17

ASITP

INSTITUTE OF THEORETICAL PHYSICS ACADEMIA SINICA

AS-ITP-93-57
August 1993

ON THE STATISTICAL ANALYSIS OF THE EXPERIMENTAL DATA COLLECTED BY CIAE WITH THE MAGNETIC SPECTROMETER AND THE NEW ν_e – MASS UPPER BOUND

electron neutrino

CHING Chengrui HO Tsohsiu
LIANG Dongqi MAO Yajun
CHEN Shiping SUN Hancheng

P.O.Box 2735, Beijing 100080, The People's Republic of China
Inst. Theor. Phys.
Telefax : (86)-1-2562587 Telephone : 2568348
Telex : 22040 BAOAS CN Cable : 6158

On the statistical analysis of the experimental data collected by CIAE with the magnetic spectrometer and the new ν_e - mass upper bound*

Ching Chengrui and Ho Tsohsiu

(Center of Theoretical Physics, CCAST(World Laboratory) and Beijing
Institute of Theoretical Physics, Academia Sinica
P.O. Box 2735, Beijing 100080)

Liang Dongqi, Mao Yajun, Chen Shiping and Sun Hancheng

(China Institute of Atomic Energy
P. O. Box 275(10), Beijing 102413) Inst. Atomic Energy

(Revised)

ABSTRACT

By comparing the results obtained using the same experimental data of CIAE but different theoretical formula fits it is pointed out that the negative value of $m_\nu{}^2$ is most likely linked to the inaccuracy of the theoretical formula of the β-spectrum. With the use of a theoretical formula with up to second order energy sum rule included, the experimental data of CIAE are re-fitted, and result in a new mass limit of 12.1 eV.

*Supported by the National Natural Science Foundation of China

In the previous paper[1] we have reported on the ν_e -mass limit obtained by the CIAE(China Institute of Atomic Energy) group as m_ν < 12.4 eV (95% C.L.). It can be compared with other results reported since 1986 by several other laboratories, namely

m_ν	<	12.4 eV	CIAE	(1992) [1]
	<	11.0 eV	Zurich	(1992) [2]
	<	9.3 eV	LANL	(1991) [3]
	<	13.0 eV	INS	(1991) [4]
	<	18.0 eV	Zurich	(1986) [5]

and also the hotly disputed result by ITEP

$$17eV < m_\nu < 40eV(1987)[6]$$

. However, a striking feature is that all the central values of $m_\nu{}^2$ are negative, and it is hard to explain it merely by the experimental uncertainties. This can be seen in Table 1, where the collection of all these results are shown. Later, the Particle Data Group (PDG) has combined the results of Ref. [2-5] and set a new world average value as

$$m_\nu < 7.3eV(90\%C.L.)[7],$$

and pointed at the same time that "Caution is urged in interpreting this result, because the m_ν^2 average is dominated by the Robertson' 91 result, which is nearly 2 σ negative." This value is also shown in the last line of Table 1.

Recently a more stringint limit of m_ν was reported by Mainz group[8] as

$$m_\nu{}^2 = -39 \pm 34 \pm 15(eV)^2$$

and

$$m_\nu < 7.2eV.$$

However, in obtaining the quoted value, the authors stressed that a specific energy interval of 137 eV was selected, and if a wider energy range is used instead, a more negative $m_\nu{}^2$ was resulted, so that the negativeness of the $m_\nu{}^2$ appears once again. Moreover, it is also interesting to note that in the experiments of Ref.[2-4] both the statistical and systematicl errors are approximately the same, nevertheless the deduced mass limit differs by as large as 3.7 eV. The origin lies in that in determining the mass limit the following assumptions are made: 1, the normal distribution is assumed to be centered at the measured central value with σ as the deviation; 2, the region with $m_\nu^2 < 0$ is abandoned as unphysical region; 3, the remaining part is normalized to unit, and 95 % of this area with the corresponding m_ν^2 is taken to be the m_ν - limit at 95 % C.L..

