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THE POST–NEWTONIAN APPROXIMATION IN THE GENERALIZED BRANS–DICKE THEORY

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The Post-Newtonian approximation in the Generalized Brans-Dicke Theory

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Abstract

The Post-Newtonian approximation method in precession of Mercury perihelion is used to investigate the generalized Brans-Dicke theory with a coupling constant β. The experiments give the limit of the coupling constant of $\beta^2 \leq 0.0015$.

In Brans-Dicke gravity, the gravitational constant $G_{eff} = \phi^{-2}$, evolves as the Brans-Dicke field ϕ evolves. Because of this, the Universe expands according to a power law during the inflation epoch, which can solve the "graceful exit" problem. It was soon found that the bubbles can lead to unacceptable distortions of the microwave background[10,11]. Then Daile La [9] considered extended inflation[7-9] in (Weyle-scaled) minimal theories of gravity in which Pauli metric was used to resolve cosmological puzzles because Brans-Dicke theory exhibits conformal(scale) invariance. Furthermore, the new interest in the Brans-Dicke(BD) theory has been stimulated. It was shown by Cho[4] that only the Pauli metric can represent the massless spin-two graviton and thus identified as physical. This theory is referred as generalized BD(GBD) theory. The GBD theory is a reinterpretation of BD theory and hence the coupling constant of its should be a new parameter to be determined by observations. In the GBD theory, the motion of a test particle deviates from geodesic, But the results of GBD theory are the same as those of Einstein's theory of relativity in weak field approximation and Newtonian limit, when we identified the gravitational constant as $G_{eff} = (1 \pm \beta^2/2)G$(not G) in GBD theory[3]. In other words, the deviation from geodesic does not violate the weak equivalent principle to the first order of M/r. Tests of general relativity fall into two cases: (1) the null tests, such as the Eötvös experiment. (2) the non-null tests, such as the relativistic rotation of Mercury perihelion. In this paper, we apply the experiment of Mercury's perihelion to give the limit of the coupling constant of GBD theory.

Firstly, we introduce our nomenclature $\overset{N}{g}_{\mu\nu} \sim v^N$, $\overset{N}{\Gamma}{}^{\mu}_{\nu\lambda} \sim v^N/r$, $\overset{N}{R}_{\mu\nu} \sim v^N/r^2$, $\overset{N}{T}{}^{\mu\nu} \sim Mv^N/r^3$. the meaning of the above symbol is as the following: $\overset{N}{g}_{\mu\nu}$ denotes $g_{\mu\nu}$ of oder v^N with $v^2 \sim 1/r$ and so on.

Then we consider the generalized Brans-Dicke Lagrangian

$$\pounds = \sqrt{-g}[\frac{-R}{2\kappa^2} \mp \frac{g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma}{2}] - e^{-a\sigma}\pounds_m \tag{1}$$

Where $\kappa^2 = 8\pi G$, the coupling constant $a = \beta\kappa$, β is an arbitrary constant which is determined in this paper, $\pounds_m = \sqrt{-g}[g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - e^{(-a\sigma)}V(\psi)]/2$ is the Lagrangian of the matter field ψ which is chosen to be a scalar field for simplicity.

We can get the following motion equations from the Lagrangian (1) :

$$R_{\mu\nu} - \frac{g_{\mu\nu}}{2}R = -\kappa^2[\pm T_{(\sigma)\mu\nu} + e^{(-a\sigma)}T_{(m)\mu\nu}] \tag{2}$$

$$\Box\sigma = \pm\frac{ae^{(-a\sigma)}T_{(m)}}{2} \tag{3}$$

$$\Box\psi = ag^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + e^{(-a\sigma)}V'(\psi) \tag{4}$$

where

$$T_{(\sigma)\mu\nu} = \partial_\mu\sigma\partial_\nu\sigma - \frac{g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma}{2} \tag{5}$$

$$T_{(m)\mu\nu} = \partial_\mu\psi\partial_\nu\psi - \frac{g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\psi\partial_\beta\psi}{2} - g_{\mu\nu}e^{(-a\sigma)}V(\psi) \tag{6}$$

$$T_{(m)} = g^{\mu\nu}T_{(m)\mu\nu} = -g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - 4e^{(-a\sigma)}V(\psi) \tag{7}$$

and$\Box \equiv \frac{\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu}{\sqrt{-g}}$, $V'(\psi) = dV(\psi)/d\psi$.

