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Analytical Approximation of Radiatively Corrected Resonant Cross section

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Abstract

An analytical approximation of the radiatively corrected cross section for the resonances in e e collision has been derived with the accuracy at the order of 0.1% for Z boson as well as J/ψ and Υ family particles within the experimentally scanned energy range. It is recommended using this analytical expression to treat the resonances which have to be folded with the accelerator beam energy spread in the experimental data analysis. This provides an alternative to the previous method of Jackson and Sharere. It gets much higher accuracy.

1 INTRODUCTION

Precise determination of the masses and widths of resonant states in e^+e^- annihilation requires proper treatment of initial state radiation. The structure function approach introduced by Kuraev and Fadin [1,2] provides a systematical method to calculate this correction to 0.1% accuracy. Their result is expressed in the integral form that the radiatively corrected cross section

$$\sigma(s) = \int dx\, \sigma_B(s(1-x))\, F(x,s) \tag{1}$$

where $\sqrt{s}$ is the C.M. energy of the colliding beam, $\sigma_B(s)$ is the cross section of Born order approximation, and

$$F(x,s) = \beta x^{\beta-1}\left[1+\frac{3}{4}\beta-\frac{\beta^2}{24}\left(\frac{1}{3}\ln\frac{s}{m_e^2}+2\pi^2-\frac{37}{4}\right)\right]\cdot(K\ factor)$$
$$-\beta\left(1-\frac{x}{2}\right)+\frac{1}{8}\beta^2\left[4(2-x)\ln\frac{1}{x}-\frac{1+3(1-x)^2}{x}\ln(1-x)-6+x\right] \tag{2}$$

where

$$\beta = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2}-1\right)$$

and (K factor) = $1+\frac{\alpha}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)$ for vector coupling and $1+\frac{\alpha\pi}{3}$ for axial vector coupling [3].

In the expression of F(x,s) given in (2), the first term is called the soft photon contribution; while the second and third terms are called hard photon contributions and they are truncated to β square order.

In this paper we are only concerned with the resonant states, so $\sigma_B(s)$ is Breit-Wigner cross section. We take the standard form

$$\sigma_B(s) = \frac{12\pi\Gamma_{ee}\Gamma_f}{(s-M^2)^2+\Gamma^2M^2} \tag{3}$$

where M and Γ are the mass and total width of the resonance. Γ_{ee} and Γ_f are the partial width to e^+e^- channel and to the final states respectively. If the resonance is coupled with photon, like J/ψ and Υ particles, then Γ_{ee} is defined as $(\Gamma_{ee})_{exp}$ to absorb the vaccum polarization [4].

If (2) and (3) are substituted into (1), then the first and the second term in F(x,s) can be integrated analytically, as demonstrated in reference [5,6], while the third term cannot. But if this third term is dropped, the accuracy gets worse substantially, as compared with the second term, the third term does not decrease as the power of β, because there is such singular function $\ln\frac{1}{x}$ in it.

In this paper, we present an analytical approximation to the cross section of resonant states, with the accuracy of 0.1% order for J/ψ, Υ and Z boson in the energy range which is scanned in the experimental situation. The material is organized as following: in section 2 we redrive F(x,s) and express it in a different form; then in section 3, with this new form of F(x,s) the approximation for the cross section (1) is made; in section 4 we disscuss the radiative correction treatment to J/ψ and Υ states and demonstrate that our formulae can get much higher accurary than the previous method used up to now [7].

2 THE NEW FORM OF F(X,S)

In this section, we redrive F(x,s) within the scheme of

structure function approach. Our calculation is parallel to the one in reference [1] but with a somewhat different mathematical technique.

The cross section of e^+e^- annihilation is

$$\sigma(s) = \iint dx_1 dx_2 \, D_{e^-}(x_1,s) D_{e^+}(x_2,s) \, \sigma_B(x_1 x_2 s) \cdot (K \text{ factor}) \tag{4}$$

where D(x,s) is the distribution function which satisfies the Lipatov equation [8]

$$D_{e^-(+)}(x,s) = \delta(1-x) + \cdot$$
$$+ \int_{m_e^2}^{s} \frac{ds'}{s'} \frac{\alpha(s')}{2\pi} \int_x^1 \frac{dz}{z} P(z) D_{e^-(+)}\left(\frac{x}{z}, s'\right)$$

with

$$\alpha(s') = \frac{\alpha}{1 - \frac{\alpha}{3\pi} \ln \frac{s'}{m_e^2}}$$

and

$$P(z) = \frac{1+z^2}{1-z} - \delta(1-z) \int_0^1 \frac{1+x^2}{1-x} dx \, .$$

The way to solve this equation is through Mellin transformation. After the inverse transformation,

$$D_{e^-(+)}(x,s) = \frac{1}{2\pi i} \int_{\delta - i\infty}^{\delta + i\infty} dn \, x^{-n} \cdot \exp\left\{ 2\eta \left[\frac{3}{4} - C - \ln n - \frac{1}{2(n+1)} + \sum_{\ell=1}^{\infty} \frac{B_{2\ell}}{2\ell} \left(\frac{1}{n}\right)^{2\ell} \right] \right\}$$

where C is Euler constant, $B_{2\ell}$ Bernoulli number, and

$$\eta = \frac{\alpha}{2\pi} \ln \frac{s}{m_e^2}$$

With the help of the formula

$$\frac{1}{2\pi i} \int_{\delta - i\infty}^{\delta + i\infty} \frac{x^{-n}}{n^j} dn = \frac{\left(\ln \frac{1}{x}\right)^{j-1}}{\Gamma(j)}$$