Then the questions are 1, why the region with $m_\nu^2 < 0$ can be eliminated and 2, why the remaining area should be normalized to unit and the confidence level is defined according to this area ? All these are not well-founded. And the inevitable consequence of this analysis is the farther the measured central value m_ν^2 is below zero, the lower the upper limit for m_ν is. Or in other words, the most improbable event determines with the highest weight the neutrino mass limit now we have.

However, we have two arguments that disfavour the results presented in Table 1. In what follows we shall show that 1, the negative central value of m_ν^2 should not be regarded as unbiased measurement resulted from statistical

fluctuation and consequently, 2, the normal distribution should not be taken as centered at this biased value. Now let us discuss these two problems in turn.

The theoretical β spectrum shape for the experimental data fit can be written down as

$$N_{th}(E) = AF(Z,E)pE_t \sum_n W_n(E_0 - E - E_{fn}) \times [(E_0 - E_{fn} - E)^2 - m_\nu^2]^{1/2} \quad (1)$$

where, A is the normalization constant; $F(Z,E)$ is the Fermi function, Z is the daughter nuclear charge; p, E and E_t are the momentum, kinetic and total energy of β rays, respectively; W_n and E_{fn} are, respectively, the relative probability and the excitation energy of the final state with $E_{f0} = 0$ in our definition; E_0 is the end point of the β-spectrum. For simplicity we have omitted in Eq.(1) all corrections resulting from a given experiment.

As is well-known, W_n and E_{fn} must be calculated based on a specific molecular model, which should be chosen to be as faithful as possible to reproduce the source used in a given experiment. The radioactive source of CIAE experiment is ^{3}T - labelled PAD(C_{14} H_{15} T_6 O_2 N_3) with tritium sitting in the C-H covalent bond of the molecule. Therefore a realistic approach is to approximate this big molecule to covalent bond such as CH_3T, CH_3-CH_2T, or CH_3-CHT-CH_3. Such approach was adopted, for instance, in Ref.[2] and Ref.[4]. For comparison, we have also tried the T_2 molecule, the T-atom, the T-nucleus as well as Valine 2, where a theoretical calculation for the latter is available[9]. All the fitted results are shown in Table 2, and the corresponding mass limits are deduced following the recipe we have just outlined. One can, however, argue in advance that it is highly unlike that ^3T

in PAD can be mimiced by a bare nucleus or an ^3T atom. In accordance with this conjecture, the least square fits presented in the last two lines of Table 2 clearly show that the bare nucleus model yields the negative m_ν^2 as big as about 3σ away from zero, (and amazingly the smallest mass limit!) and the highest χ^2 value. And the atom model also does no good. Both these two models can be rejected on the ground of the more negative m_ν^2 and higher χ^2 - values. As for the results of other molecular models, the CH_3T model with 7 levels, and the C_3H_7T with 20 levels give the smallest negative value of $m_\nu{}^2$ as well as the smallest χ^2 - value. Thus the results present in Table 2 suggest strongly that there is a correlation between the negativeness of $m_\nu{}^2$ and the precision of the theoretical formulas and the corresponding model as well. This has led us to question the precision of the theoretical formulas that were used in getting the results of Table 2.

From the theoretical point of view, and for a many-electron system, the ground state wave functions of the parent and daughter molecules can be calculated, and are calculated with better precision for most models listed in Table 2. However, it is not always the case for E_{fn} and W_n when n lies highly above the ground state . It is difficult even for simple two-electrons'molecular system such as T_2 and $(HeT)^+$. In order to see this point let us recall that there is a theoretically rigorous criterion–the sum rule, which should be fulfilled in any theoretical calculation. Therefore what we have done is to construct the first and second order energy sum rules using the latest calculations of different models, where the spectra are given on the one hand, and calculate the same quantities using the wave functions of the parent molecules on the other hand, and then to see the difference. It is appropriate

to notice here that, in principle, the higher order energy sum rules can also be constructed, but as we shall see later, the first and the second order energy sum rules enter the β -spectrum shape formula explicitly, we therefore concern only these two sum rules. In Table 3, Table 4, and Table 5 we have summarized respectively, the best calculated branching ratios and the excitation energies of 7 levels of $CH_3T \rightarrow CH_3He^+$, 20 levels of $C_3H_7T \rightarrow C_3H_7He^+$, and 12 levels of $T_2(TH) \rightarrow THe^+(HHe^+)$ known to us.