As we all know, in Post-Newtonian approximation, the components of $g_{\mu\nu}$ have the following expressions:

$$\begin{aligned} g_{00} &= -1 + \overset{2}{g}_{00} + \overset{4}{g}_{00} + \cdots \\ g_{ij} &= \delta_{ij} + \overset{2}{g}_{ij} + \overset{4}{g}_{ij} + \cdots \\ g_{i0} &= \overset{3}{g}_{i0} + \overset{5}{g}_{i0} + \cdots \end{aligned} \tag{8}$$

In this paper, we choose the harmonic coordinate because it is always possible to define the x^μ so that they obey the harmonic coordinate conditions

$$g^{\mu\nu}\Gamma^\lambda_{\mu\lambda} = 0 \tag{9}$$

Now it follows from expansions of the motion of dilation field Eq. (3) that:

$$\nabla^2 \overset{2}{\sigma} = \mp\frac{1}{2}a\,\overset{0}{T}{}^{00} \tag{10}$$

$$\nabla^2 \overset{4}{\sigma} - \frac{\partial^2 \overset{2}{\sigma}}{\partial t^2} - \overset{2}{g}_{ij}\frac{\partial^2 \overset{2}{\sigma}}{\partial x^i \partial x^j} = \pm\frac{1}{2}a^2\,\overset{2}{\sigma}\overset{0}{T}{}^{00} \pm \frac{1}{2}a(-\overset{2}{T}{}^{00} + \overset{2}{T}{}^{ii} + \overset{2}{g}_{00}\overset{0}{T}{}^{00}) \tag{11}$$

Where $\overset{N}{\sigma}$ denotes σ of order v^N. The motion of metric fields Eq. (2) are:

$$\nabla^2 \overset{2}{g}_{00} = -\kappa^2\,\overset{0}{T}{}^{00} \tag{12}$$

$$\nabla^2 \overset{2}{g}_{ij} = -\kappa^2\,\overset{0}{T}{}^{00}\,\delta_{ij} \tag{13}$$

$$\nabla^2 \overset{3}{g}_{0i} = 2\kappa^2\,\overset{1}{T}{}^{0i} \tag{14}$$

$$\nabla^2 \overset{4}{g}_{00} = \frac{\partial^2 \overset{2}{g}_{00}}{\partial t^2} + \overset{2}{g}_{ij}\frac{\partial^2 \overset{2}{g}_{00}}{\partial x^i \partial x^j} - (\nabla \overset{2}{g}_{00})^2 - \kappa^2(\overset{2}{T}{}^{00} + \overset{2}{T}{}^{ii} - a\,\overset{2}{\sigma}\overset{0}{T}{}^{00} - 2\,\overset{2}{g}_{00}\overset{0}{T}{}^{00}) \tag{15}$$

From(12), we have

$$\overset{2}{g}_{00} = -2\phi \tag{16}$$

where ϕ is Newton potential which is determined by Poisson equation$\nabla^2\phi = \kappa^2\,\overset{0}{T}_{00}/2$. Because g_{00} vanishes at infinity, then ϕ is:

$$\phi(\vec{x},t) = -\frac{\kappa^2}{8\pi}\int d^3\vec{x}'\frac{\overset{0}{T}{}^{00}(\vec{x}',t)}{|\vec{x}-\vec{x}'|} \tag{17}$$

Substituting(17) into (10) and(13), we have

$$\overset{2}{\sigma} = \mp\frac{a}{\kappa^2}\phi \tag{18}$$

$$\overset{2}{g}_{ij} = -2\delta_{ij}\phi \tag{19}$$

From Eq. (14), we get

$$\overset{3}{g}_{i0} = \xi_i \tag{20}$$

where ξ_i is a new potential which is zero at infinity

$$\xi_i(\vec{x},t) = -\frac{\kappa^2}{2\pi}\int d^3\vec{x}'\frac{\overset{1}{T}{}^{i0}(\vec{x}',t)}{|\vec{x}-\vec{x}'|} \tag{21}$$

Substituting(16)-(19) into (15), we have

$$\overset{4}{g}_{00} = -2\phi^2 - 2\psi \tag{22}$$

where ψ is the second potential

$$\psi(\vec{x},t) = -\frac{1}{4\pi}\int\frac{d^3\vec{x}'}{|\vec{x}-\vec{x}'|}[\frac{\partial^2\phi(\vec{x}',t)}{\partial t^2} + \frac{\kappa^2}{2}(\overset{2}{T}{}^{00}(\vec{x}',t) + \overset{2}{T}{}^{ii}(\vec{x}',t)) \pm \frac{a^2}{\kappa^2}\phi(\vec{x}',t)\nabla^2\phi] \tag{23}$$