We get

$$D_{e^{-(+)}}(x,s)=e^{2\eta(\frac{3}{4}-C)}\left\{\frac{\left(\ln\frac{1}{x}\right)^{2\eta-1}}{\Gamma(2\eta)}+\frac{\left(\ln\frac{1}{x}\right)^{2\eta}}{\Gamma(2\eta+1)}\cdot(-\eta)+\right.$$

$$\left.+\frac{\left(\ln\frac{1}{x}\right)^{2\eta+1}}{\Gamma(2\eta+2)}\left[\eta(1+B_2)+\frac{\eta^2}{2}\right]+\frac{\left(\ln\frac{1}{x}\right)^{2\eta+2}}{\Gamma(2\eta+3)}\left[-\eta-\eta^2(1+B_2)-\frac{\eta^3}{6}\right]+\cdots\right\}$$

Then expand

$$\left(\ln\frac{1}{x}\right)^a=\left[(1-x)+\frac{1}{2}(1-x)^2+\frac{1}{3}(1-x)^3+\cdots\right]^a$$

$$=(1-x)^a\cdot\left\{1+\frac{a}{2}(1-x)+\left(\frac{5a}{24}+\frac{a^2}{8}\right)(1-x)^2+\right.$$

$$\left.+\left(\frac{a}{8}+\frac{5a^2}{48}+\frac{a^3}{48}\right)(1-x)^3+\cdots\right\}.$$

Also following the prescription of reference [1], make the substitution

$$\eta=\frac{\alpha}{2\pi}\ln\frac{s}{m_e^2}\longrightarrow\frac{\alpha}{2\pi}\left(\ln\frac{s}{m_e^2}-1\right)=\frac{\beta}{4}$$

and add the contribution of vitual e^+e^- pairs

$$\frac{\beta}{2}(1-x)^{\frac{\beta}{2}-1}\left[-\frac{\beta^2}{288}\left(2\ln\frac{s}{m_e^2}-15\right)\right]$$

We have

$$D_{e^{-(+)}}(x,s)=\frac{\exp\left(-\frac{\beta C}{2}+\frac{3\beta}{8}\right)}{\Gamma\left(1+\frac{\beta}{2}\right)}\left\{\frac{\beta}{2}(1-x)^{\frac{\beta}{2}-1}\left[1-\frac{\beta^2}{288}\left(2\ln\frac{s}{m_e^2}-15\right)\right]+\right.$$

$$\left.+(1-x)^{\frac{\beta}{2}}\left(-\frac{\beta}{2}+\frac{\beta^2}{8}\right)+(1-x)^{\frac{\beta}{2}+1}\left(\frac{\beta}{4}-\frac{3\beta^2}{16}\right)+\cdots\right\},$$

here the actual value of Bernoulli number has been put into.

Finally substitute D(x,s) into Eq.(4), we obtain the expre-

ssion of F(x,s) in terms of a series expansion of x

$$F(x,s) = x^{\beta-1}\cdot\beta\left[1+\frac{3}{4}\beta-\frac{\beta^2}{24}\left(\frac{1}{3}\ln\frac{s}{m_e^2}+2\pi^2-\frac{37}{4}\right)\right]\cdot(K\ factor)$$

$$+\,x^{\beta}\left(-\beta-\frac{\beta^2}{4}\right)+x^{\beta+1}\left(\frac{\beta}{2}-\frac{3}{8}\beta^2\right)+O\left(x^{\beta+2}\beta^2\right) \qquad (5)$$

Notice that the terms we omitted start from $x^{\beta+2}\beta^2$, while the three terms which we kept all have β term in their coffiecients. Our first term is identical to the soft photon term of Eq.(2), while the hard photon terms of Eq.(2) are equivalent to our second and third terms in the expansion of small x. This can be easily verified if one rewrites $x^{\beta} = 1 + \beta \ln x$, $x^{\beta+1} = x + \beta x \ln x$ and $\ln(1-x) = -\left(x + \frac{x^2}{2} + \cdots\right)$. The advantage of our expression for F(x,s) lies on the fact that $0 < x < 1$ and in Eq.(1) if $\sigma_B(s(1-x))$ is Breit-Wigner form as in (3), it peaks at $x_{imp} = 1 - M^2/s$. In the actual experimental situation, the resonance is scanned near its mass, so x_{imp} is always small. This makes the expression of F(x,s) by Eq.(5) very useful to treat the radiative corrections to resonanes.

3 THE APPROXIMATION OF THE CROSS SECTION

With our expression of F(x,s), the radiatively corrected cross section of a resonant state is

$$\sigma(s) = 12\pi\Gamma_{ee}\Gamma_f\int_0^1\frac{dx}{[s(1-x)-M^2]^2+M^2\Gamma^2}\cdot$$

$$\left\{x^{\beta-1}\beta\left[1+\frac{3}{4}\beta-\frac{\beta^2}{24}\left(\frac{1}{3}\ln\frac{s}{m_e^2}+2\pi^2-\frac{37}{4}\right)\right]\cdot(K\ factor)\right. \qquad (6)$$

$$\left.+\,x^{\beta}\left(-\beta-\frac{\beta^2}{4}\right)+x^{\beta+1}\left(\frac{\beta}{2}-\frac{3}{8}\beta^2\right)+\cdots\right\}$$

