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The Quarkonium Potential Within the Short and Intermediate Range in QCD*

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Abstract

A quark-antiquark potential is calculated by involving vacuum condensates up to dimension-6 in QCD in the background fields. The underlying assumption is that a gluon (quark) propagates, not in the empty space, but through the physics vacuum, filled with the background fields. The interactions of the gluon with the background fields manifest themselves as the corrections of vacuum condensates to the free gluon propagator. It is shown that these corrections extend the potential from the short distances to the intermediate range. Indeed, with some reasonable parameters, the resulting potential is similar to those popular phenomenological potentials in shape in the region $0.1 < r < 1$fm.

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I. Introduction

It is well known that a critical feature of QCD is asymptotic freedom. Hence the strong interaction becomes weak, and even perturbatively calculable with decreasing of distances[1]. At very short distances all perturbative calculations become extremely simple and one gluon exchange diagram dominates the hard processes. Experiments at large momentum transfer, both inclusive and exclusive processes, are consistent with the perturbative QCD.

Based on the one gluon exchange, it is easy to find the Coulomb potential is responsible for the quark interaction at short distances, i.e.

$$V(q) = \frac{\alpha_s(Q^2)}{Q^2} \quad , \tag{1}$$

where $\alpha_s(Q^2)$ is the running coupling constant,

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \; log \; \frac{Q^2}{\Lambda^2}} \quad . \tag{2}$$

However we are still far from being able to reach the interaction structure at large distances from QCD. Richardson suggested[2] an interplated potential which was a simple interpolation between the asymptotic freedom and confining regions by making a replacement

$$\alpha_s(Q^2) \rightarrow \frac{4\pi}{\beta \; log(1 + \frac{Q^2}{\Lambda^2})} \quad . \tag{3}$$

It is similar to the Coulomb-plus-linear potential proposed by Cornell group[3]. Obviously, one can not expect to get the equation (3) from the perturbative QCD, since it is a consequence of non-perturbative effects.

In order to consider non-perturbative effects we assume that the physical vacuum in QCD is greatly different from the perturbative vacuum[4]. The physical vacuum can be considered as a classical average effect and described by means of background field interactions[5]. Therefore, the quarks (gluons) propagate, of course, not in an empty

space, but through the physical vacuum, filled with the non-perturbative quark and gluon fields. These non-perturbative corrections are expressed through the vacuum expection values of Lorentz invariant composite operators, such as the quark condensate $< \Omega \mid \bar{\psi}\psi \mid 0 >$ and the gluon condensate $< \Omega \mid G^2_{\mu\nu} \mid \Omega >$. One can use the background fields to account for all those nonvanishing expectation values. The perturbative effects can be described by the quantum fluctuations. In previous papers we have built a framework for a quantum chromodynamics theory in the background fields[5]. The corresponding quantum fields are quantized in the Furry representation and the physical states are defined on the physical vacuum. We hope it can provide a way to extend the pure perturbative QCD theory to the nonperturbative range. Also we have discussed the quark and gluon propagators and the Bethe- Salpeter equation[6] by introducing the perturbative and the non-perturbative contributions in this framework.

In particular, the B-S equation and the Salpeter equation for the pseudoscalar mesons in the zero-order and the first-order approximation, where only the lowest-dimension gluon and quark condensate corrections are taken into account in the gluon propagator as the kernel, are solved[7,8]. The spectrum and decay constants for the pseudoscalar mesons are obtained. The predictions from these solutions are quite reasonable. It has been shown that there are three terms in the interaction kernel[8,9]. The first term is the Coulomb potential which is from the one gluon exchange diagram. The second term is mainly the linear potential which is induced by the correction of the quark condensate. The third term is a higher power potential r^3 which is induced by the correction of the gluon condensate. However the r^3 term has an opposite sign to the linear confinement term, and the resulting potential will fall down at the large distances. In fact, the potential is consistent with the Cornell model as $r < 0.6fm$ and it provides a good agreement with data for the ground states of the pseudoscalar mesons.

It should be noted that this approach is valid when the interacting distance is still small. The higher dimension vacuum condensates have to be taken into account if one wants to extend this approach to the larger range. However, the higher dimension condensates, such as $< \Omega \mid \bar{\psi}\psi\bar{\psi}\psi \mid \Omega >$, $< \Omega \mid G\,G\,G \mid \Omega >, \ldots$ will contribute the higher-power term, like r^3, $r^5, \ldots$. It is interesting for us to calculate the quark-antiquark potential in QCD in the background fields involving higher- dimension operators. In this paper we will calculate it involving vacuum condensates up to dimension-6 as r goes to 1 fm.

