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Partial Decay width of the Z into three photons via the W loops*

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Abstract

In this paper the contribution of the W loops to the partial width of the Z decay into three photons is calculated in the standard model. The width is about 2×10^{-11} GeV.

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Recently there are three papers discussing the contribution of the W-boson loops to the Z-boson decay into three photons. M. Baillargeon and F.Boudjema[1] take a non-linear R_ξ gauge[2]. Using an approximate expansion,they get the width $\Gamma_{w\ loops}(Z \to 3\gamma) \simeq 2 \times 10^{-11} GeV$. The gauge invariant condition of the electro-magnetic interaction can be easily shown in the non-linear R_ξ gauge[3]. X.Y.Pham uses a linear R_ξ gauge and gives the expression of the fourth-rank tensor $G^{\mu\nu\rho\tau}$ in his article[4]. His conservative estimate gives $\Gamma_{w\ loops} \simeq 2eV$, which is a hundred times bigger than the one given by Baillargeon et al. We also use a linear R_ξ gauge and get the fourth-rank tensor $G^{\mu\nu\rho\tau}$ both in $Z \to 3\gamma$ case and in photon-photon scattering case[5]. In the linear R_ξ gauge the gauge invariant conditions should be carefully considered. We find that these conditions are subtly different between the fermion-loop case[6] and the W-loop case[5]. In this paper we further calculate the width $\Gamma_{w\ loops}(Z \to 3\gamma)$ in the unpolarized case. Using an approximate expansion similar to that in the reference[1] we obtain the width $= 1.94 \times 10^{-11} GeV$ when $M_z = 91.1 GeV, sin^2\theta_w = 0.23, \alpha = 1/137$ and $\alpha(M_z) = 1/128$,which is consistent with the result given by the reference [1].

The amplitude of $Z \to 3\gamma$[5] is

$$I = \epsilon_{\mu_1}(k_1,\lambda_1)\epsilon_{\mu_2}(k_2,\lambda_2)\epsilon_{\mu_3}(k_3,\lambda_3)\epsilon^*_{\mu_4}(k_4,\lambda_4)G^{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4) \qquad (1)$$

where $(k_1,\lambda_1),(k_2,\lambda_2),(k_3,\lambda_3)$ refer to the photons and (k_4,λ_4) refers to the Z-boson. k_1,k_2,k_3,k_4 are their four- momenta. $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ are their physical polarization degrees of freedom. The fourth-rank tensor $G^{\mu\nu\rho\tau}$ was calculated in our previous paper in the W-loop case. We gave two equivalent forms of $G_{\mu\nu\rho\tau}$. One of them is invariant under the permutations of $(k_1,\lambda_1),(k_2,\lambda_2)$ and (k_3,λ_3), i.e. it has S_3 symmetry. The other one $G'^{\mu\nu\rho\tau}$ is invariant under the permutation of (k_1,λ_1) and (k_2,λ_2). It has only S_2 symmetry, but it satisfies

$$k_{1\mu_1}G'^{\mu_1\mu_2\mu_3\mu_4}(1234) = 0 \qquad (2)$$

From now on we use $f(1234)$ in stead of $f(k_1,k_2,k_3,k_4)$ for simplicity. It is convenient

to calculate the decay rate in the unpolarized case. In this case one should calculate

$$T \equiv \frac{1}{3} \sum_{\lambda_1,\lambda_2,\lambda_3=1}^{2} \sum_{\lambda_4=1}^{3} |I|^2 \tag{3}$$

For the photon

$$\sum_{\lambda=1}^{2} \epsilon_\mu(k,\lambda)\epsilon_\nu(k,\lambda) = -g_{\mu\nu} - \frac{k_\mu k_\nu}{(k\cdot\eta)^2} + \frac{k_\mu\eta_\nu + \eta_\mu k_\nu}{(k\cdot\eta)} \tag{4}$$

where $\eta = (1,0,0,0)$, is a time-like unit vector[7]. For the Z boson

$$\sum_{\lambda=1}^{3} \epsilon'_\mu(k,\lambda)\epsilon'_\nu(k,\lambda) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{M_z^2} \tag{5}$$

where M_z is the mass of the Z boson. At first we sum over λ_3. Using the gauge invariant condition

$$k_{3\rho}\epsilon_\mu(k_1,\lambda_1)\epsilon_\nu(k_2,\lambda_2)\epsilon'_\tau(k_4,\lambda_4)G'^{\mu\nu\rho\tau}(1234) = 0 \tag{6}$$

we may drop terms $k_{\mu_3}k_{\nu_3}, k_{\mu_3}\eta_{\nu_3}, \eta_{\mu_3}k_{\nu_3}$ in T. Then sum over λ_1, λ_2 and λ_4. Using Eq.(2) we finally get

$$T = \frac{1}{3}(G^{\mu\nu\rho\tau}G'_{\mu\nu\rho\tau} - G^{\mu\nu\rho\tau}k_{4\tau}G'_{\mu\nu\rho\sigma}k_4^\sigma/M_z^2) \tag{7}$$