Now according to the definitions, the first and the second order energy sum rules can be written down as follows:

$$< \Psi_i \mid \Delta H \mid \Psi_i >= \sum W_n(\Delta E_{fn} + E_{f0} - E_{i0}) \tag{2}$$

and

$$< \Psi_i \mid (\Delta H)^2 \mid \Psi_i >= \sum W_n(\Delta E_{fn} + E_{f0} - E_{i0})^2 \tag{3}$$

with

$$\sum W_n = 1 \tag{4}$$

where ΔH is the difference of the Hamiltonians of the initial and final molecular systems RT and RHe^+. Ψ_i is the ground state wave function of RT, ΔE_{fn} is the excitation energy of the n^{th} -state with respect to the ground state of the daughter molecule RHe^+, so that it is identical with E_{fn} in Eq. (1) and in Tables 3 - 5. E_{i0} and E_{f0} are the ground state binding energies of RT and RHe^+ respectively.

The average excitation energy $\overline{\Delta E^*}$ is defined according to the following

equation:

$$\overline{\Delta E^{n}} = \sum W_n \Delta E_{fn} \tag{5}$$

Then with the use of Eq. (2-5) one can construct the energy dispersion function σ^2 as the following:

$$\sigma^2 \equiv < \Psi_i \mid (\Delta H)^2 \mid \Psi_i > -(< \Psi_i \mid \Delta H \mid \Psi_i >)^2 = \overline{\Delta E^{n2}} - (\overline{\Delta E^{n}})^2 \tag{6}$$

with

$$\overline{\Delta E^{n2}} = \sum W_n (\Delta E_{fn})^2 \tag{7}$$

We have calculated directly the quantities $\overline{\Delta E^{n}}$ and σ^2 using the spectras presented in Tables (3-5) and compared them with that obtained from the definitions using the initial wave functions of the parent molecules. The results are presented in Tables (6-8).

It should be noted that since in practical calculations $\sum W_n$ does not equal to 1 exactly, this leads to a small correction term in σ^2. Therefore σ^2 in Tables (6-8) is calculated, if needed, using the following formula

$$\sigma^2 = \overline{\Delta E^{n2}} - (\overline{\Delta E^{n}})^2 + (E_{i0} - E_{f0})^2 \Delta W_n \sum W_n - 2(E_{i0} - E_{f0}) \overline{\Delta E^{n}} \times \Delta W_n$$

with

$$\Delta W_n = 1 - \sum W_n$$

From Tables 6-8 it can be seen that although $\sum W_n$ in all calculations are very close to unit, and if the precision achieved in $\overline{\Delta E^{n}}$ computation is tolerable–less than a few percent–the precision in the calculated σ^2 is very poor, and it does not exceed $\approx$ 30% to 40%, and hence is not acceptable. This

shows clearly that it is really very hard to achieve reasonably high accuracy even for HT-HHe^+ or T_2-THe^+ molecular systems. In fact, the best and the most accurate calculation for T_2 and THe^+ so far was reported in Ref.[17] by W. Kolos et. al., and the 12 levels' formula was extracted based on this calculation. But if one is looking beyond the ground state, one immediately sees that the excited states were calculated not as precise as the former. The similar argument is also expressed in Ref.[15].

Then the question is: if it is sufficient to remain satisfied, as claimed in Ref.[10], with the first order energy sum rule , which is calculated with an accuracy of a few percent? Unfortunately it is not the case for β-spectrum shape. It is obvious by recalling that the β-spectrum including the final state interaction is proportional to the following expression:

$$\sum W_n (E_0 - E - \Delta E_{fn})((E_0 - E - \Delta E_{fn})^2 - m_\nu^2)^{1/2}$$

Therfore the average spectrum shape is related directly to the first and second order energy sum rule

$$N(E) \propto ((E_0 - E)^2 - 2(E_0 - E)\overline{\Delta E^{n}} + \overline{\Delta E^{n2}})$$

for $m_\nu = 0$. This point has been unfortunately neglected in most of the literatures, and the theoretical formula with first and second order energy sum rules included was derived for small neutrino mass in Ref. [16] in 1982 and later was rewritten in Ref.[14] in 1984. Now since the second order energy sum rule is reproduced with rather poor precision for all models listed above, it is natual to inquire the reliability of the theoretical formulas used in the present data analysis concerning the m_ν determination.