The reminder of this paper is organized as follows. In Sec. II we give the kernel of the Bethe-Salpeter equation which is determined by the one gluon exchange and vacuum condensates up to dimension-4 in the first order approximation. In Sec. III the quark-antiquark potential, which involves vacuum condensates up to dimension-6, is analyzed in the background field QCD. The final section is reserved for a summary and discussion.

II. The B-S Kernel and the Potential

In the previous papers we have derived the Bethe- Salpeter equation from the QCD in the background fields[6]. The equation can still be written into the homogeneous integral form,

$$\begin{aligned} \chi_{p\alpha}(x_1, x_2) &= -\int d^4y_1 d^4y_2 d^4y_3 d^4y_4 S_F(x_1 - y_1) S_F(x_2 - y_2) \\ &\quad \times G(y_1, y_2; y_4, y_3) \chi_{p\alpha}(y_3, y_4) \quad , \end{aligned} \tag{4}$$

as long as we appropriately define the complete quark propagator $S_F(x-y)$ and the kernel $G(y_1, y_2; y_4, y_3)$, which is determined by the QCD theory in the background fields. The kernel can be divided into two parts:

$$G(y_1, y_2; y_4, y_3) = G_p(y_1, y_2; y_4, y_3) + G_n(y_1, y_2; y_4, y_3) \quad , \tag{5}$$

where G_p is usually the perturbative contribution which represents the sum of all the irreducible perturbative graphs; and G_n represents the sum of all the irreducible graphs with vacuum condensates contributing to the four-point Green's function.

The effective QCD Lagrangian L_{eff} in background fields can be divided into two parts:

$$L_{eff} = L_0 + L_i \tag{6}$$

with

$$L_0 = \bar{\eta}(iD(A) - m)\eta + \frac{1}{2}\phi^a_\mu[g^{\mu\nu}D^2_{ac}(A) - (1 - \frac{1}{\alpha})(D^\nu(A)D^\nu(A))_{ac} + 2g f_{abc}G^{\mu\nu}_b]\phi^c_\nu \tag{7}$$

and

$$\begin{aligned} L_i &= g(\bar{\psi}\phi^a T^a \eta + \bar{\eta}\phi^a T^a \psi) - g^2 f^{adf} f_{abc} A^\mu_d \phi^\nu_f \phi^b_\mu \phi^c_\nu \\ &\quad - g f^{abc}(\partial_\mu \phi^a_\nu)\phi^\mu_b \phi^\nu_c - \frac{1}{4} g^2 f^{abc} f_{adf} \phi^\mu_b \phi^\nu_c \phi^a_\mu \phi^f_\nu + g\bar{\eta}\phi^a T^a \eta \ , \end{aligned} \tag{8}$$

where $A^a_\mu(x)$ and $\psi(x)$ denote gluon and quark background fields, and $\phi^a_\mu(x), \eta(x)$ are quantum fluctuations around their classical solutions, respectively. The background fields $A^a(x)$ and $\psi(x)$ satisfy the equation of motion. $T^a(a = 1, ..., 8)$ and D_μ in Eq.(7)-(8) are Gell-Mann matrices and the covariant derivatives, respectively.

The background field $A^a_\mu(x)$ has been fixed by means of the Schwinger gauge or "the fixed-point gauge"[10]

$$x^\mu \ A^a_\mu(x) = 0 \ . \tag{9}$$

The "background gauge" for the gluon quantum field $\phi^a_\mu(x)$ is chosen as

$$D^{ab}_\mu(A)\phi^\mu_b(x) = 0 \ . \tag{10}$$

It can be seen from Eqs.(6)-(7) that there exists a gluon background field $A^a_\mu(x)$ in L_0 and there are no free quark or gluon states in the eigenmodes of L_0. L_{eff} contains

not only the interactions between quantum fields and quantum fields but also the couplings between quantum fields and background fields.

Thus the B-S kernel should be determined by the QCD L_{eff} in the background fields. As an example we turn on the zero and the first-order approximation in the L_i. Thus the quark condensate and gluon condensate diagrams will contribute to the kernel G_n and the one gluon exchange diagram will contribute to the kernel G_p too.