Here the up limit of the integration is put to 1, corresponding that all the energy of the electron (or positron) can be lost to photon emission. The first term of the integration can be done analytically, as demonstrated in reference [5], with the help of the formula

$$\beta\int_0^\infty \frac{x^{\beta-1}}{x^2+2ax\cos\vartheta+a^2}\,dx = a^{\beta-2}\,\frac{\pi\beta\sin[(1-\beta)\vartheta]}{\sin\vartheta\,\sin\pi\beta}$$

To apply it to the integration in (6), the part of the integration from 1 to infinity should be subtracted. This can be done by a series expansion

$$\frac{1}{[s(1-x)-M^2]^2+M^2\Gamma^2} = \frac{1}{s^2x^2\left[1-\frac{2(s-M^2)}{xs}+\frac{(s-M^2)^2+M^2\Gamma^2}{x^2s^2}\right]}$$

$$\simeq \frac{1}{s^2x^2}\left[1+\frac{2(s-M^2)}{xs}+\frac{3(s-M^2)^2-M^2\Gamma^2}{x^2s^2}\right].$$

Here we keep only three terms which are good enough:

$$\int_0^1 \frac{\beta x^{\beta-1}}{[s(1-x)-M^2]^2+M^2\Gamma^2}\,dx = \frac{1}{s^2}\,a^{\beta-2}\,\phi(\cos\vartheta,\beta)+$$

$$+\frac{\beta}{s^2}\left\{\frac{1}{\beta-2}+\frac{2(s-M^2)}{(\beta-3)s}+\frac{3(s-M^2)^2-M^2\Gamma^2}{(\beta-4)\,s^2}\right\}$$

where

$$a^2 = \left(1-\frac{M^2}{s}\right)^2+\frac{M^2\Gamma^2}{s^2}$$

$$\cos\vartheta = \frac{1}{a}\left(\frac{M^2}{s}-1\right) \qquad (7)$$

$$\phi(\cos\vartheta,y) = \frac{\pi y\sin[\vartheta(1-y)]}{\sin\vartheta\cdot\sin\pi y}.$$

The second term in the braces of Eq.(6) can be integrated simi-

larly

$$\int_0^1 \frac{x^\beta}{[s(1-x)-M^2]^2+M^2\Gamma^2}dx = \frac{1}{(1+\beta)s^2}a^{\beta-1}\phi(\cos\vartheta,\beta+1) +$$

$$+\frac{1}{s^2}\left\{\frac{1}{\beta-1}+\frac{2(s-M^2)}{(\beta-2)s}+\frac{3(s-M^2)^2-M^2\Gamma^2}{(\beta-3)s^2}\right\} .$$

The third term in Eq.(6) is the last term we consider. Since β is small, we suggest to approximate $x^{\beta+1}\simeq x$ so that

$$\int_0^1 \frac{x^{\beta+1}\,dx}{[s(1-x)-M^2]^2+M^2\Gamma^2} \simeq \frac{1}{s^2}\left[\frac{1}{2}\ln\frac{1+2a\cos\vartheta+a^2}{a^2} -\right.$$

$$\left. -\operatorname{ctg}\theta\left(\operatorname{tg}^{-1}\frac{1+a\cos\vartheta}{a\sin\vartheta}-\frac{\pi}{2}+\vartheta\right)\right] .$$

Put together the three terms, the cross section is expressed in terms of series expansion

$$\sigma_{SE} = \frac{12\pi\Gamma_{ee}\Gamma_f}{s^2}\left\{(1+\delta)\left[a^{\beta-2}\phi(\cos\vartheta,\beta)+\beta\left(\frac{1}{\beta-2}+\frac{2(s-M^2)}{(\beta-3)s}+\frac{3(s-M^2)^2-M^2\Gamma^2}{(\beta-4)s^2}\right)\right]\right.$$

$$-\beta\left(1+\frac{\beta}{4}\right)\left[\frac{1}{1+\beta}a^{\beta-1}\phi(\cos\vartheta,\beta+1)+\frac{1}{\beta-1}+\frac{2(s-M^2)}{(\beta-2)s}+\frac{3(s-M^2)^2-M^2\Gamma^2}{(\beta-3)s^2}\right]$$

$$\left.+\left(\frac{\beta}{2}-\frac{3}{8}\beta^2\right)\left[\frac{1}{2}\ln\frac{1+2a\cos\vartheta+a^2}{a^2}-\operatorname{ctg}\vartheta\left(\operatorname{tg}^{-1}\frac{1+a\cos\vartheta}{a\sin\theta}-\frac{\pi}{2}+\vartheta\right)\right]\right\} \tag{8}$$

with

$$(1+\delta) = (K\ \text{factor}) + \frac{3}{4}\beta - \frac{\beta^2}{24}\left(\frac{1}{3}\ln\frac{s}{m_e^2}+2\pi^2-\frac{37}{4}\right) \tag{9}$$

Eq.(8) is our analytical approximation of the radiatively corrected resonant cross section. To test its accuracy, we have calculated its deviation from the exact cross section for Z boson, Υ, J/ψ and ψ''. Here by the exact cross section we mean the F(x,s) of Kuraev and Fadin in Eq.(2) convoluted numerically with Breit-Wigner cross section of Eq.(3)

$$\sigma_{KF} = \int_0^1 \frac{12\pi \Gamma_{ee}\Gamma_f \, dx}{[s(1-x)-M^2]^2 + M^2\Gamma^2} \cdot$$
$$\left\{ \beta x^{\beta-1}\left[1+\frac{3}{4}\beta - \frac{\beta^2}{24}\left(\frac{1}{3}\ln\frac{s}{m_e^2} + 2\pi^2 - \frac{37}{4}\right)\right]\cdot(K\ factor)\right.$$
$$\left. -\beta\left(1-\frac{x}{2}\right) + \frac{\beta^2}{8}\left[4(2-x)\ln\frac{1}{x} + \frac{1+3(1-x)^2}{x}\ln\frac{1}{1-x} - 6 + x\right]\right\} \qquad (10)$$

