# ON TEST THEORIES OF SPECIAL RELATIVITY 

## 4．גy甘yglר

£66l l $2 d \exists S$
日 $\forall 7$ IWyヨコ

Yuan Zhong ZHANG<br>James M．Nester

P．O．Box 2735，Beijing 100080，The People＇s Republic of China

Telefax ：（86）－1－2562587
Telex ： 22040 BAOAS CN

Telephone ： 2568348
Cable ： 6158

# On Test Theories of Special Relativity 

## Yuan Zhong ZHANG

Center of Theoretical Physics, CCAST (World Laboratory), P.O.Box 8730, Beijing Institute of Theoretical Physics, Academia Sinica, P.O.Box 2735, Beijing ${ }^{1}$ and

James M. Nester ${ }^{2}$
Department of Physics, National Central University, Chungli, Taiwan 32054

## Abstract

We review the Edwards transformation, and investigate the Robertson transformation and the Mansouri-Sexl (MS) transformation. It is shown that the MS transformation is a generalization of the Robertson transformation, just as the Edwards transformation is a generalization of the Lorentz transformation. In other words, the MS transformation differs from the Robertson transformation by a directional parameler $\mathbf{q}$, just as the case of the Edwards and Lorentz transtrosion is ony that the directional paramer a is to say that the directional parameter $q$ representing the anisotropy of the one-way speed of Rul is aber phen (Robertson) transformation(s) and the Lorentz transformation is caused by the anisotropy of the two-way speed of light and another parameter d. Therefore a physical test of the MS transfomation is a test of the two-way speed of light (or the parameter $d$ ), but not of the one-way speed of ligh

PASC number(s): 03.30.+p, 04.80.+z

Submitted to Phys. Rev. D

## 1. Permanent address.

2. Supported by the National Science Council under contract No. NSC 81-0208-M-008-03

## 1 Introduction

In Einstein's theory of special relativity [1], constancy of the speed of light is its second postulate. With this postulate, a clock located at any position in a inertial frame can be synchronized with a clock at the origin of the frame by means of a light pulse. Since that time, the clock synchronization problem has been discussed by many authors. Robertson (1949) [2] proposed a more general transformation. Reichenbach (1958) [3] and Grunbaum (1960) [4] discussed this problem in detail, and pointed out that no observable difference would result if the speed of light really were anisotropic. Ruderfer (1960) [5] held that special relativity contains an important assumption which has not and possibly cannot be tested. Edwards (1963) [6] and Winnie (1970) [7] obtained a generalized Lorentz transformation starting from the constancy of the two-way speed of light. It was concluded that the generalized Lorentz transformation predicts the same observable effects with the standard Lorentz transformation. Later, Mansouri and Sexl (1977) [8] proposed another more general transformation. After that time, many papers on this topic, such as Bertotti (1979) [9], MacArthur (1986) [10], Haugan and Will (1987) [11], Abolghasem, Khajehpour and Mansouri (1988) [12], Riis et al. $(1988,1989)$ [13], Bay and White (1989) [14], Gabriel and Haugan (1990) [15], Krisher et al. (1990) [16], and Will (1992) [17], were published. However, some ambiguities still exist in comparing the test theory with physical experiments. Thus it is necessary to analyze these kinds of test theories in detail.

In this paper, we shall first recall the Edwards transformation and its physical meaning, and then investigate the Robertson transformation and the Mansouri-Sexl transformation. We give the physical meaning of these transformations, and show the connections among the transformations under consideration. We propose a method for constructing a covariant dynamics.

## 2 One-Way Speed and Two-Way Speed

Consider a Cartesian coordinate frame whose origin is the point $O$. Let $P$ denotes a point with coordinates $(x, y, z)$, and $\mathbf{r}=(x, y, z)$ indicates a radial vector. $c_{r}$ and $c_{-r}$ refer to the one-way speed of light in the direction of $r / r$ and in the opposite direction, respectively. We define the two-way speed of light along the path $l_{O P}+l_{P O}$
as $\tilde{c}_{r}=\left(l_{O P}+l_{P O}\right) /\left(t_{O P}+t_{P O}\right)$, where $l_{O P}=l_{P O}=r, t_{O P}=r / c_{r}$ is the time lapse between the emission of the light pulse at $O$ and its arrival at $P$, and $t_{P O}=r / c_{-r}$ is the time interval spent by the pulse from $P$ back to $O$. So that the two-way speed of light can be expressed as

$$
\begin{equation*}
\bar{c}_{r}=\frac{2 c_{c_{r}} c_{-r}}{c_{r}+c_{-r}} . \tag{2.1}
\end{equation*}
$$

