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Thermal Gas Models and Particle Production in Heavy Ion Collisions

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ABSTRACT

A systematic study of particle production in nuclear S-S and S-W collisions at 200 GeV/A is presented within the context of an equilibrium interacting hadron gas model. It is shown that the results for strange particle multiplicities and for non-strange baryons obtained in the NA35 and WA85 experiments can be well described in terms of the considered model.

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1 Introduction

Relativistic heavy ion reactions provide a useful tool for studying matter at high densities and temperatures. Sufficient energy is available to excite new collective modes such as a quark gluon plasma [1]. It has been suggested that a remnant of the quark gluon plasma might be observable as an enhancement of the production of strange particles as compared with that seen in p-p or p-A collisions [2]. Recent experiments have therefore concentrated on the measurement of the strange hadron multiplicities in relativistic heavy ion collisions [3] [4]. An enhancement of the strange hadrons has been observed in all of these experiments. The measured $\Lambda, \overline{\Lambda}$ and K^0_s multiplicities in the NA35 experiment for central S-S collisions at 200 GeV/A [3] exceed the re-scaled p-p productions by a factor of ~ 1.5. An increase in the $\overline{\Xi}/\Xi$ ratio has also been observed for central S-W collisions in the WA85 experiment [4]. Also, an enhancement has been seen in the Λ/h^- ratio as a function of charged particle multiplicity h^- [3]. In order to ascertain the origin of this strange particle enhancement it is crucial to understand to what extent these results can be explained in terms of an equilibrated interacting relativistic hadron gas model. In such a model, strange particles are produced thermally without the requirement of the initial formation of a quark gluon plasma.

Recently, different hadron gas models [9][5] [6] [7] [8] have been used to explain the observed strange particle enhancement. However, a systematic study of all experimental data has not been undertaken.

In this paper we present a detailed study of thermal particle production in a relativistic hadron gas model which is consistent with the present experimental data with the exception of the pion multiplicity. Both baryon number and strangeness conservation have been taken into account which is crucial in obtaining a consistent description of the available experimental data.

2 The Thermal Model

Equilibrium hadron gas models describe hadronic matter in terms of a gas of hadrons with different masses, where the individual mass states are populated according to equilibrium distributions (Fermi-Dirac or Bose-Einstein). The conservation of quantities such as baryon number and strangeness is achieved either within the grand canonical (GC) ensemble via the introduction of appropriate chemical potentials [6] [11], or in the canonical ensemble [12] [13] [14] [15]. Although the limitations of the GC formulation

for small systems should be noted, its computational simplicity makes it the prefered choice for our calculations. In what follows, we shall thus have two chemical potentials, μ_b and μ_s, corresponding to the conservation of baryon number and strangeness respectively.

Hadrons are strongly interacting particles, and any realistic model must include the effects of these interactions in some way. While these interactions should ideally be derived from first principles within the framework of QCD, this approach is at present unfeasible. It is therefore necessary to resort to a phenomenological description of the particle interactions in the framework of a hadron gas model.

Since we will apply hadron gas models to situations of high temperatures ($T > 100$ MeV) and densities, the important part of the hadron-hadron interactions will be the short range repulsive piece. While there are certainly attractive interactions, to explain the stability of nuclear matter at normal nuclear matter density ($n_0 = 0.17$ fm^{-3}), the effect of these interactions at the temperatures considered will be negligible. In this paper we will therefore only concentrate on the repulsive hadron-hadron interactions.

There are several approximations for these interactions, including the excluded volume approximation, in which the hadrons are given a hard-core volume in a van der Waal's fashion [5] [16], and the mean field approximation, which we will use in the present work [17] [18] [19]. The central idea of the mean field approximation is to start with some two body interaction, and to calculate the self-energy correction to the single particle energy in the Hartree approximation. In this approximation the self-energy correction is proportional to the number density of the hadron species [20], so that the single particle energy ϵ_i of a hadron of mass m_i and momentum $\vec{p}$ is given by

$$\epsilon_i = \sqrt{\vec{p}^2 + m_i^2} + K_i n_i. \tag{1}$$

Here K_i is the strength of the repulsive potential and n_i is the average number density of hadron species i. This then leads to a simple transcendental equation for the number density n_i; for example, for a Maxwell-Boltzmann gas of component i (ignoring chemical potentials for simplicity) we obtain

$$n_i = \left[d_i \int_0^\infty dp\, g(p)\, \exp[-\beta\sqrt{\vec{p}^2 + m_i^2}]\right] \exp[-\beta K_i n_i] \tag{2}$$

where d_i is a degeneracy factor such as spin-isospin, $g(p)$ is the density of single particle momentum states, and $\beta = 1/T$ is the inverse temperature.