In order to re-analyse the β-spectrum properly our principle is the following: since the ground state wave functions are calculated with highest precision, we therefore rely only on the ground state branching ratio, and the first and the second energy sum rules, which are evaluated using only the initial wave functions. The theoretical β-spectral shape is given as the following:

$$\begin{aligned} N(E) = \; & AF(Z,E)pE_\iota\{W_1(E_0-E)[(E_0-E)^2-m_\nu^2]^{1/2}\theta(E_0-E-m_\nu) \\ & +(1-W_1)\times[(E_0+<\Delta H>_1-E)^2+<\Delta H^2>_1-<\Delta H>_1{}^2 \\ & -m_\nu^2/2]\times\theta(E_0+<\Delta H>_1-E-m_\nu)\} \end{aligned} \tag{8}$$

where

$$<\Delta H>_1=\overline{\Delta E^\bullet}/(1-W_1)$$

and

$$<\Delta H^2>_1=\overline{(\Delta E^\bullet)^2}/(1-W_1)$$

Eq.(8) is the so-called two-levels' formula with the ground state transition treated exactly while the contribution from all excited state transitions is estimated using the first and second energy sum rules. For a formula with more transitions treated exactly we refer the reader to Ref.[14].

Now using formula Eq.(8) the CIAE data are re-fitted. The results are shown in Table 9. Two remarkable features from Table 9 can be seen, namely, 1, in all models without exception the least square fits with closure formula (8) lead to smaller negative values of m_ν^2 and smaller χ^2 values, particularly for C_3H_7T, the m_ν^2 is $+4(eV)^2$ with the χ^2 equal to 1.091, the smallest one; and 2, all the obtained m_ν^2 values using formula (8) are compatible with zero within one standard deviation. This result clearly demonstrates that

the quality of the fitting as well as the m_ν^2 value itself rely heavily on the precision of the theoretical spectrum. and the negativeness of m_ν^2 seems at least alleviated. It is also interesting and appropriate to note that the sum rule approach for both C_3H_7T and T_2 models here leads to very similar results. This is related to the fact that incidently these two models have roughly the same values of W_1 and $\overline{\Delta E^\bullet}$, as well as σ^2.

Now let us turn to the question of how to obtain the upper limit of ν_e-mass. It is well-known that unless a tachyon is considered, the neutrino mass must be real. The results presented in Table 2 and Table 9 suggest strongly that there is a correlation between the negative m_ν^2 value and the theoretical spectrum shape. And the more precise the theoretical shape, the smaller the negative m_ν^2 value. So that we may take the absolute value of negative m_ν^2 as a measure of the theoretical uncertainty aroused from our incomplete knowledge of the radiactive source. In other words, the absolute value of negative of m_ν^2 can be regarded as a systematic error in theory, which has not been included in the previous data processing. With this in mind, and given the fact that the whole body of the existing experimental evidences so far obtained in laboratories and astrophysics observations favours a very small mass. We assume that ν_e mass is small, say, less than few eV above zero, so that the normal distribution can be taken to be centered at 0, with the standard deviation consisting of statistical, systematic and the theoretical systematic errors. And this 95% normal distribution determines a limit of m_ν^2, and hence a limit of m_ν at 95% C. L. In Table 10 the newly determined uppers limits for $\bar{\nu}_e$ mass are shown and compared to their previous values.

It is noted that although the newly deduced upper limits are higher in most cases than that obtained previously using the same experimental data. But it is, according to the arguments given in this paper, more reasonable. Thus as a conclusion, we are led to the following result: the best fit of CIAE experimental data using the two-level's formula with closure gives the electron neutrino mass limit as $m_\nu < 12.1$ eV (95% C.L.).

REFERENCES

[1] Sun Hancheng, Liang Dongqi, Chen Shiping et al.,
To appear in Chinese Journ.Nucl.Phys.