Combining Eqs. (18), (19) with Eq. (11) , we get

$$\overset{4}{\sigma} = \mp\frac{a}{\kappa^2}\psi \mp a\Delta$$

where

$$\Delta \equiv \int\frac{\overset{2}{T}{}^{ii}(\vec{x}',t)}{4\pi|\vec{x}-\vec{x}'|}d^3\vec{x}'$$

because Δ is a small term relative to ψ, so we omit it. Then

$$\overset{4}{\sigma} = \mp\frac{a}{\kappa^2}\psi \tag{24}$$

The coordinate condition Eq. (9) imposes on ϕ and ξ the further relation

$$4\frac{\partial\phi}{\partial t} + \vec{\nabla}\cdot\vec{\xi} = 0 \tag{25}$$

Now we consider the motion of a freely falling material particle or photon in the gravitational fields $g_{\mu\nu}$ and σ:

$$\frac{d^2x^\mu}{dp^2} + \Gamma^\mu_{\lambda\nu}\frac{dx^\lambda}{dp}\frac{dx^\nu}{dp} = \frac{a\partial^\mu\sigma}{2} + \frac{a}{2}\frac{dx^\mu}{dp}\frac{d\sigma}{dp} \tag{26}$$

By definition, to order v^4,

$$(\frac{d\tau}{dt})^2 = 1 + 2\phi - v^2 + 2[\phi^2 + \psi - \vec{\xi}\cdot\vec{v} + \phi v^2]$$

Combining the Post-Newtonian metric we have obtained with the geodesic Eq. (26) gives the equation of motion:

$$\frac{d^2\vec{x}}{dt^2} = -(1 \pm \frac{\beta^2}{2})\vec{\nabla}(\phi + \psi) - \vec{\nabla}(2\phi^2) - \frac{\partial\vec{\xi}}{\partial t} + \vec{v} \times (\vec{\nabla} \times \vec{\xi}) + (3 \mp \frac{\beta^2}{2})\vec{v}\frac{\partial\phi}{\partial t} + 4\vec{v}(\vec{v} \cdot \vec{\nabla})\phi - (1 \mp \frac{\beta^2}{2})v^2\vec{\nabla}\phi \mp 2\beta^2\phi\vec{\nabla}\phi \tag{27}$$

Let us calculate the gravitational field far away from an arbitrary finite distribution of energy and momentum. Let $T^{\mu\nu}$ vanishes for $r > R$, where $r \equiv |\vec{x}|$. We expand the denominators of Eqs. (17), (23) in terms of the inverse powers of r/R.

$$|\vec{x} - \vec{x'}|^{-1} \to \frac{1}{r} + \frac{\vec{x} \cdot \vec{x'}}{r^3} + \cdots$$

Then we have

$$\phi \to -\frac{G\overset{0}{M}}{r} - \frac{G\vec{x} \cdot \overset{0}{\vec{D}}}{r^3} + o(\frac{1}{r^3}) \tag{28}$$

$$\psi \to -\frac{G\overset{2}{M}}{r} - \frac{G\vec{x} \cdot \overset{2}{\vec{D}}}{r^3} + o(\frac{1}{r^3}) \tag{29}$$

where

$$\overset{0}{M} \equiv \int \frac{\kappa^2}{8\pi G} \overset{0}{T^{00}}\, d^3x, \qquad \overset{0}{\vec{D}} \equiv \int \frac{\kappa^2}{8\pi G} \vec{x}\, \overset{0}{T^{00}}\, d^3x$$

$$\overset{2}{M} \equiv \frac{1}{4G\pi} \int [\frac{\kappa^2}{2}(\overset{2}{T^{00}} + \overset{2}{T^{ii}}) \pm \frac{a^2}{\kappa^2}\phi\nabla^2\phi] d^3x$$

$$\overset{2}{\vec{D}} \equiv \frac{1}{4G\pi} \int \vec{x}[\frac{\kappa^2}{2}(\overset{2}{T^{00}} + \overset{2}{T^{ii}}) \pm \frac{a^2}{\kappa^2}\phi\nabla^2\phi + \frac{\partial^2\phi}{\partial t^2}] d^3x$$

The field ψ has physical effects only through its presence in the expression of g_{00} , Evidently, we can take account of ψ by simply replacing ϕ everywhere with $\psi + \phi$. That is, within the accuracy of the Post-Newtonian approximation we may write

$$g_{00} = -1 - 2(\phi + \psi) - 2(\phi + \psi)^2 + \cdots \tag{30}$$

the physical significant field

$$\phi + \psi \to -\frac{GM}{r} - \frac{G\vec{x} \cdot \vec{D}}{r^3} + o(\frac{1}{r^3}) \tag{31}$$

where $M = \overset{0}{M} + \overset{2}{M}$, $\vec{D} = \overset{0}{\vec{D}} + \overset{2}{\vec{D}}$.