The integration (10) has been evaluated by Gaussian method with a CERN library subroutine DGAUSS. For the first term in the braces of (10), a partial integration was done to eliminate the singularity of $x^{\beta-1}$ before Gaussian method was applied. For illustration and comparision, we also include three other approximations of the radiatively corrected cross section of resonances. The first is the soft photon approximation which is commonly used in the experimental data annalysis of J/ψ and Υ states

$$\sigma_{soft} = \int \frac{12\pi \Gamma_{ee}\Gamma_f \, dx}{[s(1-x)-M^2]^2 + M^2\Gamma^2} \cdot$$
$$\left\{ \beta x^{\beta-1}\left[1+\frac{3}{4}\beta - \frac{\beta^2}{24}\left(\frac{1}{3}\ln\frac{s}{m_e^2} + 2\pi^2 - \frac{37}{4}\right)\right]\cdot(K\ factor)\right\} \qquad (11)$$

Compared with Eq.(10), it only keeps the first term in the integrand. The second approximation we consider here is suggested by

Cahn [5] :

$$\sigma_{Cahn} = \int_0^1 \frac{12\pi \Gamma_{ee}\Gamma_f \, dx}{[s(1-x)-M^2]^2 + M^2\Gamma^2} \cdot$$

$$\left\{ \beta x^{\beta-1} \left[1 + \frac{3}{4}\beta - \frac{\beta^2}{24}\left(\frac{1}{3} \ln \frac{s}{m_e^2} + 2\pi^2 - \frac{37}{4} \right) \right] \cdot (K\ factor) - \beta\left(1 - \frac{x}{2}\right) \right\} \tag{12}$$

Compared with Eq.(10), it drops the β^2 term in the hard photon contribution, so that the remaining terms can be integrated analytically if the upper limit of the integration is extended to infinity for the soft photon contribution. But here we took the upper limit to be 1 and use the numerical evaluation as we did for the exact cross section (10). The third approximation which we include is a simplified version of Eq.(8)

$$\sigma_{sim} = \frac{12\pi \Gamma_{ee}\Gamma_f}{s^2} \left\{ (1+\delta) a^{\beta-2} \phi(\cos\vartheta, \beta) + \right.$$
$$\left. + \left(-\beta + \frac{3}{4}\beta^2\right) a^{\beta-1} \phi(\cos\vartheta, \beta+1) \right\} \tag{13}$$

This expression can already achieve high precision in the application to some resonances like J/ψ .

In the calculation, we put the K factor = $1 + \frac{\alpha\pi}{3}$ for Z boson, corresponding $\sin^2\theta_W = 0.25$, so the coupling is purely axial vector, and K factor = $1 + \frac{\alpha}{\pi}(\frac{\pi^2}{3} - \frac{1}{2})$ for J/ψ, ψ'' and Υ . Also we put M_Z = 92.4GeV, Γ_Z = 2.6GeV; $M_{J/\psi}$ = 3096.9MeV, $\Gamma_{J/\psi}$ = 0.068MeV; M_Υ =9460.3MeV, Γ_Υ = 0.052MeV; $M_{\psi''}$ = 3769.9MeV, $\Gamma_{\psi''}$ = 25MeV. The deviation of our approximation (8) from the exact cross section (10), as well as three other approximations (11), (12), (13), are expressed in terms of four errors which are defined as

$$E_1 = 1 - \frac{\sigma_{soft}}{\sigma_{KF}} \qquad E_2 = 1 - \frac{\sigma_{cahn}}{\sigma_{KF}}$$
$$E_3 = 1 - \frac{\sigma_{sim}}{\sigma_{KF}} \qquad E_4 = 1 - \frac{\sigma_{SE}}{\sigma_{KF}} \tag{14}$$

Here σ_{SE}, σ_{KF}, σ_{soft}, σ_{cahn} and σ_{sim} are defined by Eq.(8), (10), (11), (12) and (13). The calculated results of these four errors are depicted by the four curves in Fig.(1a), (1b) (1c) and (1d) for Z boson, Υ, J/ψ and ψ'' states respectively.

In the figures (1a) to (1d), all four curves have a mimimum at the energy of the resonance mass. This is due to that near the resonance mass, the soft photon contribution dominates, while the hard photon emission is negligible. The approximations (11) and (12) include the same form of soft photon term as Eq.(10) does; while in the approximation (8) and (13), the analytical expression for the integration of the soft photon term achieves very high precision. Furthermore, the narrower the resonance is, the more prominent of the soft photon domination at the energy of the resonance mass. So for very narrow resonances like J/ψ and Υ, the minimum is a very sharp dip; while for wide resonances like Z boson and ψ'', it varies smoothly. In Fig.(1a), the error of our approximation (8), i.e. the curve 4 has a dip down to 0 at the energy above the resonance mass, this is due to that the error changes its sign at that point.