For simplicity, we make the following three assumptions: i) We do not try to solve Schwinger-Dyson equation for quark self-energy instead of the constituent quark mass as parameters in the quark propagators; ii) Only three light quark condensates are involved in the kernel G; iii) Only the lower condensate diagrams in the gluon propagator are taken into account and all of higher orders and higher dimension condensates are neglected. For the leading terms we have

$$< \Omega \mid \psi^j_\beta(x)\bar{\psi}^i_\alpha(0) \mid \Omega > \sim \frac{1}{12}\{\delta_{ij} - \frac{1}{4}m\ x_\mu\gamma^\mu_{ij}\}\ \delta_{\alpha\beta} < \Omega \mid \psi\bar{\psi} \mid \Omega > \tag{11}$$

and

$$A_\mu(x) \sim \frac{1}{2}x^\nu G_{\nu\mu}(0)\ .$$

Now the B-S kernel can be determined by the gluon propagator in this framework

$$\begin{aligned} D^{ab}_{\mu\nu}(q) &= \int d^4x e^{iq\cdot x} < \Omega|T(\phi^a_\mu(x)\phi^b_\nu(0))|\Omega > \\ &= -ig_{\mu\nu}\ D(q^2)\delta^{ab} \end{aligned} \tag{12}$$

in the Feynman gauge. $D(q^2)$ is defined as,

$$\begin{aligned} D(q^2) = \frac{1}{q^2} &+ \frac{1}{6}g^2 \sum_{f=u,d,s} \frac{m_f < \Omega \mid \psi_f\bar{\psi}_f \mid \Omega >}{q^2(q^2-m_f^2)}[\frac{1}{q^2} + \frac{1}{q^2-m_f^2}] \\ &+ \frac{3}{8}g^2\frac{< \Omega \mid G^2 \mid \Omega >}{q^6}\ . \end{aligned} \tag{13}$$

$< \Omega \mid \psi_f\bar{\psi}_f \mid \Omega >$ and $< \Omega \mid G^2 \mid \Omega >$ in Eq.(13) denote the quark condensate with flavor f and the gluon condensate. They are parameters which have been determined

phenomenologically[4,11]

$$
\begin{aligned}
&< \Omega \mid \frac{\alpha_s}{\pi} G^2 \mid \Omega > \sim [(360 \pm 20) MeV]^4 \ , \\
&< \Omega \mid d\bar{d} \mid \Omega > = < \Omega \mid u\bar{u} \mid \Omega > = [(225 \pm 25) MeV]^3 \ , \\
&< \Omega \mid s\bar{s} \mid \Omega > = 0.8 < \Omega \mid u\bar{u} \mid \Omega > \ .
\end{aligned} \tag{14}
$$

It was pointed out[8,9] that the first term in Eq.(13) is the Coulomb potential in the coordinate space and the second term is mainly the linear potential, which is induced by the correction of the quark condensates. The third term is induced by the correction of the gluon condensate which gives a higher power term r^3. However, the r^3 term has an opposite sign to the linear term. Thus, the resulting potential in the coordinate space becomes

$$
U(r) = -\frac{g^2}{3\pi r} + \sum_f \frac{g < \Omega \mid \psi_f \bar{\psi}_f \mid \Omega >}{36\pi m_f} \left(\frac{1}{m_f} e^{-m_f r} + r \right) - \frac{g^4 < \Omega \mid G^2 \mid \Omega >}{192\pi} r^3, \tag{15}
$$

which can compare with the phenomenological potentials[3,12,13] in the region $r < 0.6 fm$. It is the reason why this B-S kernel can give a reasonable pseudoscalar spectrum and decay constants of the ground states.

