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Observable Mixing Angles and Phases In MSSM

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Abstract

We examine the number of observable mixing angles and phases in minimal supersymmetric standard model. Their total number for three family MSSM including quark (squark), lepton (slepton) and gaugino sectors exceeds one hundred. Furthermore taking into account the physical masses increases the total number of free parameters above two hunderds. This extremely large number indicates that SUSY should be realized at a more fundamental (preonic) level. It is of interest that even a single family MSSM covers observable angles and phases which induce too large contribution to neutron electric dipole moment unless supressed by two or three orders of magnitude.

As is well known the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] is a standard candidate for the explanations of the origin of the CP violation. If the number of families is three or more a CP violating phase can naturally be introduced into the quark sector of the standard electroweak theory. The number of the observable mixing angles N_θ and complex phases N_ϕ for the n-generation standard model are given by the following well-known formulas :

$$N_\theta = \frac{n(n-1)}{2} \quad , \quad N_\phi = \frac{(n-1)(n-2)}{2} \tag{1}$$

On the other hand, various extensions of the standard model (SM) are extensively discussed in the literature. Among them the minimal supersymmetric extension of SM (MSSM) [2] is considered to be internally consistent and so far flexible enough to avoid conflict with any experiment. The principal theoretical motivations for SUSY can be stated as follows: i) solution to the hierarchy problem, ii) unification of gauge coupling constants at GUT or Planck scale, iii) agreement with the standard model precision tests, iv) consistency with the value of $sin^2\theta_W$. On the other hand there are several negative aspects to be mentioned. First of all the model contains too many parameters. Secondly, CP and $FCNC$ data restrict severely them and finally there is no experimental evidence for a SUSY particle yet. In this brief report we will examine some of these issues.

In MSSM additional phases may appear due to the complexity of the soft SUSY breaking couplings [3]. In principle it is possible to calculate the number of these extra phases and angles when the SUSY parameters are generally taken to be complex. For example quark-squark-neutralino and the quark-squark-gluino interactions can bring many new mixing angles and phases; their total numbers, which include also the usual CKM mixings, are found to be [4] :

$$N_\theta = n(5n-3) \quad , \quad N_\phi = n(5n-3) \tag{2}$$

As can be seen from the above expressions the MSSM with three families contains (in the quark-squark sector) $N_\theta = 36$ observable angles and $N_\phi = 36$ observable phases. The interesting point is that MSSM includes two mixing angles and two observable complex phases even in the simplest case of one family; $N_\theta = N_\phi = 2$ for $n = 1$.

This analysis can be repeated in the lepton-slepton sector of the MSSM, assuming that the neutrinos are massive Dirac particles, which gives new parameters whose numbers are exactly analogous to those appearing in Eqn.(2). The possibility of treating neutrinos as Majorana particles will further increase these numbers.

In SUSY models CP violating phases and angles appear also in the mixing between the supersymmetric partners of the charged $SU(2)_L \times U(1)$ gauge bosons and Higgs; $\tilde{W}^{\pm}, \tilde{H}^{\pm}$. It has been shown that the diagonalization of their mass matrix to obtain the charginos $\tilde{\chi}_1^{\pm}$, $\tilde{\chi}_2^{\pm}$ give two angles and four phases after suitable redefinitions of the parameters [5].

As to the neutral Majorana fermions, $\tilde{W}_3, \tilde{B}, \tilde{H}_1^0, \tilde{H}_2^0$, they will mix in principle, with non-absorbable phases inducing possible CP-violation effects [6]. Their mass matrix can be taken, in general, to be complex symmetric and therefore diagonalization through unitary transformations on the fields will introduce 6 angles and 10 phase factors. All these considerations show us that in a minimal supersymmetric extension of the standard model a great number of observable new angles and CP-violating phases appear. Specifically these are $N_\theta \geq 36 + 36 + 2 + 6 = 80$ mixing angles and $N_\phi \geq 36 + 36 + 4 + 10 = 86$ phases for three family MSSM. One should add to these the following 45 mass parameters: 6 for quarks, 6 for leptons (if the neutrinos are Dirac particles), 12 for squarks, 12 for sleptons, 2 for charginos, 4 for neutralinos and finally 3 for the Higss sector. Consequently the number of physically observable parameters put by hand exceed two hundreds for three family MSSM.