It is important to note that the inclusion of interactions in a hadron gas model can require modifications to the grand potential Ω in order to ensure that the interaction is not double counted. This requirement is known as thermodynamic consistency. It is easy to show that the grand potential for a single component hadron gas of species i with interactions treated in the mean field approximation is given by

$$\Omega_i = d_i \frac{T}{a_i} \int_0^\infty dp\, g(p) \ln\{1 - a_i \exp[-\beta(\epsilon_i - \mu_b b_i - \mu_s s_i)]\} - F_i^c \qquad (3)$$

where $a_i = 1$ for bosons, $a_i = -1$ for fermions and b_i and s_i are the baryon number and strangeness respectively of hadron species i. The function F_i^c originates in the energy density and corrects for double counting of the interaction. For interactions of the form (1), $F_i^c = \frac{1}{2} K_i n_i^2 V$, where V is the volume of the system. It should be noted that thermodynamic consistency for models in which the hadronic interactions are treated in the excluded volume approximation is more subtle, requiring either the replacement of the total volume as a free parameter by the effective volume (*i.e.* the total volume minus the volume occupied by the hadron hard cores) [16], or the modification of the chemical potentials [21] [22].

In theory, of course, there should be a K_i for each hadron species. However, since there are approximately 150 known hadron states (stable and resonances) with masses below 2 GeV, this is not viable. Also, attempts might be made to take interactions between hadrons of different types into account in a model, *e.g.* meson-baryon interactions. As the form of these interactions is not generally known due to spin and isospin dependences, the following assumptions are usually made [17] [23]:

- Hadrons of a particular type (meson, baryon or antibaryon) interact only with hadrons of the same type.
- The interactions between all species of a particular hadron type are identical, so that the self-energy correction (in the Hartree approximation) becomes $K_\alpha n_\alpha$ where K_α is the potential strength for hadrons of type α and n_α is the total average number density of hadrons of type α.
- Antibaryons interact among themselves in the same way as baryons, *i.e.* $K_b = K_{\bar{b}}$.

With these assumptions, the grand potential for a gas of hadrons of different species becomes

$$\Omega = \sum_i d_i \frac{T}{a_i} \int_0^\infty dp\, g(p) \ln\{1 - a_i \exp[-\beta(\epsilon_i - \mu_i)]\} - \frac{1}{2} \sum_\alpha K_\alpha N_\alpha^2 / V, \qquad (4)$$

where the sum over i runs over hadron species, and the sum over α is taken over hadron types.

In our subsequent calculations, we take into account all hadron species with masses less than 2 GeV. Previous calculations, in which the higher mass resonances were included in the form of a continuum level density [23] indicate, that for temperatures below 200 MeV, the contribution of these states to thermodynamic observables is of the order of 1-2%. In the light of the other approximations made in the model described here, the neglect of the higher mass resonances is justified. However, the contribution of the hadronic states with masses in the range 1-2 GeV have been found to be important in the calculation of particle multiplicities.

We thus have to find two effective potential strengths, K_m and K_b, which fulfill the role of compressiblity coefficients for the meson and baryon sectors respectively. A reasonable value for K_m is that obtained from Weinberg's effective Lagrangian for the $\pi - \pi$ interaction [24], which yields a value of around 600 MeV fm^3. This value has been quite widely used, and should be an acceptable approximation for a general meson-meson repulsive interaction since the pion, being the meson with the lightest mass, is expected to dominate the total meson number density.