However, $U(r)$ will fall down as $r > 0.8 fm$ since the r^3 makes a cancellation of the linear confinement potential. By carefully studying the potential $U(r)$, it can be easily found that the potential will rise up if higher dimensional condensates are involved. Thus we should calculate all higher condensates and all loop contributions to get the full gluon propagator. In fact it is impossible to sum over all such contributions at present. In principle, it is conjectured that the $D(q^2)$ may be written as

$$
D(q^2) = \frac{C}{q^2} + \sum_{m,n} \frac{A_{mn}}{q^{2m}(q^2 - m_f^2)^n} + \sum_{n=2} \frac{B_{2n}}{q^{2n+2}} \tag{16}
$$

where the coefficients C, A_{mn} and B_{2n} can be determined by the QCD theory in the background fields which includes the contribution from the perturbative and non-perturbative condensate diagrams. In this paper we are not going to obtain the full gluon propagator and we only consider the next higher dimensional condensates, up

to dimension-6, to see what's happen with the potential behavior since we can not really go to the large distances in this way. Also we ignore all loop contributions. Thus A_{mn} and B_{2n} are determined simply by the vacuum condensates up to dimension-6, they are

$$
\begin{aligned}
O_4^{\psi} &= m < \Omega|\bar{\psi}\psi|\Omega > && \text{quark condensate,} \\
O_4^{G} &= \alpha_s < \Omega|G_a^{\mu\nu}G_{\mu\nu}^a|\Omega > && \text{gluon condensate,} \\
O_6^{\psi} &= < \Omega \mid \bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi \mid \Omega > && \text{four quark condensate.} \\
O_6^{G} &= g f_{abc} < \Omega|G_a^{\mu\nu}G_{\nu\rho,b}G_{\mu c}^{\rho}|\Omega > && \text{triple gluon condensate,} \\
O_6^{G\psi} &= mg < \Omega|\bar{\psi}\sigma^{\tau\kappa}\tfrac{\lambda^a}{2}G_{\tau\kappa}^a\psi|\Omega > && \text{mixed condensate.}
\end{aligned}
$$

Here ψ denotes u, d or s quark. The corresponding Feynman diagrams of the gluon propagator at tree level are shown in Fig.1.

If Eq.(14) and the following assumption about vacuum condensate parameters are employed[4,14]

$$g^3 f_{abc} < \Omega|G_a^{\mu\nu}G_{\nu\rho}^b G_{\mu c}^{\rho}|\Omega > = 0.045\ GeV^6, \tag{17}$$

$$g < \Omega|\bar{\psi}\sigma^{\tau\kappa}\frac{\lambda^a}{2}G_{\tau\kappa}^a\psi|\Omega > = M_0^2 < \Omega|\psi\bar{\psi}|\Omega >^2, \quad M_0^2 = 0.2\ GeV^2 \tag{18}$$

$$< \Omega|\bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi|\Omega > = \frac{1}{144}[T_r\Gamma_1 T_r\Gamma_2 - T_r(\Gamma_1\Gamma_2)] < \Omega|\psi\bar{\psi}|\Omega >^2 . \tag{19}$$

$D(q^2)$ can be determined by the Fig.1.

III. Vacuum Condensate Corrections to the Potential

As the first step we calculate the vacuum condensate corrections to the potential (up to dimension-6) at tree level.

From Fig.1 it may be seen that the vacuum expectation values of the background fields are the multiple-point functions of the positions. They can be expanded in terms of vacuum condensates, such as those in eq.(11). For our aim, the relevant formulas, in the fixed-point gauge[10], are