In the literature there are several other methods to count the physical parameters. Recently Dimopoulos and Sutter [7] have also reexamined the supersymmetric flavour problem and pointed out that it is not possible to simultaneously satisfy the constraints from both flavour mixings and naturalness. To count the physical mixing angles and phases in MSSM they studied the breaking of the $U(3)^5$ global flavour symmetry of the gauge part of the Lagrangian by the Yukawa and triscalar (sfermion-sfermion-Higgs) interactions and scalar mass terms. Their analysis indicates that MSSM has 110 physical parameters in its flavour sector: 30 masses, 39 real mixing angles and 41 phases.

Actually these number could be much larger because any SUSY extension of the standard model posseses eleven flavour matrices rather than the three Yukawa matrices of the standard model because one should treat right and left sfermions for each generation independently:

$$\begin{aligned} L_{triscalar} = \tilde{q} A_{u_L} \tilde{\bar{u}}_L H_u + \tilde{q} A_{u_R} \tilde{\bar{u}}_R H_u + \tilde{q} A_{d_L} \tilde{\bar{d}}_L H_d + \tilde{q} A_{d_R} \tilde{\bar{d}}_R H_d \\ + \tilde{l} A_{e_L} \tilde{\bar{e}}_L H_d + \tilde{l} A_{e_R} \tilde{\bar{e}}_R H_d + \tilde{l} A_{\nu_L} \tilde{\bar{\nu}}_L H_u + \tilde{l} A_{\nu_R} \tilde{\bar{\nu}}_R H_u \end{aligned} \tag{3}$$

where A's are 3x3 complex matrices in flavour space. These additional eight flavour matrices involve couplings to squarks and sleptons and, have not been directly probed experimentally. However the rare processes such as the $K_L - K_S$ mass difference provide experimental constraints on these flavour mixing matrices. These too many observable mixing angles and CP violating phases might be seen as an indication of the fact that SUSY must be realized at a more fundamental level.

Predictions of CP violating effects in supersymmetric extensions due to these complex phases are widely discussed in the literature. In the usual MSSM scenario CP violating phases occur in two different sectors: in Yukawa couplings where they would be responsible for CP violation in K and B physics, and in SUSY-breaking terms (gaugino masses and scalar trilinears) where they would induce too large neutron and electron electric dipole moments (d_n and d_e) compared to the experimantal values unless a mechanism for the suppression by two or three orders of magnitude. To avoid this usually the scalar quarks and scalar leptons are taken heavier than a few TeV or the phases are somehow reduced just the right amount so that the dipole moments remain below the present experimental upper bounds. However it is pointed out recently [8] that the naive quark model may not be adequate to describe the contributions of the quark electric dipole operators to the neutron electric dipole moment. In particular a contribution from the strange quark could be significant, and there might be a systematic cancellation between the u,d and s contributions. This would mean that previous constraints on CP-violating parameters in MSSM derived from upper limits on d_n should be relaxed.

As we pointed out above it is interesting that MSSM contains two observable complex phases in the quark-squark sector even in the simplest case of one family. In fact these CP-violating phases were indirectly pointed out before [5] in connection with the discussion of the neutron electric dipole moment. Let us consider this point in more detail. For $n = 1$ family the $\tilde{u}$ type squark mass matrix in $(\tilde{u}_L, \tilde{u}_R)$ basis is given by :

$$M_{\tilde{u}}^2 = \begin{pmatrix} m_{\tilde{u}_L}^2 + m_u^2 + m_Z^2(1/2 - e_u \sin^2\theta_W)\cos 2\beta & m_u(A_u - \mu cot\beta) \\ m_u(A_u - \mu cot\beta) & m_{\tilde{u}_R}^2 + m_u^2 + m_Z^2 e_u \sin^2\theta_W \cos 2\beta \end{pmatrix} \tag{4}$$

Here A_u is the soft-SUSY breaking A-parameter, μ is the Higgs mass parameter and $tan\beta = v_2/v_1 = O(1)$, where v_1 and v_2 are vacuum expectation values of the Higgs fields H_1 and H_2 respectively. The mass matrix for the $\tilde{d}$ type squark is similar. To diagonalize these matrices one should introduce the following squark mass eigenstates through unitary transformations:

$$\begin{pmatrix} \tilde{u}_L \\ \tilde{u}_R \end{pmatrix} = \begin{pmatrix} \cos\alpha_u e^{i\phi_1} & \sin\alpha_u e^{i\phi_2} \\ -\sin\alpha_u e^{i\phi_3} & \cos\alpha_u e^{i\phi_4} \end{pmatrix} \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} \tilde{d}_L \\ \tilde{d}_R \end{pmatrix} = \begin{pmatrix} \cos\alpha_d e^{i\psi_1} & \sin\alpha_d e^{i\psi_2} \\ -\sin\alpha_d e^{i\psi_3} & \cos\alpha_d e^{i\psi_4} \end{pmatrix} \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix} \tag{6}$$