The figures (1a) to (1d) demonstrate the excellence of our approximation (8). For Z boson and ψ'', within the energy range ($M - 4\Gamma$, $M + 4\Gamma$), the error is within 0.1%. For J/ψ the error is also less than 0.1%, and for Υ, it is slightly larger, within the appropriate energy interval. Here in the experiments of these very narrow resonances like J/ψ and Υ, the scanned energy interval depends on the collider beam energy spread. Usually it is

no more than $(M - 10\Delta, M + 20\Delta)$, with Δ is the stanard deviation of the beam resolution function. In Fig.(1b) and (1c), the energy interval in which we did the calculation was taken from the planned experiments on Beijing Electron Positron Collider ($\Delta = 0.88$MeV) for J/ψ and from the experiments on DORIS at DESY ($\Delta = 8$MeV) for Υ [9].

In this paper, the soft photon terms which we consider are identical to the ones in the original work of reference [1]. In the literature, additional terms have been calculated for Z boson [2]. These results can be straightforwardly incorporated into our expression (8). Also we only considered standard Breit-Wigner formula (3), some authors suggested the energy-dependent resonant width in the Breit-Wigner formula [5,6]. The mathematical technique in this paper can be easily extended to such considerations.

4 TREATMENT TO THE OBSREVED NARROW RESONANCES

In this section, we discuss the radiative correction treatment to the experimentally observed resonances of J/ψ and Υ families. Many experiments have been done on them with e^+e^- collider. The main feature of these experiments is that those resonnat states below the charmed meson or beauty meson production threshold have widths which are narrower than the beam energy resolution of the e^+e^- collider. So the measured cross section is the cross section (1) folded with the beam energy resolution function $G(W,W')$ where W equals to the C.M. energy of the $e^+ e^-$

collider.

$$\sigma_{exp}(W) = \int_0^{\infty} G_T(W,W')\,\sigma(W')\,dW' \tag{15}$$

G(W,W') is usually taken as a Gaussion function.

With the expression (8) for $\sigma(W')$, Eq.(15) contains a one-dimensional integration which can be performed numerically. Actually, the simplified approximation for $\sigma(W')$ given by Eq.(13) already gets accurary of 0.1% for J/ψ and 0.2% for Υ . So in the following, we take the simplified expression of the observed cross section, although the complete formula (8) can be put into:

$$\sigma_{obs}(W) = 12\pi\Gamma_{ee}\Gamma_f \int_0^{\infty} dW' \cdot G(W,W')\,\frac{1}{W'^4}\cdot \left\{(1+\delta)\,a^{\beta-2}\phi(\cos\vartheta,\beta) + \left(\frac{3}{4}\beta^2-\beta\right)a^{\beta-1}\phi(\cos\vartheta,\beta+1)\right\} \tag{16}$$

with

$$G(W,W') = \frac{1}{\sqrt{2\pi}\,\Delta}\exp\left[-\frac{(W-W')^2}{2\Delta^2}\right]$$

here a, $\cos\vartheta$ and $\phi(\cos\vartheta,x)$ are defined in Eq.(7) with $\sqrt{s}$ = W' , and δ in Eq.(9).

Up to now, all the experimental data are analyzed with a method developed by Jackson and Scharre [7]. Their original work mistreated the radiative correction contribution, but here we have made the neccesary modifications based on Kuraev and Fadin's work [1] and present it in the following:

The experimentally observed cross section expressed by Jackson and Sharre is

$$\sigma_{JS} = (1+\delta)\int_{-\infty}^{+\infty} dW'\,\sigma_B(W')\,G_R(W,W') \tag{17}$$

where $G_R(W,W')$ is the so-called radiatively corrected resolution function

$$G_R(W,W') = \left(\frac{2\Delta}{W}\right)^{\beta} \frac{1}{\Delta} F\left(\frac{W-W'}{\Delta}, \beta\right)$$

in which Δ is the standard deviation of the beam energy resolution function. If $G(W,W')$ is in the form of Gaussian, then $F(z,\beta)$ can be expressed in terms of Weber's parabolic cylinder function $D_{-\nu}(x)$:

$$F(z,\beta) = \frac{\Gamma(1+\beta)}{\sqrt{2\pi}} e^{-z^2/4} D_{-\beta}(-z)$$

At this step, most of the experimental data annalysis based on Eq.(17) make an additional approximation on the Breit-Wigner formula:

$$\sigma_B(W) = \frac{12\pi^2 \Gamma_{ee}\Gamma_f}{\Gamma M} \delta(W^2 - M^2)$$

so that with this delta function approximation, the observed cross section is

$$\sigma_\delta(W) = \frac{6\pi^2 \Gamma_{ee}\Gamma_f}{\Gamma M^2} \left(\frac{2\Delta}{W}\right)^{\beta} \frac{1}{\Delta} F\left(\frac{W-M}{\Delta}, \beta\right)(1+\delta) \qquad (18)$$