Eq.(2.1) implies that the choices of $c_{r}$ and $c_{-r}$ are restricted in such a way that the sense of cause is preserved. In other words, a light signal starting at $O$ cannot reach $P$ before it leaves $O$. Since top and $t_{P O}$ must be positive, so must $c_{r}$ and $c_{-r}$ be positive. Thus Eq.(2.1) leads to the restriction

$$
\begin{equation*}
\frac{\bar{c}_{r}}{2} \leq c_{r}\left(c_{-r}\right) \leq \infty . \tag{2.2}
\end{equation*}
$$

It is convenient to introduce a directional parameter q as follows

$$
\begin{equation*}
c_{r}=\frac{\bar{c}_{r}}{1-q_{r}}, \quad c_{-r}=\frac{\bar{c}_{r}}{1+q_{r}} . \tag{2.3a}
\end{equation*}
$$

Using Eqs.(2.3a) in (2.2), we get the limit on the directional parameter

$$
\begin{equation*}
-1 \leq q_{r} \leq+1 . \tag{2.4a}
\end{equation*}
$$

In particular, along the $x-, y$ - and $z$-axes, we have

$$
\begin{align*}
& c_{i}=\frac{\bar{c}_{i}}{1-q_{i}}, \quad c_{-i}=\frac{\bar{c}_{i}}{1+q_{i}}  \tag{2.3b}\\
& -1 \leq q_{i} \leq+1, \quad i=x, y, z \tag{2.4b}
\end{align*}
$$

Let us discuss the relation between $q_{r}$ and $q_{i}$. Consider the following "loops" of light:

$$
\begin{equation*}
l_{+}=l_{O A}+l_{A B}+l_{B P}+l_{P O}, \quad l_{-}=l_{O P}+l_{P B}+l_{B A}+l_{A O}, \tag{2.5}
\end{equation*}
$$

where $l_{O A}$ is the distance between $O$ and $A$, and so on. Coordinates of the points $O, A, B$ and $P$ are $(0,0,0),(x, 0,0),(x, y, 0)$ and $(x, y, z)$, respectively. Let $t_{+}$and $t_{-}$ denote the time intervals spent by the light pulse traveling along $l_{+}$and $l_{-}$, respectively, i.e.,

$$
\begin{equation*}
t_{+}=t_{O A}+t_{A B}+t_{B P}+t_{P O}, \quad t_{-}=t_{O P}+t_{P B}+t_{B A}+t_{A O} \tag{2.6}
\end{equation*}
$$

Substituting $t_{O A}=x / c_{x}, t_{A O}=x / c_{-x}, t_{A B}=y / c_{y}, t_{B A}=y / c_{-y}, t_{B P}=z / c_{x}, t_{P B}=$ $z / c_{-z}, t_{O P}=r / c_{r}, t_{P O}=r / c_{-r}$ into Eq. (2.6), we have

$$
\begin{equation*}
t_{+}=\frac{x}{c_{x}}+\frac{y}{c_{y}}+\frac{z}{c_{z}}+\frac{r}{c_{-r}}, \quad t_{-}=\frac{x}{c_{-x}}+\frac{y}{c_{-v}}+\frac{z}{c_{-z}}+\frac{r}{c_{r}} . \tag{2.7}
\end{equation*}
$$

Using the definition, Eq.(2.3), we obtain from Eq.(2.7)

$$
\begin{equation*}
t_{+}-t_{-}=2 r\left[\frac{q_{r}}{\bar{c}_{r}}-\left(\frac{q_{x}}{\bar{c}_{x}} \cos \alpha+\frac{q_{y}}{\bar{c}_{y}} \cos \beta+\frac{q_{z}}{\bar{c}_{x}} \cos \gamma\right)\right], \tag{2.8}
\end{equation*}
$$

where $\cos \alpha=x / r, \cos \beta=y / r, \cos \gamma=z / r$, and $\cos ^{2} \alpha+\cos ^{2} \beta+\cos ^{2} \gamma=1$. Assuming $t_{+}=t_{-}$, from Eq.(2.8) we obtain

$$
\begin{equation*}
\frac{q_{r}}{\bar{c}_{r}}=\frac{q_{x}}{\bar{c}_{x}} \cos \alpha+\frac{q_{v}}{\bar{c}_{y}} \cos \beta+\frac{q_{x}}{\bar{c}_{x}} \cos \gamma . \tag{2.9}
\end{equation*}
$$