Here $\tilde{u}_{1,2}$ and $\tilde{d}_{1,2}$ are squark mass eigenstates and due to the unitarity $\phi_1+\phi_4 = \phi_2+\phi_3$ and $\psi_1+\psi_4 = \psi_2+\psi_3$ hold. This transformation to mass eigenstates should also be made in the interaction terms. Consider quark-squark-gluino interactions:

$$L_{q\tilde{q}\tilde{g}} = -\sqrt{2}g_s T^a_{jk} \sum_i (\bar{\tilde{g}}^a P_L q^k_i \tilde{q}^{*j}_{iL} + \bar{q}^j_i P_R \tilde{g}^a \tilde{q}^k_{iL} - \bar{\tilde{g}}^a P_R q^k_i \tilde{q}^{*j}_{iR} - \bar{q}^j_i P_L \tilde{g}^a \tilde{q}^k_{iR}) \tag{7}$$

where the sum is over $i = u, d$, $P_{L,R} = 1/2(1 \pm \gamma_5)$ and j, k are color indices. The phase angles can be absorbed into the squark fields through the redefinitions such as $\tilde{u}_1 \to e^{-i\phi_1}\tilde{u}_1$, $\tilde{u}_2 \to e^{-i\phi_2}\tilde{u}_2$, $\tilde{d}_1 \to e^{-i\psi_1}\tilde{d}_1$ and $\tilde{d}_2 \to e^{-i\psi_2}\tilde{d}_2$, but at the end two phases $\delta_u = \phi_1 - \phi_3$ and $\delta_d = \psi_1 - \psi_3$ reappear.

On the other hand the EDM of a quark gets contribution at the one-loop level from the Feynman diagrams in which propagate squarks and gluinos (there are also contributions coming from the other diagrams where charginos or neutralinos replace gluinos). The EDM of neutron can be estimated to be [5]:

$$d^e_n \simeq \frac{2e\alpha_s m_{u,d}}{9\pi m^2_{\tilde{g}}} \sin\delta_{u,d}\eta_{mixing} \tag{8}$$

Here η_{mixing} stands for the left-right mixing in the squark line and in terms of the mixing angles appearing in Eqs.(5-6) it is given by $\eta_{mixing} = \sin\alpha_u \cos\alpha_u$ for the u-type quarks. For d quarks α_d replaces α_u. For a value of $m_{\tilde{g}} = 150GeV$ this gives $10^{-22}\delta_{u,d}e.cm$ which is much higher than the experimental limit, $4 \times 10^{-25}e.cm$ unless the CP-violating phases are tiny; $\delta_{u,d} \leq 0.002$.

On the other hand if we assume that a SUSY model cannot contain fine tunings or large mass scales then one must require that SUSY breaking mechanism give real soft breaking parameters in which case MSSM has no CP violation other than from CKM matrix. However in non-minimal supersymmetric extensions of SM with real soft breaking parameters a non-standard CP violating phase is shown to arise due to the one-loop corrections to the scalar potential [9]. In supersymmetric theories additional box diagrams appear during the calculation of the $K_L - K_S$ transition, and to achieve agreement with the experiment one should have a SUSY GIM (Glashow- Iliopoulos-Maiani) cancellation, which requires $\tilde{m}^2_u - \tilde{m}^2_c \approx m^2_c$. This looks unnatural since experimentally $\tilde{m}^2_u, \tilde{m}^2_c > (100GeV)^2$ and these squark mass terms are introduced into the Lagrangian by hand in MSSM with soft SUSY-breaking mechanism. Furthermore, it is important to ensure the absence of flavour-changing neutral gaugino interactions, which otherwise would make disastrously large contributions to the $K_1 - K_2$ mass difference. So this also leads to still more stringent conditions on the degeneracy of squarks belonging to different generations. There are no reasonable arguments for small values of mixing angles and mass degeneration in MSSM.

Finally, considering so many arbitrary parameters put by hand and the severe requirements, in our opinion SUSY must be realized on the preonic level; therefore the aim should be to construct a realistic SUSY preonic model and extracting its consequences at lepton- quark level.

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