In our previous paper we have proved that $G'^{\mu\nu\rho\tau}$ can be expressed in terms of four Lorentz scalars, e.g. $A_{12}(123), A_{13}(123), A'_4(123)$ and $B_5(123)$, which are given in the reference [5]. Hence one can explicitly write

$$G'^{\mu\nu\rho\tau}(123) = \sum_{i=1}^{8} Q_i(123)P_i^{\mu\nu\rho\tau}(123) + ((k_1,\mu) \leftrightarrow (k_2,\nu)) \tag{8}$$

where $Q_1(123) = A_{12}(123), Q_2(123) = A_{12}(132), Q_3(123) = A_{13}(123), Q_4(123) = A_{13}(132), Q_5(123) = A_{13}(321), Q_6(123) = A'_4(132), Q_7(123) = A'_4(321), Q_8(123) =$

$B_5(321)$ and

$$
\begin{aligned}
P_1^{1234} &= -(2112) + d_3(34,12)g^{12} - d_2(12,21)g^{34}/2 + d_2d_3g^{12}g^{34}/2 \\
P_2^{1234} &= (3111) + (3112) - d_2(24,11)g^{13} + d_2(24,12)g^{13} - d_3(13,31)g^{24} + d_2d_3g^{13}g^{24} \\
&\quad - (3311)d_3/d_1 + (24,31)g^{13}d_2d_3/d_1 \\
P_3^{1234} &= (2111) + (2112) - d_3(34,11)g^{12} - d_3(34,12)g^{12} + d_2(12,21)g^{34} - d_2d_3g^{12}g^{34} \\
P_4^{1234} &= -(3112) + d_2(24,12)g^{13} + d_3(13,31)g^{24} - d_2d_3g^{13}g^{24} \\
P_5^{1234} &= -(3321) + d_1(14,31)g^{23} + d_2(23,32)g^{14} - d_1d_2g^{23}g^{14} \\
P_6^{1234} &= (3121) - d_3(14,31)g^{23} - d_1(24,11)g^{13} + d_1(23,11)g^{14} + d_3(24,31)g^{13} \\
&\quad -d_3(23,31)g^{14} + d_2(23,12)g^{14} + d_2d_3g^{23}g^{14} \\
P_7^{1234} &= -(3122) + d_2(34,22)g^{12} + d_1(24,12)g^{13} - d_2(13,22)g^{24} + d_3(13,32)g^{24} \\
&\quad +d_1(13,21)g^{24} - d_1(34,12)g^{12} - d_1d_3g^{13}g^{24} \\
P_8^{1234} &= [(23,31)g^{14} - (24,31)g^{13} - (12,33)g^{34}/2]d_3/d_2 - (34,21)g^{12} + (23,12)g^{14} \\
&\quad +(14,22)g^{23} + (14,21)g^{23} + (12,23)g^{34} - (13,22)g_{24} - d_1g^{12}g^{34}/2 - d_3g^{14}g^{23}
\end{aligned}
\tag{9}
$$

where $P_i^{1234} \equiv P_i^{\mu_1\mu_2\mu_3\mu_4}, (ijml) \equiv k_i^{\mu_1}k_j^{\mu_2}k_m^{\mu_3}k_l^{\mu_4}, (ab,ij) \equiv k_i^{\mu_a}k_j^{\mu_b}, g^{ab} \equiv g^{\mu_a\mu_b}, d_1 = k_2 \cdot k_3, d_2 = k_1 \cdot k_3, d_3 = k_1 \cdot k_2$.