To find a value for K_b, we make use of the $N - N$ interaction. However, very different values for K_b are obtained depending on the two-body interaction used. For example, on the grounds that it costs approximately 500 MeV of energy to force two nucleons to overlap completely [25], a rather arbitrary parameterization of the $N - N$ interaction of the form $V(r) = 500 \exp[-m_\omega r]$ MeV, where m_ω is the ω-meson mass, leads to $K_b = 200$ MeV fm^3 [19]. The use of a repulsive ω-exchange potential, *i.e.* $V(r) = 36\pi \exp[-m_\omega r]/r$ yields $K_b = 1700$ MeV fm^3 [26]. Between these two extremes, the Reid soft core potential [27] gives $K_b = 680$ MeV fm^3 [18]. In our subsequent calculations, we will use this last value. This choice is motivated by the success of the Reid potential in studies of nuclear systems. As it turns out, the results of calculations are (fortunately) not too sensitive to the precise value of the potential strengths; the important point is that there should be some non-negligible repulsive interactions in the calculations.

In applying hadron gas models to heavy ion collisions, it is important that the effect of small system size be taken into account when performing the integrals over phase space. A simple method of including such effects is through a modification of the density of states $g(p)$. A careful analysis of the derivation of $g(p)$ (see for example [28]) shows that the form of $g(p)$ depends on both the surface area of the system and its linear dimension; the exact dependence is determined by the boundary conditions imposed on the single particle states.

If the single particle states obey Dirichlet or von Neumann boundary conditions, the density of states $g(p)$ must be modified to [28]

$$g(p) = \frac{V}{2\pi^2}p^2 \pm \frac{A}{8\pi}p + \frac{L}{8\pi}, \quad (5)$$

where A and L are the surface area and linear dimension of the system of volume V respectively. For spherical systems, which we will consider here, $V = 4\pi R^3/3$, $A = 4\pi R^2$ and $L = 2\pi R$ where R is the radius of the system. The positive and negative signs in (5) refer to von Neumann and Dirichlet conditions respectively. Clearly, in the limit $R \to \infty$ the volume-dependent term dominates $g(p)$, as would be expected.

The use of (5) requires some care, since the error implicit in its derivation is of the same order of magnitude as the last term in (5) [28]. One should therefore check that this term is small compared to both the volume- and surface area-dependent terms, and, in the case of Dirichlet boundary conditions, their difference.

Another important point which should be included in a hadron gas model when applied to heavy ion collisions is the influence of resonance decays on the measured hadron multiplicities. After the hadrons freeze out, they free stream towards the detectors. The high mass resonances will then decay, increasing the number of low mass hadrons which are experimentally observed. To take this into account in a hadron gas model, we allow the high mass hadrons to decay according to the measured branching ratios [29]. The total multiplicity of a particular low mass hadron species is then the sum of the thermal contribution and the decay products of heavier resonances.

3 Particle Production

Having established a thermodynamical model of hadronic matter produced in heavy ion collisions, we can now calculate different particle multiplicities. It is clear that the thermal multiplicities depend on the set of freeze-out parameters T, μ_b, μ_s and V. In order to compare the thermal particle production with that measured in the NA35 experiment, we have to fix the above four parameters from experimental data. Two of those parameters can be determined from the requirement of strangeness and baryon number conservation. We have assumed that the total strangeness of equilibrated hadronic matter is equal to zero. Thus we have not allowed for strange particle production in the pre-equilibrium state which could escape from the system before

thermalization.

The central trigger in the NA35 S-S collisions was a zero degree calorimeter, which ensured that only events in which less than 1/6 of the incoming energy was deposited in the forward direction were selected. This also implies that only 4-6 of the 32 projectile nucleons are spectators. Thus the lowest value of the baryon number for S-S collisions in NA35 experiment would be ~ 52 [3] [30]. On the other hand, the baryon number cannot be larger that 64. In our actual calculation we have fixed the baryon number $B \sim 58$, which is the average between the minimum and maximum values expected in the experiment.

By constraining the average strangeness and baryon number to the values indicated above we can determine two parameters relevant for the estimation of mean values of particle multiplicities. The two parameters left we fix by equating the thermal averages of lambda and anti-lambda particles to the experimental mean values $\Lambda = 8.2$ and $\overline{\Lambda} = 1.5$, as measured in the NA35 experiment for central S-S collisions.