We should note that the quantity $\vec{D}$ does not represent an effect of physical importance, but rather just a displacement of the whole fields. Now we consider the precession of planetary orbits in the actual solar system, taking into account other planets, solar rotation, solar oblateness, and so on. The potential $\phi + \psi$ that determines g_{00} is overwhelmingly dominated

by the spherically symmetric part $-GM_\odot/r$ of the sun's contribution, so it is convenient to write

$$\phi + \psi = -\frac{GM_\odot}{r} + \epsilon(\vec{x}, t) \tag{32}$$

With ϵ including not only the Newtonian potential of the other planets but also any quadrupole or higher terms in the sun's contribution to $\phi + \psi$. The equation of motion (27) now reads

$$\frac{d\vec{v}}{dt} = -\frac{G_{eff}M_\odot}{r^3}\vec{x} + \vec{\eta} + o(v^6) \tag{33}$$

with

$$\vec{\eta} = -\vec{\nabla}[(1 \pm \frac{\beta^2}{2})\epsilon + 2\phi^2] - \frac{\partial\vec{\xi}}{\partial t} + \vec{v} \times (\vec{\nabla} \times \vec{\xi}) + (3 \mp \frac{\beta^2}{2})\vec{v}\frac{\partial\phi}{\partial t} + 4\vec{v}(\vec{v} \cdot \vec{\nabla})\phi - (1 \mp \frac{\beta^2}{2})v^2\vec{\nabla}\phi \mp 2\beta^2\phi\vec{\nabla}\phi \tag{34}$$

a small perturbation. where $G_{eff} = (1 \pm \frac{1}{2}\beta^2)G$.

By far the most convenient technique for calculating precession of perihelion is to compute the rate of change of the Runge-Lenz vector

$$\vec{A} = -M_\odot G_{eff}\frac{\vec{x}}{r} + \vec{v} \times \vec{h} \tag{35}$$

Here $\vec{v} \equiv d\vec{x}/dt$, and $\vec{h}$ is the orbital angular momentum per unit mass:

$$\vec{h} \equiv \vec{x} \times \vec{v} \tag{36}$$

If the perturbation η in Eq. (33) were absent, then the orbit would be an ellipse. The $\vec{h}$ would be a constant vector normal to the orbit, with magnitude

$$|\vec{h}| = \sqrt{LM_\odot G_{eff}} \tag{37}$$

and $\vec{A}$ would be a constant vector pointing toward perihelia, with magnitude

$$|\vec{A}| = eM_\odot G_{eff} \tag{38}$$

Where e the eccentricity, L the semilatus rectum.

Thus, the rate of precession of perihelion $d\phi_0/dt$ caused by any perturbation is just the component of the change $d\tilde{\vec{A}}/dt$ in the unit vector $\tilde{\vec{A}} \equiv \frac{\vec{A}}{|\vec{A}|}$ along a direction perpendicular to both $\vec{A}$ and $\vec{h}$, which is

$$\frac{d\phi_0}{dt} = (\vec{h} \times \vec{A}) \cdot \frac{d\vec{A}/dt}{|\vec{h}|\vec{A}^2} \tag{39}$$

A straightforward calculation gives for the rate of change of $\vec{A}$ produced by the perturbation $\vec{\eta}$ in Eq. (33) the value

$$\frac{d\vec{A}}{dt} = \vec{\eta} \times \vec{h} + \vec{v} \times (\vec{x} \times \vec{\eta}) \tag{40}$$

Note that $d\vec{A}/dt$ and hence $d\phi_0/dt$ is linear in η. So $d\phi_0/dt$ is conveniently calculated by adding up the precessions produced by each small term in $\vec{\eta}$.