$$
\begin{aligned}
< \Omega \mid \bar{\psi}_{i\alpha}(x)\psi_{j\beta}(0) \mid \Omega > \approx & \frac{1}{12}\delta_{\alpha\beta}\{(\delta_{ij} + \frac{i}{4}mx^{\mu}(\gamma_{\mu})_{ji}] < \Omega \mid \bar{\psi}\psi \mid \Omega > \\
& -\frac{1}{16}x^2[\delta_{ij} + \frac{i}{6}mx^{\mu}(\gamma_{\mu})_{ji}]\, g < \Omega \mid \bar{\psi}\sigma_{\mu\nu}G_a^{\mu\nu}\frac{\lambda^a}{2}\psi \mid \Omega > \\
& +\frac{i}{288}x^2x^{\mu}(\gamma_{\mu})_{ji}g^2 < \Omega \mid \bar{\psi}\gamma_{\rho}\frac{\lambda^a}{2}\psi\sum_f \bar{\psi}^f\gamma_{\rho}\frac{\lambda^a}{2}\psi^f \mid \Omega >\} , \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
& < \Omega|\bar{\psi}_{i\alpha}(x)A_{\rho}^a(z)\psi_{j\beta}(0)|\Omega > \\
\approx \; & \frac{z^{\mu}}{384}(\frac{\lambda^a}{2})_{\beta\alpha}\{[\sigma_{\mu\rho} - \frac{m}{2}(x_{\mu}\gamma_{\rho} - x_{\rho}\gamma_{\mu}) + \frac{i}{2}mx^{\nu}\gamma_{\nu}\sigma_{\mu\rho}]_{ji} < \Omega|\bar{\psi}\sigma_{\tau\kappa}G_a^{\tau\kappa}\frac{\lambda^a}{2}\psi|\Omega > \\
+ \; & [(-\frac{2}{3}z_{\mu}\gamma_{\rho} + \frac{2}{3}z_{\rho}\gamma_{\mu}) + \frac{i}{2}x^{\nu}\gamma_{\nu}\sigma_{\mu\rho}]_{ji}g < \Omega|\bar{\psi}\gamma^{\lambda}\frac{\lambda^a}{2}\psi\sum_f \bar{\psi}_f\gamma_{\lambda}\frac{\lambda^a}{2}\psi_f|\Omega >\} , \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
& < \Omega|G_{\mu\rho}^a(x)G^{b\rho}{}_{\nu}(y)|\Omega > \simeq -\frac{1}{32} < \Omega|G^2|\Omega > \delta^{ab}g_{\mu\nu} \\
& +\frac{g^2}{384} < \Omega|\sum_{f_1}\bar{\psi}_{f_1}\gamma^{\rho}\frac{\lambda^a}{2}\psi_{f_1}\sum_{f_2}\bar{\psi}_{f_2}\gamma_{\rho}\frac{\lambda^a}{2}\psi_{f_2}|\Omega > (x-y)^2g_{\mu\nu}\delta^{ab} \\
& +\frac{1}{192}g^2 < \Omega|\sum_{f_1}\bar{\psi}_{f_1}\gamma^{\rho}\frac{\lambda^a}{2}\psi_{f_1}\sum_{f_2}\bar{\psi}_{f_2}\gamma_{\rho}\frac{\lambda^a}{2}\psi_{f_2}|\Omega > (x-y)_{\mu}(x-y)_{\nu}\delta^{ab} , \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
< \Omega|\bar{\psi}_{i\alpha}(x)G_{\mu\nu}^a(y)\psi_{j\beta}|\Omega > & \simeq < \Omega|\bar{\psi}_{i\beta}G_{\mu\nu}^a\psi_{j\alpha}|\Omega > \\
& = \frac{1}{192}(\frac{\lambda^a}{2})_{\beta\alpha}[\sigma_{\mu\nu}]_{ji} < \Omega|\bar{\psi}\sigma_{\tau\kappa}G^{\tau\kappa}\psi|\Omega > , \qquad (23)
\end{aligned}
$$

$$
< \Omega|\bar{\psi}(x)\Gamma_1\psi(y)\bar{\psi}(z)\Gamma_2\psi(0)|\Omega > \simeq < \Omega|\bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi|\Omega > , \qquad (24)
$$

and

$$
\begin{aligned}
< \Omega|G_{\mu\nu}^a(x)G^{b\nu}{}_{\rho}(y)G^{c\rho}{}_{\lambda}(j)|\Omega > & \simeq < \Omega|G_{\mu\nu}^aG^{b\nu}{}_{\rho}G^{c\rho}{}_{\lambda}|\Omega > \\
& = \frac{1}{96}g_{\mu\lambda}f^{abc}f^{def} < \Omega|G_{\rho\nu}^dG^{e\nu\tau}G_{\tau\rho}^f|\Omega > . \qquad (25)
\end{aligned}
$$

The method of deriving these formulas were given in Ref.[15].

Using these formulas, through a straightforward and long calculation, we obtain the gluon propagator in the momentum space,