In Table 1 we show the results of our model for the values of the four relevant thermal parameters at freeze-out. These results have been obtained as explained above to satisfy baryon number and strangeness conservation as well as to reproduce the experimentally measured multiplicities of Λ and $\overline{\Lambda}$ particles. The radius at freeze-out in Table 1 is calculated assuming that the system has spherical symmetry. Decreasing the baryon number to its lowest possible value would slightly change the temperature whereas μ_b would decrease by $\sim 10\%$ compared to the results indicated in the above table.

In order to investigate how relevant our model could be in explaining the particle production observed in the NA35 experiment, some other particle multiplicities have to be calculated with the parameters indicated in Table 1, and then compared with available experimental data. The model predictions and the experimental results are shown in Table 2 and 3. As seen in the tables, we obtain excellent agreement for K_s^0, K^+, K^- and proton production between the model and the NA35 data. We have also shown in Table 3 the value of the $\overline{\Xi}/\Xi$ ratio as measured in the WA85 S-W experiment. This value agrees with the model. The experimental ratio, however, is obtained for the central kinematic region. The 4π value, which should be compared with the results of the thermal model, is not available. If the $\overline{\Xi}/\Xi$ behaves in the same way as the $\overline{\Lambda}/\Lambda$ ratio, which increases with the extrapolation from the central region to 4π [3], the agreement between the experimental and calculated values for the $\overline{\Xi}/\Xi$ ratio may be even better than shown in the table.

For completeness we have quoted in Table 2 the results for some other strange particle multiplicities (calculated with the model using the parameters indicated in Table 1) which has not been experimentally measured until now.

It is clear from Table 3 that the pion multiplicity calculated from the thermal model is $\sim 30\%$ less than the experimental value. Several mechanisms have already been suggested in the literature [7] [10] [31] to explain the discrepancies of measured multiplicities and predictions of different thermodynamical models. In particular, the low p_t enhancement of pion spectra has been found to be well reproduced by assuming a positive chemical potential for pions or a non-thermal pion distribution [10]. A fit to the pion p_t distribution with two different temperatures, corresponding to pion gas and quark gluon plasma contributions also gives good results [31]. The decay of higher resonance states into pions has been found to have important consequences for the soft pion spectrum [7]. Some of these calculations have reproduced almost all available experimental data on the p_t spectra of measured particles [7]. In our calculations, as mentioned above, we could not reproduce the experimental results for pion multiplicities by requiring full agreement with all available experimental data and assuming strangeness and, in particular, baryon number conservation.

The lack of agreement between experiment and the model for the pion multiplicity could be related to our assumption that the system reaches complete chemical equilibrium. Recently, however, it has been argued [2] that one could not expect to reproduce the abundance of strange particles in central heavy ion collisions by applying hadron gas model. It would therefore be interesting to check how sensitive our results are to the degree of chemical equilibration. In particular one could expect that by overestimating the strange particle multiplicities by taking their equilibrium values, the entropy content of pions would be underestimated. Consequently, the pion multiplicity decreases.

The analysis of particle production in a non-equilibrium gas model would require a detailed study based on kinetic theory. In order, however, to test how important non-equilibrium factors could be in explaining the experimental data we make a simple model. Denoting by a factor $\gamma < 1$ the deviation from the absolute chemical equilibrium abundance of strange and anti-strange particles, i.e replacing the fugacities $\lambda_s = \exp[\mu_s/T]$ and λ_s^{-1} by $\gamma\lambda_s$ and $\gamma\lambda_s^{-1}$ in the grand potential in (4) we have constructed an effective non-equilibrium model. This model contains an additional parameter γ accounting for non-equilibrium effects in the strange particle sector. We can now ask whether it is possible to find a set of five parameters which will reproduce the experimental data of the NA35 Collaboration. With strangeness conservation and by requiring that the total $\Lambda, \overline{\Lambda}$, n_{π^-} and K_s^0 multiplicities be equal to their experimentally measured values [3], we have found a consistent set of parameters. Values of

$\gamma \sim 0.5$, $T \sim 180$ MeV and $\mu_b \sim 280$ MeV are required. However, when calculating the baryon number with this set of parameters, we find $B \sim 100$. This evidently violates baryon number conservation, as the maximal value of the baryon number allowed for S-S collisions is $B = 64$.