[2] E. Holzschuh, M. Fritschi and W. Kundig, *Phys. Lett.*, B 287 (1992) 381.

[3] R.G.H. Robertson, T.J. Bowles, G.J. Stephenson et al., *Phys. Rev. Lett.*, 67(1991) 957.

[4] H. Kawakami, S. Kato, T. Ohshima et al., *Phys. Lett.*, B 256 (1991) 105.

[5] E. Holzschuh, M. Fritschi, W. Kundig et al., *Phys. Lett.*,
B 137 (1986) 485.

[6] S. Boris, A. Golutvin, L. Laptin et al., *Phys. Rev. Lett.*, 58 (1987) 2019.

[7] Particle Data Group, *Phys. Rev.* D 45 No. 11 (1992) Part 2, P. VI.6.

[8] Ch. Weinheimer et. al., *Phys. Lett.*, B 300 (1993) 210,

[9] I.G. Kaplan, G.V. Smelov and V.N. Smutny, *Phys. Lett.*,
B 161 (1985) 389.

[10] S. Schafroth, *Doctoral Thesis*, Ab Initio Calculations on ^{3}T Sustituted

Molecules, p.69.

[11] R.L. Martin and J.S. Cohen, *Phys. Lett.*, A 110 (1985) 95.

[12] I.G. Kaplan and G.V. Smelov, *Nuclear β Decay and neutrinos*
Proc. Intern. Symp., Osaka, Japan, June 1986. Editors:
T. Kotani et.al., World Scientific, p.354.

[13] J. Arafune et.al., *J. phys. Soc. Japan*, 55 (1986) 3806

[14] Ching Chengrui and Ho Tsohsiu, *Phys. Reports*, 112, No.1 (1984) 1.
see also X.L.Zhao,C.R.Ching,*Commun. Theor.Phys.*,7,(1987) 195.

[15] R.L. Martin, Presentation at 11th Intern. Workshop on Weak
Interactions, Santa Fe, NM (1987)

[16]C.R. Ching and T.H. Ho, *Commun. Theor. Phys.*, 1 (1981) 11,
C.R. Ching, T.H. Ho and H.L. Zhao, *ibid*, 1 (1982) 267.

[17]W. Kolos et.al., *Phys. Rev.*, A 31(1985)551,
O. Fackler et.al., *Phys. Rev. Lett.*, 55 (1985) 1388.

Table 1: m_ν^2 and m_ν - upper limit

Reference	m_ν^2 (eV^2)	Error (eV)^2		Upper limit
		stat.	syst.	for m_ν(95%C.L.)
CIAE -92 [1]	-31	± 75	± 48	12.4 eV
Zurich-92 [2]	-24	± 48	± 61	11.0 eV
LANL -91 [3]	-147	± 68	± 41	9.3 eV
INS -91 [4]	-65	± 85	± 65	13.0 eV
Zurich-86 [5]	-11	± 63	± 178	18.0 eV
ITEP -87 [6]	919	± 60	± 150	$17 < m_\nu < 40$
PDG -92 [7]	-107	± 60		7.3(90%C.L.)

Table 2: m_ν^2, m_ν-limit and χ^2-value for CIAE data fit[1]

Model	No. of level	m_ν^2	m_ν	χ^2	E_0-18500
		(eV)^2	(eV)		(eV)
CH_3T	7	-31± 75	12.4	1.141	78.3
CH_2=CHT	2	-51± 75	12.0	1.145	79.7
CH_3-CHT-CH_3	2	-43± 75	12.2	1.144	79.9
CH_3-CHT-CH_3	20	-9± 75	12.9	1.134	79.4
VALINE II	2	-141± 75	10.3	1.140	78.9
T-molecule	2	-68± 75	11.7	1.148	79.9
T-molecule	12	-177± 75	9.7	1.145	77.6
T-atom	2	-191± 75	9.4	1.146	75.2
T-nuclei	1	-237± 75	8.9	1.158	67.2

Table 3: The W_N and E_{FN} in the 7 levels transition of CH_3T molecule.[9]

W_n	E_{fn}(eV)	W_n	E_{fn}(eV)
0.6056	0.00	0.017	57.50
0.084	22.50	0.075	72.50
0.141	32.50	0.044	91.33
0.033	47.50		

