent with measured width for leptonic decay it would be more reasonable to explain $\psi(4160)$ as state $\psi(4S)$. By varying parameters properly (e.g., increasing the parameter α_s in set II) one can change the Coulomb term to a great extent, and thus there only four S-wave states below 4.40 GeV will appear in the solution. In this case, $\psi(4160)$ would be interpreted as 2D-state and $\psi(4415)$ as the 4S-state just as identified in calculations with linearly confining potential.

Of course, how to explain the angular momentum of these energy levels will remain to be solved by the experimental measurements. However, we may still get some information from experiment for the leptonic decays of $J^{PC} = 1^{--}$ charmonium states. Table 2 shows a comparison of theoretical values with experimental data for e^+e^- decay widths of $J^{PC} = 1^{--}$ charmonium states. In this table there are given absolute values as well as relative ratios of the decay widths. To date theoretical calculations for widths themselves still have a certain inaccuracy, for this reason the relative ratio might be regarded as a more reliable physical quantity. Experimental values of six energy levels and their leptonic decay widths as well as the relative ratios are presented in the left columns of Tab.2. Theoretically, the partial width for decay of nth S-state into e^+e^- is represented approximately by

$$\Gamma_{nS}(e^+e^-) = \frac{16\pi e_q^2 \alpha^2}{M_{nS}^2} \mid \psi_{nS}(0) \mid^2 \tag{8}$$

where M_{nS} is the mass of corresponding heavy quarkonium, e_q is effective charge of

the quark (in unit of electron charge), α is fine structure constant, $\psi_{nS}(0)$ is the wave function of nS-state at the origin. The values of Γ_{nS} evaluated according to above formula in cases of parameter set I and II are shown in the middle of this stable. The data plotted in its right columns are the results obtained by A. Barchielli et al.[5] using relativistically corrected $q\bar{q}$ potential with two choices of their parameters. One can see from these data that with regard to the linearly confining potential, when energy level $\psi(4415)$ is explained as 4S-state the deviation of theoretical width for decay to e^+e^- from the experimental one is large, even though some relativistic corrections to the potential have been considered. Furthermore, the measured widths of e^+e^- decay for $\psi(4040)$ and $\psi(4160)$ are basically the same, but theoretically the leptonic (e^+e^-) decay width for D-wave state is smaller than that for S-wave state by almost an order of magnitude. For this reason, a serious discrepancy between theory and experiment in the problem of leptonic decay will occur if energy level $\psi(4160)$ is interpreted as 2D-state. This disagreement cannot be removed even by including the mixing of 2D-and 3S-states. However, if in a confining potential the color screening effect is taken into account the $\psi(4160)$ may be considered as 4S-state and $\psi(4415)$ as 5S-state in calculations with our parameters set I, and in this way the problem of discrepancy in the e^+e^- decay widths for $J^{PC} = 1^{--}$ energy levels could be well solved.

Our results show that the color screening effects may have at least two significant consequences for high-lying states: first, it reduces the energy of high level states; second, it increases the distribution of wave functions at larger distances and consequently decreases the values of wave functions at the origin, so that the leptonic decay widths could be much closer to the experimental value. This effect evidently appears even in our calculations with parameter set II. Even though $\psi(4415)$ is also interpreted here as 4S-state, its e^+e^- decay width still becomes much smaller than the results obtained by A. Barchielli et al., so an improvement has also been observed in this case.

We have also calculated corresponding energy spectrum for the bottomonium Υ. Because of larger mass of b-quark the color screening effect on the $b\bar{b}$ is not as remarkable as on $c\bar{c}$ system. It may make an improvement only in the width for leptonic decay of high excited states.

As discussed above, in the frame of nonrelativistic potential model we investigated basically two problems in which the color screening effects may explicitly manifest themselves, namely the identification for orbital momentum of $J^{PC} = 1^{--}$ quarkonium states and their leptonic decay widths. Obviously, the validity of our results remains to be tested by the experiment. More accurate theoretical calculations and their comparison with experimental data for other physical quantities of heavy quarkonium will be reported in future papers.

Authors would like to thank Professors Zhang Zongye, He Zuoxiu, Qing Chengrui and Shen Pengnian for their useful discussions.

References

[1] E. Laermann et al., Phys. Lett. B173(1986)437.