An increase of pion multiplicity in the thermal model under consideration can only be achieved by increasing the freeze-out volume as the temperature should not be larger than the critical deconfinement value T_c, where $150 < T_c < 250$ MeV. In taking a reasonable value for the temperature and adjusting the volume to reproduce the pion multiplicity $n_{\pi^-} \sim 100$ within a thermodynamical hadron gas model, we will therefore most likely violate baryon number conservation.

An increase in the pion multiplicity can also be achieved by allowing for a positive chemical potential $\mu_\pi < m_\pi$ for pions, as suggested in [10]. In terms of our model only 25% of the pions have a thermal origin, with all others being due to the decay of higher mass resonances. Thus if one were to include an additional factor $\exp(\mu_\pi/T)$ in the calculation of the pion multiplicity, then with $\mu_\pi \sim 130$ MeV one could get at most 15 additional pions in the final state. This is still not sufficient to reproduce the experimental value.

The above discussion suggests that the non-equilibrium effects of strangeness production in hadronic matter may not be sufficient to reproduce all the experimental NA35 data. Also the non-equilibrium pion production calculated in a hadronic model which allows pions to be produced from the decay of higher mass resonances does not seem to be sufficient to explain these data.

Besides the enhancement of strangeness production there are also other interesting features of the experimental data reported by the NA35 Collaboration which could suggest a non-trivial behaviour. The Λ^{acc} and $(K_s^0)^{acc}$ multiplicities (the superscript "acc" indicates that the multiplicities are taken within some fiducial acceptance region) divided by the multiplicity of negative hadrons h^- ($\sim$95% of which are π^-), shows a rapid increase with increasing h^-. Here h^- is the parameter which measures the centrality of the collisions [3]. If S-S collisions were a sum of independent p-p interactions then all multiplicities would be proportional to the average number of such collisions, *i.e.* the ratios Λ^{acc}/h^- and $(K_s^0)^{acc}/h^-$ would be independent of h^-. The observed increase of these ratios with charge particle multiplicity is a clear sign of some non-trivial collective phenomenon in S-S collisions.

This behaviour of the ratios has recently been explained using a picture based on quark gluon plasma formation [32]. It is interesting to see to what extent a thermal gas model can reproduce this property. There are, however, immediate difficulties when trying to understand the dependence of Λ^{acc}/h^- and $(K_s^0)^{acc}/h^-$ ratios on the charged particle multiplicity. Firstly, the particle multiplicities in the experimentally measured ratios have been taken in some acceptance region. Thermal models are appropriate for bulk particle production but are less reliable in the calculation of particle production in a given kinematical window. Secondly, the model considered does not correctly reproduce the pion multiplicity, so that we cannot make a direct comparison with the experimental results. We can, however, assume that extending the Λ and K_s^0 multiplicities in the Λ^{acc}/h^- and $(K_s^0)^{acc}/h^-$ ratios to their full phase space values one would expect to see the same relative increase of these ratios when going from periferal to central collisions. In fact, restricting the h^- multiplicity to the acceptance region appropriate to Λ or K_s^0 leads to a similar rapid increase in the $(\Lambda/h^-)^{acc}$ and $(K_s^0/h^-)^{acc}$ ratios when going from periferal to central collisions [3].

We therefore proceed as follows. We first perform a minimum χ^2 fit to the experimental multiplicities as functions of h^-. This step is necessary due to the large error bars on the experimental data [3]. We then assume the same proportional decrease in the negative hadron multiplicity in our calculations as has been observed in the experimental results for the transition from central to periferal collisions. We then fit parameters to reproduce the Λ, K_s^0 and h^- multiplicity appropriate for each set of experimental conditions.