Table 4: The W_N and E_{FN} in the 20 levels transition of C_3H_7T molecule.[10]

W_n	E_{fn}(eV)	W_n	E_{fn}(eV)
.571036	0.00	.045685	63.740
.117594	23.105	.012837	65.565
.073401	35.655	.007596	73.222
.012191	38.874	.053237	78.616
.008831	42.572	.005876	82.119
.020535	44.795	.002271	86.099
.007183	48.285	.001970	92.813
.012122	51.384	.003415	97.807
.008235	55.890	.011496	106.532
.015856	58.777	.001496	120.988

Table 5: The W_N and E_{FN} in the 12 levels transition of T_2 molecule.[11]

W_n	E_{fn}(eV)	W_n	E_{fn}(eV)
.5822	0.00	.0089	41.75
.1675	27.29	.0143	46.03
.0787	33.89	.0166	51.71
.0081	37.96	.0789	65.28
.0001	38.82	.0297	75.45
.0092	39.38	.0061	88.07

Table 6: A comparison of $\overline{|DeltaE^{\bullet}}$ and σ^2 between the direct calculation and the sum rule approach for 7 levels in CH_3T model.

	Direct calcul.	Sum rule	Deviation
W_n	1	1	very small
$\overline{\Delta E^{\bullet}}$(eV)	18.51	18.98[12]	2.5%
$\sigma^2(eV)^2$	744.07	1207.6[12]	$\approx$ 40%

Table 7: A comparison of $\overline{\Delta E^{\bullet}}$ and σ^2 between the direct calculation and the sum rule approach for 20 levels in C_3H_7T model.

	Direct calcul.	Sum rule	Deviation
W_n	.9929	1.00	.7%
$\overline{\Delta E^{\bullet}}$(eV)	20.56	19.1$\pm$.4[3,10]	7.5%
$\sigma^2(eV)^2$	795.69	1231.14[13]	$\approx$ 30%

Table 8: A comparison of $\overline{\Delta E^{\bullet}}$ and σ^2 between the direct calculation and the sum rule approach for 12 levels in T_2 model.

	Direct calcul.	Sum rule(HT)	Sum rule(T_2)	Deviation
W_n	0.9997	1.00	1.00	.3%
$\overline{\Delta E^{\bullet}}$(eV)	17.67	18.62[12]	18.80[14]	5-6%
$\sigma^2(eV)^2$	566.50	1109.5[12]	1045.9[14]	43-50%

Table 9: **Re-fitted m_ν^2 using the closure formula (8) and the comparison with that using formula (1).**

Model	No. of levels	Formula(1) or(8)	W_1	E_0-18500 (eV)	m_ν^2 $(eV)^2$	χ^2
CH_3T	2	(8)	0.6056	79.7	-21	1.128
CH_3T	7	(1)	0.6056	78.3	-31	1.141
CH_3-CHT-CH_3	20	(1)	0.5710	79.4	-9	1.134
CH_3-CHT-CH_3	2	(8)	0.5710	82.2	+4	1.091
T-molecule	2	(8)	0.5820	78.2	1	1.098
T-molecule	12	(1)	0.5822	77.6	-177	1.145

Table 10: ν_e - **mass limit**

Model	Experiment (Formula)	No. of levels	Δ m_ν^2 (errors)	m_ν limit, old 95%C.L.	m_ν limit, new 95%C.L.
CH_3T	Zurich-86	7 (1)	$\pm 11 \pm 63 \pm 178$	18.0	17.6
CH_3T	CIAE-92	2 (8)	$\pm 21 \pm 75 \pm 48$	12.8	12.2
CH_3-CHT-CH_3	INS-91	9 (1)	$\pm 65 \pm 85 \pm 65$	13.0	14.4
CH_3-CHT-CH_3	Zurich-92	20 (1)	$\pm 24 \pm 48 \pm 61$	11.0	11.6
CH_3-CHT-CH_3	CIAE-92	2 (8)	$\pm 4 \pm 75 \pm 48$	12.3	12.1
T_2	LANL-91	?12 (1)	$\pm 147 \pm 68 \pm 41$	9.3	16.6