To get an idea about the accuracy of this method, we have calculated the cross section by (17) with the integration evaluated numerically as well as (18), then compared them with the outcome by Eq.(16) which is also integrated numerically for J/ψ and Υ. In the calculation, the same values of mass and width

were used as in section 3. Also we took the standard deviation of the collider energy resolution function $\Delta = 0.88$ Mev for J/ψ and $\Delta = 8$Mev for Υ. The results are depicted in Fig. 2a, 2b for J/ψ and Fig. 2c, 2d for Υ respectively. It can be seen from these figures that the approximation (18) deviates from the correct value very badly at the energy $(6\sim 8)\Delta$ below the resonance peak, in the way that it dips too sharply. Besides, (18) also deviates from the correct value at the resonance peak. To show this more clearly, as well as the discripency of (17) with (16), we calculated the errors of (18) and (17) which are defined as

$$E_{\delta} = 1 - \frac{\sigma_{\delta}}{\sigma_{obs}}, \qquad E_{JS} = 1 - \frac{\sigma_{JS}}{\sigma_{obs}} \tag{19}$$

where σ_{obs} and σ_{JS} are calculted from Eq.(16) and (17) with the integration evaluated numerically, and σ_{δ} is from Eq.(18). The results are depicted in Fig.(3a) and (3b) for J/ψ and Υ respectively. They show that the error of Eq. 17) is at the order of 1% for J/ψ and Υ. In the figures, the dips down to 0 are due to the changing of the sign.

For future experiments when the accuracy needs to be better than 1% level, we recommend that the Eq.(16) to be used as the theoretical curve to fit the measured data, with the integration in it done numerically. Thus in the determination of the mass, width and leptonic partial width, the systematic uncertainty due to the treatment of radiative correction is pushed down to 0.2%. (The complete formula of Eq.(8) can be put into (15) to achieve the accuracy of 0.1%.) Besides the improved precision, an additional advantage of this method is that it does not depend on the

condition $\Gamma \ll \Delta$ as Eq.(18) does, so it can be used for whatever values of Γ and Δ . One more advantage of it is that with the integration done numerically, one can fit Γ and Γ_{ee} simultaneously, if the statistics of the data is good enough, and sufficient number of points are scanned in the appropriate energy range; whereas with the Eq.(18), one can only obtain the product $\Gamma_{ee}\Gamma_f/\Gamma$, as most of the previous experiments so far have done.

5 CONCLUTION

We have derived an analytical approximation for radiatively corrected resonant cross section presented in Eq.(8) which gets accuracy at the order of 0.1% for all the interested resonant states within the appropriate experimentally scanned energy range. For the experiments in which the collider beam energy resolution has to be taken into account, we recommend the use of the numerically integrated Eq.(15) as the theoretical curve to fit the data of the measured cross section. By this way, the mass, total width and leptonic partial width can be fitted simultaneously. The precision of the radiative correction treatment is at the order of 0.1%.

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References

[1] E.A.Kuraev and V.S.Fadin, Sov.J.Nucl.Phys.41(1985)466.

[2] G.Altarelli and G.Martinelli, Yellow Report CERN 86-02(1986)47;
O.Nicrosini and L.Trentadue, Phys.Lett.B196(1987)551.

[3] E.A.Kuraev, Private communication.

[4] T.S.Tsai, SLAC-PUB-3129(1983).

[5] R.N.Cahn, Phys.Rev.D36(1987)2666.

[6] F.A.Berends et al, CERN-TH 5468(1989).

[7] J.D.Jackson and Scharre, Nucl.Instrum.Methods128(1975)13.

[8] L.N.Lipatov Yad.Fiz.20(1974)181.
[Sov.J.Nucl.Phys.20(1975)94].

[9] W.Buchmuller and S.Cooper, MIT-LNS-159(1987).

Figure Captions

Fig. 1. The errors of approximations (11), (12), (13) and (8), which are shown by curve 1, 2, 3 and 4 respectively, (a) is for Z boson, (b) is for Υ , (c) is for J/ψ , and (d) is for ψ''. The errors are defined in Eq.(14). Here the parameters used are:

$M_Z = 92.4$GeV, $\Gamma_Z = 2.6$GeV; $M_\Upsilon = 9.4603$GeV, $\Gamma_\Upsilon = 0.052$MeV;

$M_{J/\psi} = 3.0969$GEV, $\Gamma_{J/\psi} = 0.068$MeV; $M_{\psi''} = 3.7699$GeV, $\Gamma_{\psi''} = 25$MeV.

Fig. 2. The experimentally observed cross section calculated by Eq.(18), (17) and (16), they are shown by curve 1, 2 and 3 respectively. (a) and (b) are for Υ , (c) and (d) are for J/ψ . The parameters used are:

$M_\Upsilon = 9.4603$GeV, $\Gamma_\Upsilon = 0.052$MeV, $\Delta = 8$MeV;

$M_{J/\psi} = 3.0969$GeV, $\Gamma_{J/\psi} = 0.068$MeV, $\Delta = 0.88$MeV.

Fig. 3. The errors of approximations (18) and (17), which are shown by curve 1 and 2 respectively. (a) is for Υ , and (b) is for J/ψ . The errors are defined in Eq.(19). The parameters used are the same as in Fig. 2.