The results of this procedure are plotted in Figures 1 to 3 where we show the fitted parameters as functions of the negative hadron multiplicity, normalized to its value for central collisions, *i.e.* $h^-/h^-_{central}$. As one might expect, the freeze-out temperature decreases steadily as one moves from central to periferal collisions, due to the lower energy deposition in the periferal collisions. The chemical potentials change smoothly during the transition.

As mentioned above, the experimental measurements of the particle multiplicities, particularly for the minimum-bias, non-periferal events, have large error bars. To reflect this, we have also shown in each of Figures 1 to 3 the parameters for fits to the experimental mean values. The chemical potentials show the most change, while the temperature displays a rather small variation with the change from the minimum χ^2 fit to the experimental means.

Since we are able to find sets of physically reasonable parameters which reproduce the experimental behaviour of the hadron multiplicities with changing centrality, we may conclude that the thermal model considered in this paper reproduces in a qualitative way the experimentally observed dependence of neutral strange particle multiplicities as functions of h^-.

4 Conclusions

We have made use of a thermodynamical model, with the assumption of complete thermal and chemical equilibrium, to describe particle production in relativistic heavy ion collisions. The main purpose of this paper was to investigate whether such an approach can be used to understand the enhancement of the strange particle production which was recently reported by the NA35 and WA85 Collaborations. Since we require only baryon number and strangeness conservation, two experimental particle multiplicities are needed to specify the model. All the remaining hadron multiplicities can then be determined and compared with the experimental results.

We have shown that the thermodynamical model gives excellent agreement with all experimental results for strange particles and non-strange baryon multiplicities measured by the NA35 and WA85 Collaborations (see Table 3). We have also shown that the observed increase of Λ/h^- and K_s^0/h^- ratios with increasing centrality can be explained within this model.

A discrepancy between the thermodynamical gas model and the experimental results occurs when one compares the pion yields. We have suggested that this discrepancy cannot be explained by simply including non-chemical equilibrium effects in the thermal hadron gas model. However, the lack of agreement on the level of pion multiplicities does not necessarily invalidate our approach. Pions can be produced in rescattering processes in the initial and final states. We therefore conclude that the predictions of the thermal gas model considered in this work are consistent with the presently available experimental data.

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Central ^{32}S - ^{32}S Freeze-out Parameters						
T (GeV)	R (fm)	μ_b (GeV)	μ_s (GeV)	n_m (fm^{-3})	n_B (fm^{-3})	B
0.170	5.09	0.257	0.076	0.194	0.104	58

Table 1: *Thermodynamic parameters at freeze-out (temperature T, radius R, baryon- and strange chemical potentials μ_b and μ_s, meson density n_m, baryon number density n_B, baryon number B) obtained from fits to measured Λ and $\overline{\Lambda}$ multiplicities for the NA35 central ^{32}S - ^{32}S experiment.*

Central ^{32}S - ^{32}S Hadron Multiplicities							
Mesons							
	π^+	π^-	K^+	K^-	K_s^0	η	
	57.1	56.9	14.25	7.15	10.7	2.90	
Baryons							
p	$\bar{p}$	Λ	$\overline{\Lambda}$	Ξ	$\overline{\Xi}$	Ω	$\overline{\Omega}$
23.3	1.82	8.20	1.50	1.85	0.83	0.12	0.13

Table 2: *Calculated hadron multiplicities for the NA35 central ^{32}S - ^{32}S experiment.*

Central ^{32}S - ^{32}S Hadron multiplicities							
	K^0_s	K^+	K^-	Λ	$\bar{\Lambda}$	p	π^-
Model	10.7	14.2	7.15	8.2‡	1.5‡	23.2	56.9
Experiment	10.7 ± 2.0	12.5 ± 0.4	6.9 ± 0.4	8.2 ± 0.9	1.5 ± 0.4	22 ± 2.5	92.7 ± 4.5
Hadron production ratios							
		$\bar{\Lambda}/\Lambda$	K^0_s/Λ	$\bar{\Xi}/\Xi$	π^-/p	K^0_s/π^-	
Model		0.18	1.3	0.45	2.5	0.19	
Experiment		0.183 ± 0.053	1.304 ± 0.283	0.39 ± 0.07*	4.2 ± 0.8	0.11 ± 0.02	

‡ Experimental values from NA35 Collaboration used for fitting.
* Central rapidity value from WA85 Collaboration, see text for details.