$$D^{ab}_{\mu\nu}(q) = d_{\mu\nu}(q)\delta_{ab} \tag{26}$$

with

$$\begin{aligned}
d_{\mu\nu}(q) \quad = \quad & -\frac{i}{q^2}g_{\mu\nu} \\
& + \sum_{f=u,d,s} \{ \frac{g^2 m_f < \Omega|\psi_f\bar{\psi}_f|\Omega >}{6} [\frac{-i}{q^4}\frac{1}{q^2-m_f^2} + \frac{-i}{q^2(q^2-m_f^2)^2}]g_{\mu\nu} \\
& + \frac{g^3 m_f^3 < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{6}\frac{i}{q^4(q^2-m_f^2)^3}g_{\mu\nu} \\
& + [\frac{g^3 m_f < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{12} + \frac{g^4 < \Omega|\bar{\psi}_f\psi_f|\Omega >^2}{81}] \\
& \times \frac{i}{q^2}[\frac{1}{(q^2-m_f^2)^2} - \frac{10q^2}{3(q^2-m_f^2)^3} + \frac{2q^4}{(q^2-m_f^2)^4}]g_{\mu\nu} \\
& + \frac{m_f g^3 < \Omega|\psi_f\bar{\psi}_f|\Omega >}{3q^4(q^2-m_f^2)^2}q_\mu q_\nu \\
& + [\frac{g^3 m_f^3 < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{12} + \frac{g^4 < \Omega|\bar{\psi}_f\psi_f|\Omega >^2}{81}] \\
& \frac{i}{q^4}[\frac{16q_\mu q_\nu}{3(q^2-m_f^2)^3} - \frac{4q^2 q_\mu q_\nu}{(q^2-m_f^2)^4}]\} \\
& + \frac{3}{8}g^2 < \Omega|G^2|\Omega > \frac{-i}{q^6}g_{\mu\nu} + \sum_f \frac{1}{18}g^4 < \Omega|\bar{\psi}_f\psi_f|\Omega >^2 \frac{-i}{q^8}g_{\mu\nu} \\
& + \sum_f \frac{1}{4}g^4 < \Omega|\bar{\psi}_f\psi_f|\Omega >^2 \frac{i}{q^{10}}q_\mu q_\nu \\
& + \sum_f [\frac{g^3 m_f < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{96} - \frac{g^4 < \Omega|\bar{\psi}_f\psi_f|\Omega >^2}{216}]\frac{i}{q^4}[\frac{g_{\mu\nu}}{(q^2-m_f^2)^2} \\
& - \frac{4q_\mu q_\nu}{3(q^2-m_f^2)^3} - \frac{2q^2 g_{\mu\nu}}{3(q^2-m_f^2)^3}] \\
& + \sum_f \frac{g^4 < \Omega \mid \bar{\psi}_f\psi_f|\Omega >^2}{324}\frac{-i}{q^4}[\frac{g_{\mu\nu}}{(q^2-m_f^2)^2} + \frac{2q_\mu q_\nu}{3(q^2-m_f^2)^3} - \frac{2q^2 g_{\mu\nu}}{3(q^2-m_f^2)^3}] \\
& + \sum_f \frac{g^3 m_f < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{8}\frac{i}{q^6}\frac{1}{q^2-m_f^2}g_{\mu\nu}
\end{aligned}$$

$$+\sum_f \frac{g^4}{108} < \Omega|\bar{\psi}_f\psi_f|\Omega >^2 \frac{-i}{q^6}\frac{1}{(q^2-m_f^2)^2}(-5m_f^2 - 16q^2)g_{\mu\nu}$$
$$+\sum_f \frac{g^4}{54} < \Omega|\bar{\psi}_f\psi_f|\Omega >^2 \frac{i}{q^6}\frac{1}{(q^2-m_f^2)^2}q_\mu q_\nu$$
$$+\frac{9}{2}g^3 f_{abc} < \Omega|G_a^{\mu\nu}G_{\nu\rho,b}G^{\rho}_{\mu,c}|\Omega > \frac{i}{q^8}g_{\mu\nu} \tag{27}$$

Here, the vacuum condensates are the fixed parameters showed in eqs.(15, 17, 18); the three light quark masses and the strong coupling constant are free parameters.

Transforming eq.(25) from momentum space into coordinate space in the instantaneous approximation, we get the quark-antiquark potential,

$$\begin{aligned} V(r) &= \frac{4}{3}g^2\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}d_{00}(q)|_{q_0=0} \\ &= -\frac{4}{3}g^2\frac{1}{4\pi r} + \sum_f A_f e^{-m_f r} + \sum_f B_f(\frac{1}{r} - \frac{1}{r}e^{-m_f r}) \\ &+ \sum_f C_f r e^{-m_f r} + Dr + \sum_f E_f e^{-m_f r}r^2 + Fr^3 + Gr^4 \end{aligned} \tag{28}$$