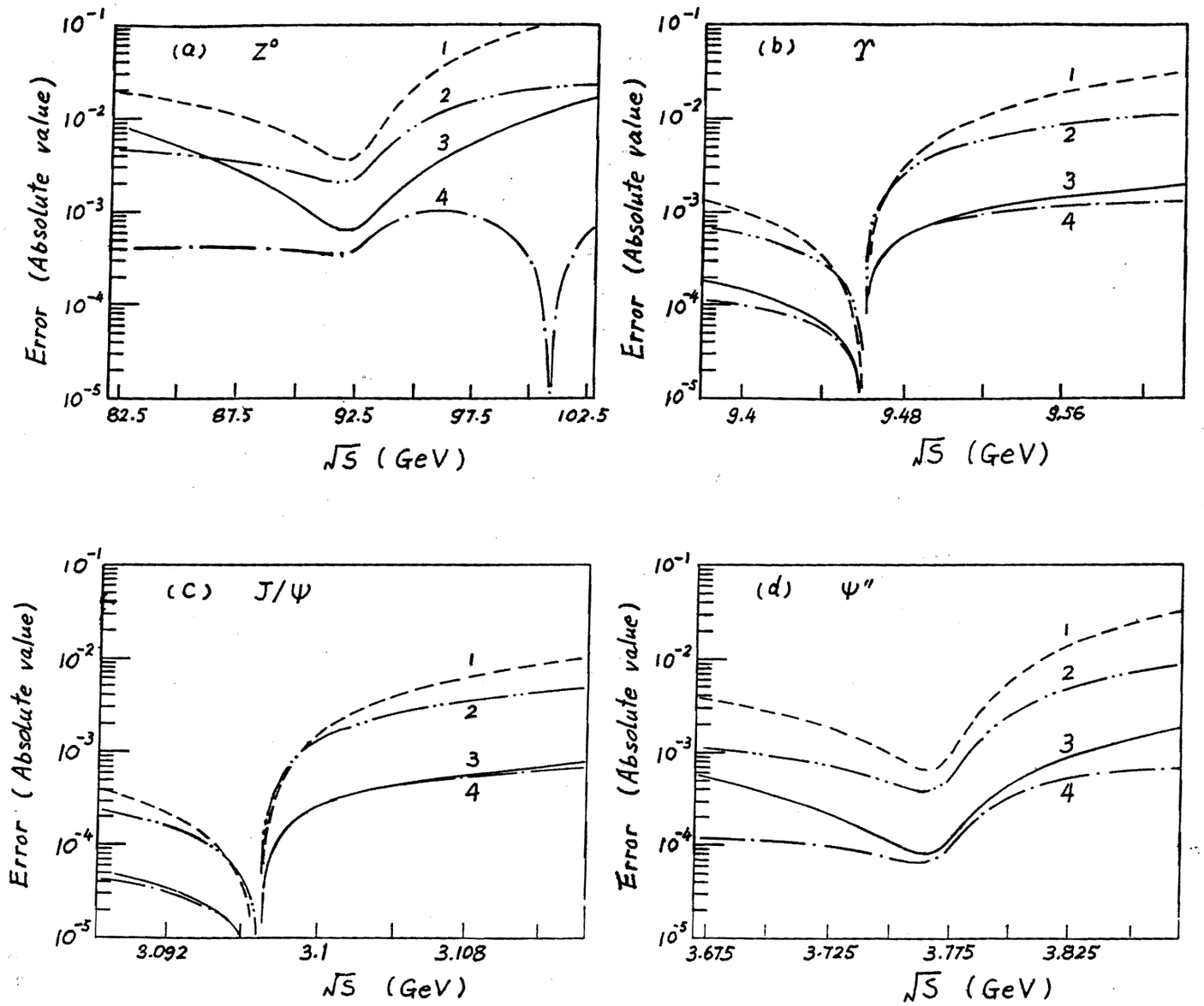
(a) Z°
(b) ϒ
(c) J/ψ
(d) ψ''
Error (Absolute value)
√S (GeV)
1
2
3
4
82.5
87.5
92.5
97.5
102.5
9.4
9.48
9.56
3.092
3.1
3.108
3.675
3.725
3.775
3.825