Note that the coefficients A_{12}, A_4' and B_5 are further constrained by [5]

$$d_2A_{12}(123) = d_1A_{12}(213) \tag{10.1}$$

$$d_2B_5(312) = d_1B_5(321) \tag{10.2}$$

$$B_5(123) - B_5(321) = d_2[A_4'(312) - A_4'(132)] \tag{10.3}$$

By definition , A_4' satisfies

$$A_4'(123) + A_4'(231) + A_4'(312) = 0 \tag{10.4}$$

The next step of calculation is tedious. We use Reduce to calculate T. But further numerical calculation is still tremendous. Fortunately,there is an approximate method. There are quantities in the reference [5]

$$M_i^2 = M_w^2 - 2\sum_{m=1}^{3} d_m t_{im} \quad (i = 1,2,3)$$

where $t_{11} = t_{22} = t_{31} = (1-b)c, t_{12} = t_{21} = t_{23} = -dc, t_{13} = t_{23} = t_{32} = -db$ and d, b, c are Feynman integration parameters[5]. Note that $0 \leq t_{im} \leq 1/4$ and $k_i \cdot k_j \geq 0$ $(i,j = 1,2,3)$,so

$$0 \leq \frac{2}{M_w^2} \sum_{m=1}^{3} d_m t_{im} \leq \frac{M_z}{4M_w} \sim \frac{1}{3} \tag{11}$$

where we use $2(d_1 + d_2 + d_3) = M_z^2$. Now in general, we can expand

$$Q(123) = \alpha_1 s^2 \{Q_0(s) + s[d_1 Q_{11}(s) + d_2 Q_{12}(s)]/M_z^2 + \cdots\}/M_z^4 \tag{12.1}$$

$$B_5(312) = \alpha_1 s\{B_0(s) + s[d_1 B_1(s) + d_2 B_2(s)]/M_z^2 + s^2[d_1^2 B_{11}(s) + d_1 d_2 B_{12}(s) + d_2^2 B_{22}(s)]/M_z^4 + \cdots\}/M_z^2 \tag{12.2}$$

where $s = 1/\cos^2\theta_w$, Q is one of A_{12}, A_{13}, A_4' and $\alpha_1 = -e^3 g_2 \cos\theta_w/(4\pi)^2$. Now from Eqs.(10.2) and (12.2) one has

$$B_0(s) = B_2(s) = B_{22}(s) = 0, \; B_{11}(s) = B_{12}(s) \tag{13}$$

$$B_5(312) = \alpha_1 s^2 d_1[B_1(s) + B_{11}(s)s(d_1 + d_2)/M_z^2 + \ldots]/M_z^4 \tag{14.1}$$

Using the tables in the reference [5], to the order of s^3,

$$B_5(312) = \alpha_1 s^2 d_1[\frac{54}{5} + \frac{101}{63}s - \frac{22s}{63M_z^2}(d_1 + d_2)]/M_z^4 \tag{14.2}$$

which checks that to the order of $s^2, B_0(s) = 0$,to the order of $s, B_2(s) = 0$ and $B_{22}(0) = 0, B_{11}(0) = B_{12}(0)$. In the same way,using Eqs.(10.1) and (12.1) one gets

$$A_{120}(s) = A_{1212}(s) = 0 \tag{15}$$

$$A_{12}(123) = \alpha_1 s^3 d_1 A_{1211}(s)/M_z^6 + \cdots \tag{16.1}$$

To the order of s^3,

$$A_{12}(123) = \frac{4\alpha_1 s^3 d_1}{105 M_z^6} \tag{16.2}$$

which also checks to the order of $s, A_{120}(s) = 0$ and $A_{1212}(0) = 0$.

As for $A_{13}(123)$ there is no any constraint from Eq.(10). So it has the general form of Eq.(12.1). To the order of s^3

$$A_{13}(123) = \alpha_1 s^2[\frac{2}{5} - \frac{41}{315}s + s(\frac{26}{105}d_1 + \frac{188}{315}d_2)/M_z^2]/M_z^4 \tag{17}$$

The general form of A'_4 is

$$A'_4(123) = \alpha_1 s^2\{A'_{40}(s) + s[d_1 A'_{41}(s) + d_2 A'_{42}(s)]/M_z^2 + \cdots\}/M_z^4 \tag{18}$$