Table 3: *Comparison of calculated and measured hadron multiplicities and production ratios for the NA35 central ^{32}S - ^{32}S and WA85 central ^{32}S - ^{184}W experiments.*

Figure Captions

Figure 1

The freeze-out temperature T as a function of the negative hadron multiplicity h^- normalized to its value for central collisions $h^-_{central}$. The solid curve is determined using the minimum χ^2 hadron multiplicities (see text). The points marked with $\times$ are determined using the experimental mean values for the hadron multiplicities.

Figure 2

The baryon chemical potential at freeze-out μ_b as a function of the negative hadron multiplicity h^- normalized to its value for central collisions $h^-_{central}$. The solid curve is determined using the minimum χ^2 hadron multiplicities (see text). The points marked with $\times$ are determined using the experimental mean values for the hadron multiplicities.

Figure 3

The strange chemical potential at freeze-out μ_s as a function of the negative hadron multiplicity h^- normalized to its value for central collisions $h^-_{central}$. The solid curve is determined using the minimum χ^2 hadron multiplicities (see text). The points marked with $\times$ are determined using the experimental mean values for the hadron multiplicities.

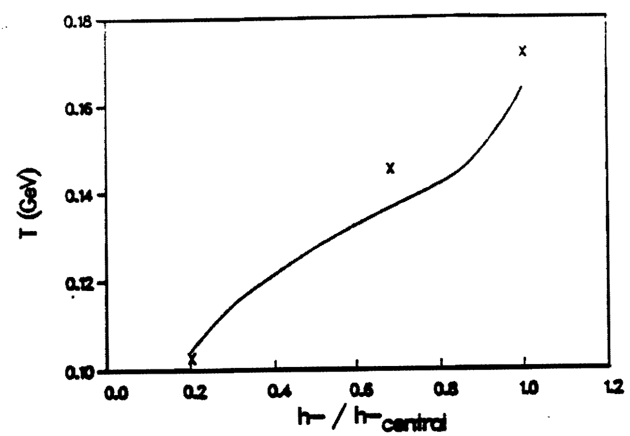

Fig.1

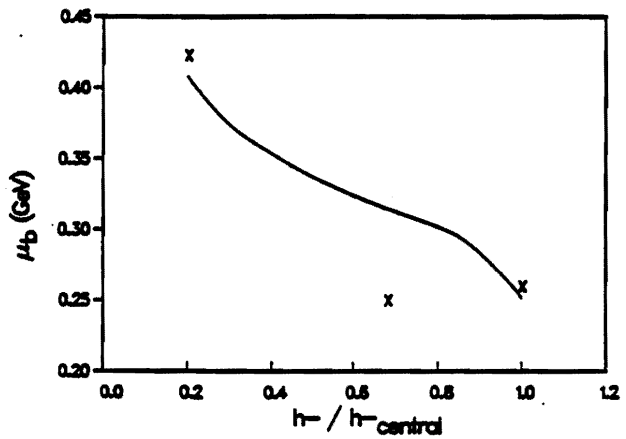

Fig.2

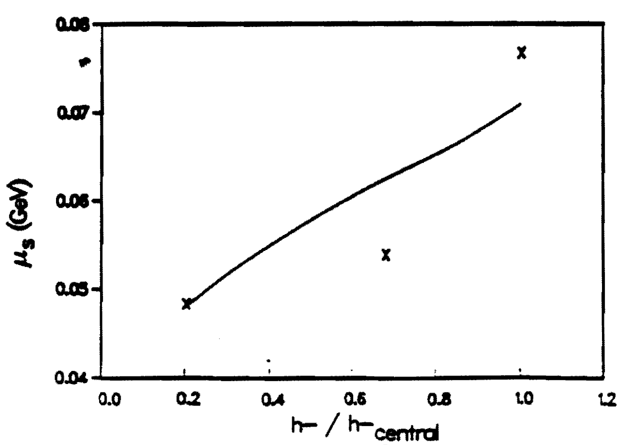

Fig.3