FIG. 1

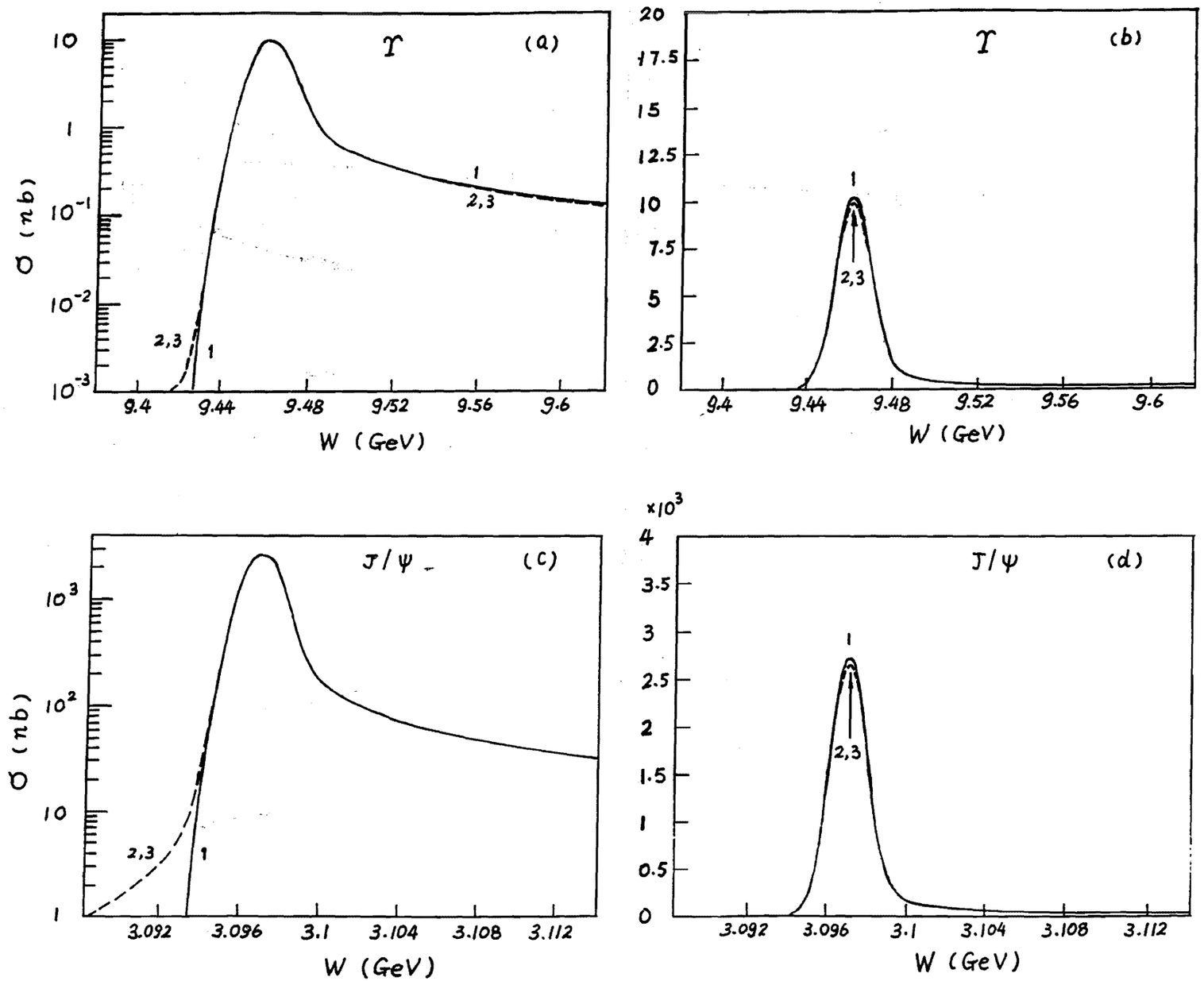

FIG. 2

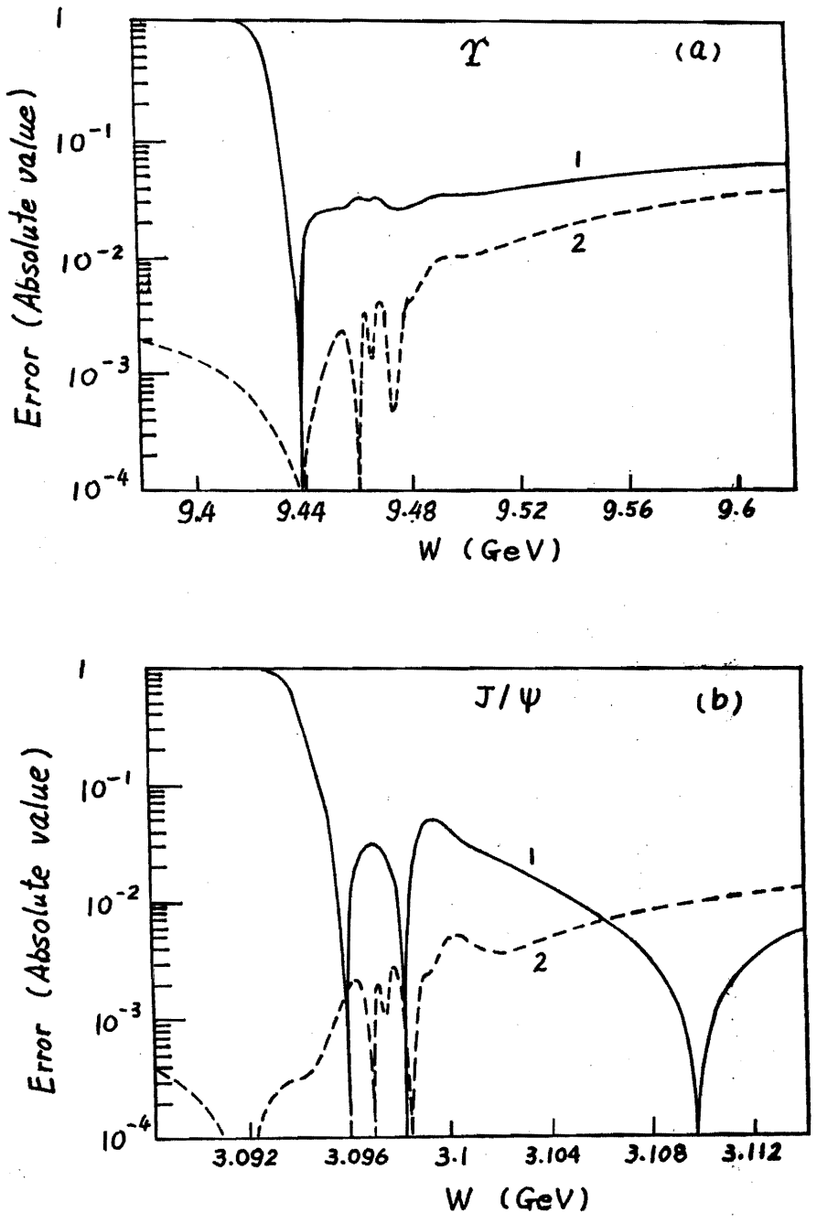
Υ
(a)
1
2
Error (Absolute value)
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
9.4
9.44
9.48
9.52
9.56
9.6
W (GeV)
J/ψ
(b)
1
2
Error (Absolute value)
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
3.092
3.096
3.1
3.104
3.108
3.112
W (GeV)

FIG. 3