In general let $T(s) = \sum_{n=1}^{\infty} T_n s^n$, Eq.(10.3) gives

$$B_{11,0} = A'_{41,0} - A'_{42,0} \tag{19.1}$$

Eq.(10.4) gives

$$A'_{40,0} \equiv A'_{40}(0) = 0 \tag{19.2}$$

$$3A'_{40,1} + (A'_{41,1} + A'_{42,1})M_z^2/2 = 0 \tag{19.3}$$

Finally to the order of s^3

$$A'_4(123) = \alpha_1 s^3[\frac{232}{315}(d_1 - M_z^2/6) + \frac{122}{315}(d_2 - M_z^2/6)] \tag{20}$$

which also checks Eqs.(19). Using Eqs.(14.2),(16.2),(17) and (20) to the order of s^3 we finally get

$$\begin{aligned} T = {} & \tfrac{4}{3}\alpha_1^2 s^4[49518M_z^6 d_3^2 + 49455M_z^6 d_2 d_3 + 49518M_z^6 d_2^2 \\ & -198072M_z^4 d_3^3 - 296982M_z^4 d_3^2 d_2 - 296982M_z^4 d_3 d_2^2 \\ & -198072M^4 d_2^3 + 198072M_z^2 d_3^4 + 396144M_z^2 d_3^3 d_2 \\ & +594216M_z^2 d_3^2 d_2^2 + 396144M_z^2 d_3 d_2^3 + 198072M_z^2 d_2^4 \\ & +s(12780M_z^6 d_3^2 + 11383M_z^6 d_3 d_2 + 12780M_z^6 d_2^2 \\ & -51120M_z^4 d_3^3 - 60846M_z^4 d_3^2 d_2 - 60846M_z^4 d_3 d_2^2 \\ & -51120M_z^4 d_2^3 + 51120M_z^2 d_3^4 + 50080M_z^2 d_3^3 d_2 \\ & +75120M_z^2 d_3^2 d_2^2 + 50080M_z^2 d_3 d_2^3 + 51120M_z^2 d_2^4 \\ & +52160d_3^4 d_2 + 104320d_3^3 d_2^2 + 104320d_3^2 d_2^3 \\ & +52160d_3 d_2^4)]/(1575M_z^{10}) \end{aligned} \tag{21}$$

T has apparent S_2 symmetry. It is not difficult to see it also has S_3 symmetry,which in a way checks our results because we start with $G'^{\mu\nu\rho\tau}$ which has only S_2 symmetry. The differential decay rate can be calculated by the formula in the reference [7]. Using the momenta conservation one may integrate one of $\vec{k_i}$ $(i = 1,2,3)$,e.g. $\vec{k_3}$. When ω_1, ω_2 are fixed,one has

$$\omega_3 d\omega_3 = \omega_1\omega_2 d\cos\theta \tag{22}$$

where θ is the angle between $\vec{k_1}$ and $\vec{k_2}$; ω_i $(i = 1,2,3)$ are the energies of the photons. Note that

$$d_1 = M_z^2(1 - 2\omega_1/M_z)/2 \tag{23.1}$$

$$d_2 = M_z^2(1 - 2\omega_2/M_z)/2 \tag{23.2}$$

Let $x_1 = 2\omega_1/M_z, x_2 = 2\omega_2/M_z$. We finally get

$$\begin{aligned}\Gamma_{w\ loops}(Z \to 3\gamma) &= \frac{M_z}{256\pi^3 3!}\int_0^1 dx_1 \int_0^{1-x_1} dx_2 T(d_1 = x_1 M_z^2/2, d_2 = x_2 M_z^2/2) \\ &= \frac{M_z}{3!3}\alpha(M_z)\alpha^3 X(s)\end{aligned} \tag{24}$$

where

$$\begin{aligned}X(s) &\simeq \frac{1}{256\pi^3 \sin^2\theta_w \cos^6\theta_w}(346479 + 90547/\cos^2\theta_w)/441000 \\ &\simeq 1.26257 \times 10^{-3}\end{aligned} \tag{25}$$

where $\sin^2\theta_w = 0.23$. The result of the reference [1] gives $X(s) \simeq 1.3 \times 10^{-3}$,which is consistent with ours.

The ratio of the second term and the first term of $X(s)$ in the Eq.(25) $\sim$ 1/3,which is nearly the one estimated in the Eq.(11). This gives

$$\Gamma_{w\ loops}(Z \to 3\gamma) \simeq (0.0194 \pm 0.0040)\ eV \tag{26}$$

when $M_z^2 = 91.1\ GeV$, $\sin^2\theta_w = 0.23$, $\alpha = 1/137$ and $\alpha(M_z) = 1/128$. Here we give a conservative error estimation of twenty percents. This result is quite small in comparison with the $\Gamma_{fermion\ loops}(Z \to 3\gamma)$,which is about 0.7 eV for a large top quark mass[8]. The result is somewhat unexpected,because the W contribution largely dominates the fermionic one(top quark included) in the decays $Z \to W + f + \bar{f}'$[9] and $Z \to \gamma + H$[10],where H is the physical Higgs scalar meson.

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