[2] Dong Yu-bing and Yu You-wen, IHEP-TH-92-27;
Zhang Zong-ye et al., IHEP-TH-92-28.

[3] Jian-guo Bian and Tao Huang, Commun. Theor. Phys. 16(1991)337; Shen Peng-nian et al., Commun. Theor. Phys. 16(1991)443.

[4] S. Ono and F. Schoberl, Phys. Lett. B118(1982)419.

[5] A. Barchielli et al., Nuovo Cimento 103A(1990)59; Yu- qi Chen and Yu-ping Kuang, Phys. Rev. D46(1992)1165;
Y. B. Ding, J. He, S. O. Cai, D. H. Qin and K. T. Chao, in "Particles and Nuclear Physics", Proceedings of the International Symposium, Beijing, China, 1985, edited by N. Hu and C.-S. Wu (World Scientific, Singapore, 1987), p88.

[6] Particle Data Group, Phys. Rev. D45, N0 11-II, (1992)1.

Fig.1. Charmonium Spectrum. Solid lines denote experimental values[6], dashed and dot-dashed lines represent theoretical ones calculated with set I and II of parameters, respectively.

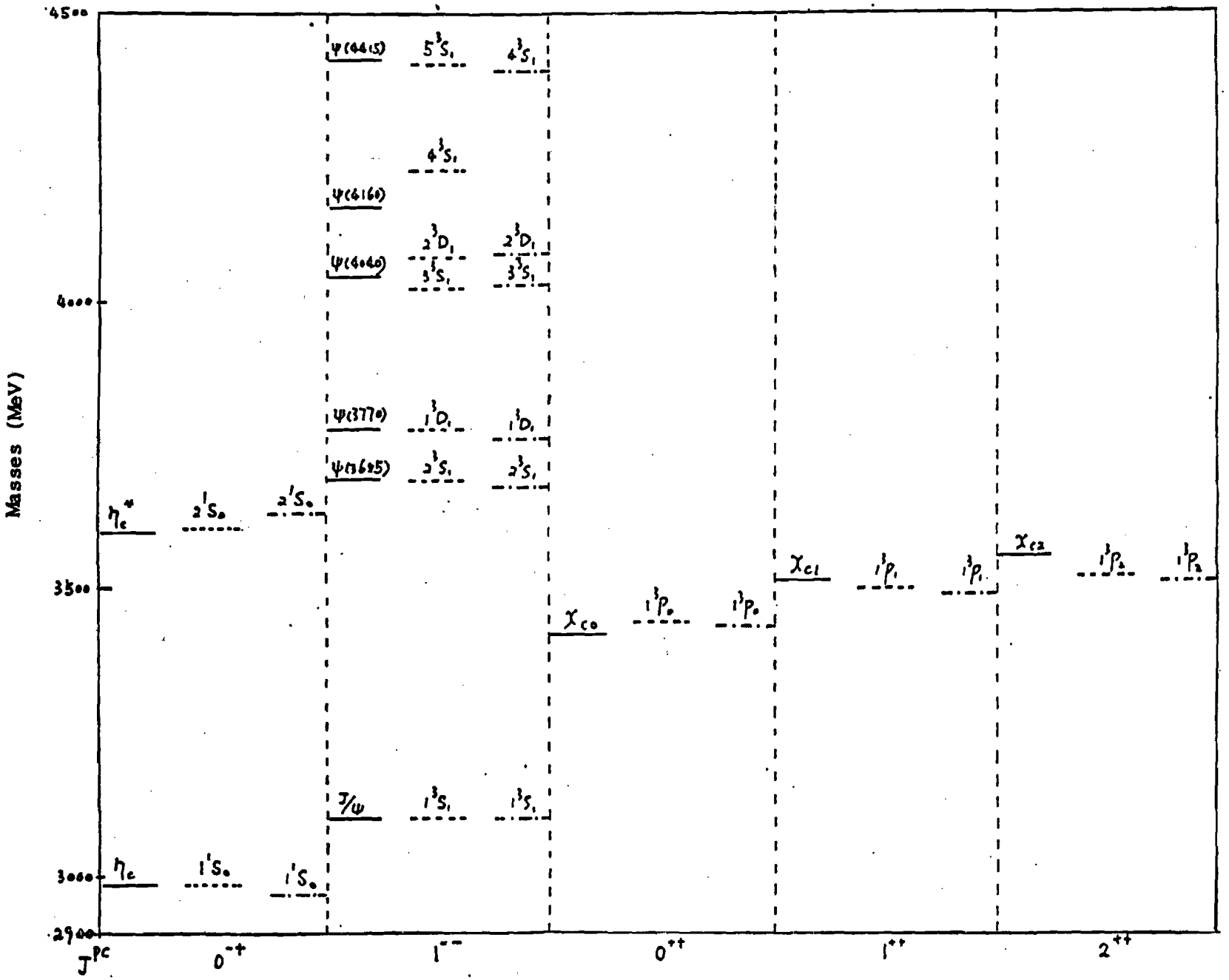

Table 1. Energy levels for $J^{PC} = 1^{--}$ states charmonium. (set I and II of parameters are specified in the text)

Experiment[6](MeV)	A.Barchielli[5]	Y-Q Chen et al.,[5]	$V_v \cdot erf(\mu_0 r)$	
			set I	set II
$J/\psi(1S) = J/\psi(3097)$	$J/\psi(1S) = J/\psi(3097)$	$J/\psi(1S) = J/\psi(3097)$	$J/\psi(1S) = J/\psi(3097)$	$J/\psi(1S) = J/\psi(3097)$
$\psi(2S) = \psi(3685)$	$\psi(2S) = \psi(3685)$	$\psi(2S) = \psi(3686)$	$\psi(2S) = \psi(3688)$	$\psi(2S) = \psi(3673)$