with

$$\begin{aligned} A_f &= \frac{4}{3}g^2[\frac{g^2 < \Omega|\psi_f\bar{\psi}_f|\Omega >}{48\pi m_f^2} - \frac{55g^2 < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi|\Omega >}{4608\pi m_f^4} \\ &- \frac{313g^4 < \Omega \mid \psi_f\bar{\psi}_f|\Omega >^2}{10386\pi m_f^5}] \\ B_f &= \frac{4}{3}g^2[-\frac{13g^2 < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{288\pi m_f^5} + \frac{10g^4 < \Omega|\psi_f\bar{\psi}_f|\Omega >^2}{81\pi m_f^6}] \\ C_f &= \frac{4}{3}g^2[-\frac{5g^2 < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{4608\pi m_f^3} - \frac{5g^4 < \Omega|\psi_f\bar{\psi}_f|\Omega >^2}{3456\pi m_f^4}] \\ D &= \sum_f \frac{4}{3}g^2[\frac{g^2 < \Omega|\psi_f\bar{\psi}_f|\Omega >}{48\pi m_f} - \frac{19g^3 < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{768\pi m_f^3} \\ &+ \frac{181g^4 < \Omega|\psi_f\bar{\psi}_f|\Omega >^2}{5196\pi m_f^4}] \\ E_f &= \frac{4}{3}g^2[-\frac{g^3 < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{1152\pi m_f^2} - \frac{g^4 < \Omega|\psi_f\bar{\psi}_f|\Omega >^2}{7776\pi m_f^3}] \\ F &= \frac{4}{3}g^2[-\frac{g^2 < \Omega|G^2|\Omega >}{256} - \sum_f(\frac{g^3 < \Omega|\bar{\psi}_f\sigma_{\tau\kappa}G^{a\tau\kappa}\frac{\lambda^a}{2}\psi_f|\Omega >}{768\pi m_f} \end{aligned}$$

$$-\frac{5g^4 < \Omega|\psi_f\bar{\psi}_f|\Omega >^2}{10368\pi m_f^2})]$$

$$G = \frac{4}{3}g^2[\sum_f \frac{-g^4 < \Omega|\psi_f\bar{\psi}_f|\Omega >}{51840\pi} + \frac{g^3}{640\pi}f_{abc} < \Omega|G^a_{\tau\kappa}G^{b\kappa\rho}G^{c\tau}_{\rho}|\Omega >] \quad (29)$$

The potential (28) is the combination of the short, intermediate and large range interactions. However it is not reliable in the large range for lack of the contributions from higher vacuum condensates. In particular, the coefficents $D > 0$, $F < 0$ and $G > 0$ are found after putting (14) and (17 - 19) into Eq.(29).

The short range interaction is essentially a coulomb-type one, a little corrected by the exponential decreasing terms which decay fast than the former with increasing of the quark-antiquark separation. Among the vacuum condensates, the two and three gluon condensates only give the contributions to the large range interaction, the rest give the contributions to all the kinds of interactions. The quark masses, as mentioned in Sec.II, are necessary to produce the intermediate distance interaction. With the various parameter sets, the potential is shown in Fig.2 in the region $0.1 < r < 1$fm where the phenomenological potentials are well fixed by the data. We take the constituent quark masses as input parameters, because our interest is mainly in the meson spectra. In order to show the tendency for our potential to vary with increasing of the vacuum condensate dimension, we compare it with Cornell and a QCD motivated potential[13]. Their comparison is drawn in Fig.2.

IV. Summary and Discussion

Up to now, we have calculated the quark-antiquark potential by involving vacuum condensates (up to dimension-6) at tree level in QCD theory in the background field framework. It has been shown that this approach give a way to reach the quark-antiquark potential in QCD.

As well known, it is a Coulomb potential if we only consider the one gluon ex-

change in the perturbative QCD theory. It is impossible to fit the hadronic spectra since the hadronic structure is mainly determined by the QCD theory at the large distances. It is believed that the QCD vacuum, which differs greatly from the perturbative QCD vacuum, should be related to real physical states. Therefore could lose some very important physics if based on the perturbative vacuum one. However there is no an explicit solution to non-perturbative QCD at present. In this paper, we have made a try to approach the quark-antiquark potential by involving the vacuum condensate effect.