The largest term in $\vec{\eta}$ is the part of $-(1 \pm \beta^2/2)\vec{\nabla}\epsilon$ arising from the Newtonian potentials of the other planets. We shall make no attempt to compute this term. The next largest term is obtained from the relativistic corrections in Eq. (34), Setting ϕ and ξ equal the values they would have for a spherical nonrotating sun:

$$\phi_\odot = -\frac{GM_\odot}{r}, \qquad \xi_\odot = 0 \tag{41}$$

Combining with Eqs. (34)-(41), we get the precession

$$\begin{aligned}\frac{d\phi_0}{dt} &= \frac{8GM_\odot\sqrt{LM_\odot G_{eff}}}{L^3}\sin^2(\phi-\phi_0)[1+e\cos(\phi-\phi_0)]^2 \\ &-(1\pm\tfrac{\beta^2}{2})\frac{4G^2M_\odot^2}{eL^2\sqrt{LM_\odot G_{eff}}}\cos(\phi-\phi_0)[1+e\cos(\phi-\phi_0)]^3 \\ &+(1\mp\tfrac{\beta^2}{2})\frac{GM_\odot\sqrt{LM_\odot G_{eff}}}{eL^3}\cos(\phi-\phi_0)[1+e\cos(\phi-\phi_0)]^2[1+e^2+2e\cos(\phi-\phi_0)]\end{aligned} \tag{42}$$

Since ϕ_0 changes slowly, the change in ϕ_0 in one revolution can be determined by integrating $d\phi_0/dt$ over one period, keeping ϕ_0 fixed in the integrand. This gives for the precession per revolution

$$\begin{aligned}\delta\phi_0 &= \int_0^{2\pi}\frac{d\phi_0}{dt}\frac{dt}{d\phi}d\phi = \delta\phi_E + \delta\phi \\ &= \frac{6\pi MG_{eff}}{L} \mp \frac{8\pi\beta^2 G_{eff}M}{L(2\pm\beta^2)}\end{aligned} \tag{43}$$

Where $\delta\phi_E = 6\pi MG_{eff}/L$ is the result of Einstein theory of relativity.

It has been shown by Zhang[3] that the value of G_{eff} (not G) represent the measurement value of Newton gravitational constant. After the perturbing effects of the other planets have been accounted for, the excess shift is now known as $\delta\phi = 42''.98$ per century with accuracy of 0.1%. Then, we can get the value of β^2[2]:

$$\beta^2 \leq 0.0015 \tag{44}$$

It is shown[3] that GBD theory gives the same results as the Einstein's theory of gravity in weak field approximation and Newtonian limit. Although the motion of a test particle deviates from geodesic motion because of existence of an additional force exerted by the dilation field. However, there is no way to distinguish the new force from that exerted by the metric field to first order M/r. So the effects of strong field should be considered to give a limit on the coupling constant β in GBD theory. One of us[1] gives the limit $\beta^2 < 0.3$ for the coupling constant using inflation model in GBD theory. But the limit is not satisfied, our result applying the post-Newtonian approximation gives smaller upper limit on the coupling constant. However, let us note the relation $\beta^2 = 2/(2\omega+3)$[1] and $\omega \geq 500$[5] which give the value $\beta^2 \leq 0.002$. So our result is more reliable and consistent with the usual BD parameter. In this paper, we consider the determination of the Sun's gravitational quadrupole moment to

be most reliably determined at present from helioseismology, implying a negligible value. If we apply the value of the Sun's gravitational quadrupole moment given by Dicke et. al. [6] which contributed about 4 second per century to Mercury perihelion shift, thus it would put general relativity in serious disagreement with the observations, while on the other hand supporting the GBD theory with a value $\beta^2 \leq 0.1$. Furthermore, Our results give some constraints on the scalar-tensor theories.

References

[1] Y. Z. Zhang, Inflation in Generalized Jordan-Brans-Dicke Theory, Beijing, ITP preprint(92)

[2] Krisher, Anderson and Taylor, Astro. J. V. 373, 665(1990)

[3] Y. Z. Zhang, On the Generalized Jordan-Brans-Dicke Theory, ICTP Internal reports(92); to be appear in Phys. Rev. Lett.

[4] Y. M. Cho, Phys. Rev. Lett. V. 68, 3133(1992)

[5] R.D. Reasenberg et.al. , Astro. J. V. 234 L219-221(1979)

[6] R.H. Dicke, Science V. 184 419-429(1974)

[7] A. L. Berkin, K. Maeda and J. Yokoyama, Phys. Rev. Lett. V. 65, 144(1990)

[8] A. L. Berkin, K. Maeda, Phys. Rev. D 44, 1691(1991)

[9] D. La, Phys. Rev. D 44, 1680(1991)

[10] E. Weinberg, Phys. Rev. D40, 3950(1989)

[11] D. La., P. J. Steinhardt, and E. W. Bertschinger, Phys. Lett. B231, 231(1989)