$\psi(3770)$	$\psi(1D) = \psi(3766)$	$\psi(1D) = \psi(3803)$	$\psi(1D) = \psi(3769)$	$\psi(1D) = \psi(3763)$
$\psi(4040)$	$\psi(3S) = \psi(4056)$	$\psi(3S) = \psi(4098)$	$\psi(3S) = \psi(4013)$	$\psi(3S) = \psi(4024)$
			$\psi(2D) = \psi(4069)$	
$\psi(4160)$	$\psi(2D) = \psi(4158)$	$\psi(2D) = \psi(4176)$	$\psi(4S) = \psi(4220)$	$\psi(2D) = \psi(4067)$
$\psi(4415)$	$\psi(4S) = \psi(4407)$	$\psi(4S) = \psi(4443)$	$\psi(5S) = \psi(4408)$	$\psi(4S) = \psi(4396)$

Table 2. Decay widths $\Gamma_{nS}(e^+e^-)$ and their relative ratios for $J^{PC} = 1^{-1}$ states charmonium (choices I and II of parameters are specified in the text)

Experimental values[6]			$V_\mu er\,f(\mu_0 r)$				A.Berchielli et al.,[5]			
			I		II		I		II	
Energy levels(MeV)	$\Gamma(e^+e^-)(KeV)$	$\Gamma_\psi/\Gamma_{J/\psi}$	$\Gamma_{nS}(e^+e^-)$	Γ_{nS}/Γ_{1S}	$\Gamma_{nS}(e^+e^-)$	Γ_{nS}/Γ_{1S}	$\Gamma_{nS}(e^+e^-)$	Γ_{nS}/Γ_{1S}	$\Gamma_{nS}(e^+e^-)$	Γ_{nS}/Γ_{1S}
$J/\psi = J/\psi(3097)$	$5.36\ ^{+0.29}_{-0.28}$		5.48(1S)		3.79(1S)		5.02(1S)		3.12(1S)	
$\psi(2S) = \psi(3685)$	2.14±0.27	0.40	2.45(2S)	0.45	1.70(1S)	0.45	2.40(1S)	0.48	1.65(2S)	0.53
$\psi(3770)$	0.24±0.05	0.04								
$\psi(4040)$	0.75±0.15	0.14	1.41(3S)	0.26	0.99(3S)	0.26	1.60(3S)	0.32	1.10(3S)	0.35
$\psi(4160)$	0.77±0.23	0.14	0.87(4S)	0.16						
$\psi(4415)$	0.47±0.1	0.09	0.55(5S)	0.094	0.49(4S)	0.13	1.24(4S)	0.25	0.892(4S)	0.29