It was found in the previous papers[8,9] that the linear potential, appeared in the quark-antiquark potential, was induced by the correction of the quark condensate $< q\bar{q} >$. However the lowest gluon condensate $< G^2 >$ gives the r^3 term which has an opposite sign to the linear confinement term. Although the resulting potential is consistent with the Cornell form as $r < 0.6fm$, it will fall down at the large distances. The reason is that the interacting distance is still small and one has to take into account for the higher dimension vacuum condensates to extend to the larger range. Indeed we show that the vacuum condensates with dimension-6 contribute to the higher power term, r^5, which has an opposite sign to the r^3 term and makes the potential rising up again. Thus it can be conjectured that the higher dimension condensates will give the higher power term with the alternative sign for larger distances,

$$V(r) = -\frac{\alpha}{r} + \sum_{n=0}^{\infty}(-)^n \; C_{2n+1} \; r^{2n+1} \tag{30}$$

and eventually $V(r)$ will rise up. Now it is rather hard to answer a question how does $V(r)$ like at large distances? We will study it in the future and try to get a real potential in the QCD theory.

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Figure Captions

Fig 1. Gluon propagator involving the vacuum condensate corrections at the tree level. ψ is the quark background field; A_μ is the gluon background field; $G_{\mu\nu}$ is the gluon background field strength.

Fig 2. Comparision with the phenomenological potentials. Solid line represents the Cornell potential. Dashed line represents potential which was given by Ref.[13]. Dotted line $U(r)$ and $V(r)$ is calculated by Eq.(15) and (28) respectively. (with $\alpha_s = 0.54$, $m_u = m_d = 0.35\,\mathrm{GeV}$ and $m_s = 0.53\,\mathrm{GeV}$).

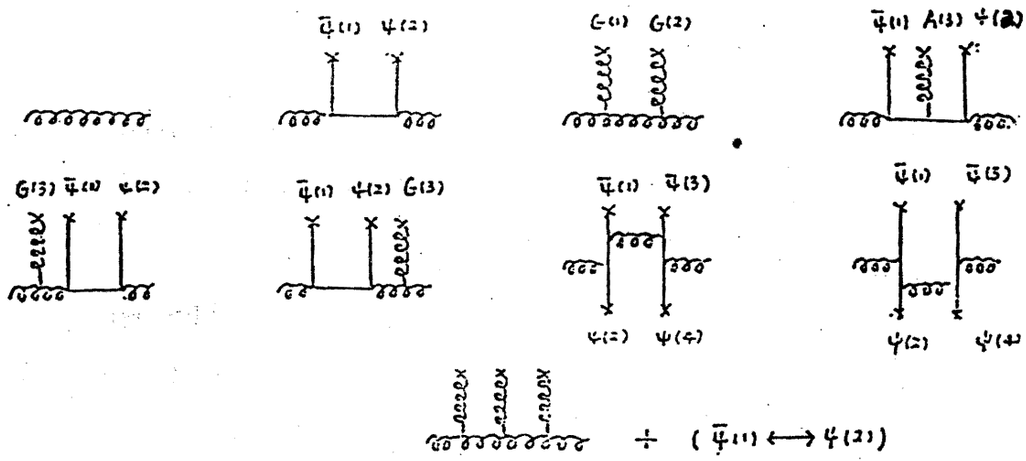

Fig 1.

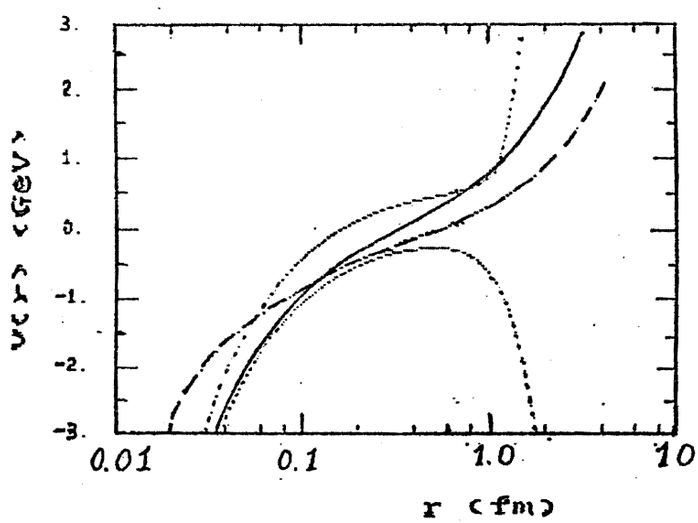

Fig 2.