## 3 The Edwards Transformation

Let us recall a generalized Lorentz transformation as an example in illustration of how to compare a test theory of special relativity with physical experiments. Ed wards (1963) [6] assumed the constancy of two-way speed of light, and modified Einstein's second postulate as: the two-way speed of light in a vacuum as measured in two coordinate systems moving with constant relative velocity is the same constant regardless of any assumptions concerning the one-way speed. The constancy of two-way speed of light implies $\bar{c}_{r}=\bar{c}_{r^{\prime}}=c=$ constant. For simplicity let $q \equiv q_{x} \neq 0, \quad q^{\prime} \equiv q_{x^{\prime}} \neq 0, \quad q_{y}=q_{y^{\prime}}=q_{z}=q_{z^{\prime}}=0$, so we have from Eq.(2.3)

$$
\begin{gather*}
c_{x}=\frac{c}{1-q}, \quad c_{-x}=\frac{c}{1+q}, \quad c_{y}=c_{-y}=c_{z}=c_{-x}=c, \\
c_{x^{\prime}}=\frac{c}{1-q^{\prime}}, \quad c_{-x^{\prime}}=\frac{c}{1+q^{\prime}}, \quad c_{y^{\prime}}=c_{-y^{\prime}}=c_{x^{\prime}}=c_{-z^{\prime}}=c \tag{3.1}
\end{gather*}
$$

From the constancy of the two-way speed, Edwards (1963) [6] obtained the following generalized Lorentz transformation:

$$
\begin{gather*}
t=\frac{1}{\sqrt{\left(1+\frac{v}{c} q^{\prime}\right)^{2}-\frac{v^{2}}{c^{2}}}}\left\{\left[1+\frac{v}{c}\left(q+q^{\prime}\right)\right] t^{\prime}-\left[\frac{v}{c}\left(1-q^{\prime 2}\right)+\left(q-q^{\prime}\right)\right] \frac{x^{\prime}}{c}\right\}, \\
x=\frac{1}{\sqrt{\left(1+\frac{v}{c} q^{\prime}\right)^{2}-\frac{v^{2}}{c^{2}}}}\left(x^{\prime}-v t^{\prime}\right), \\
y=y^{\prime}, \\
z=z^{\prime},
\end{gather*}
$$

where $v$ is the velocity of the inertial frame $S(t x y z)$ with respect to $S^{\prime}\left(t^{\prime} x^{\prime} y^{\prime} z^{\prime}\right)$. In the case $q^{\prime}=0$, the frame $S^{\prime}$ is a "preferred" reference system to be denoted by $\Sigma(T X Y Z)$. In this case, the Edwards transformation Eq.(3.2) reduces to

$$
\begin{gather*}
t=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}\left[\left(1+\frac{v}{c} q\right) T-\left(\frac{v}{c}+q\right) \frac{X}{c}\right], \\
\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}(X-v T) \\
y=Y \\
z=Z \tag{3.3}
\end{gather*}
$$

How are applications of the Edwards transformation (3.2) to physical experiments made? It is noted that the coordinate $t$ and $t^{\prime}$ are not directly observable because they depend upon the definition of simultaneity (an observable time should be a proper time), and hence all quantities associating with $t$ and $t^{\prime}$, such as $v=d x / d t$ and $v^{\prime}=$ $d x^{\prime} / d t^{\prime}$, are also not directly observable. On the other hand, distant clocks in physical experiments are generally synchronized by means of Einstein simultaneity, i.e. the constancy of the one-way speed of light. Thus in order to compare the mathematical quantities in the test theory with data given in physical experiments, a relation between a general clock synchronization and Einstein clock synchronization is needed. Let quantities with a subscript " 0 " correspond to Einstein simultaneity. Consider a light signal traveling from $O$ to $P$. Let $t_{0}$ be the departure time at $O$, and $t$ or $t_{0}$ be the arrival time at $P$. A general clock synchronization implies

$$
\begin{equation*}
t=t_{0}+\frac{x}{c_{x}}+\frac{y}{c_{y}}+\frac{z}{c_{z}} \tag{3.4}
\end{equation*}
$$

On the other hand, Einstein clock synchronization gives

$$
\begin{equation*}
t_{0}=t_{0}+\frac{x}{c}+\frac{y}{c}+\frac{z}{c} . \tag{3.5}
\end{equation*}
$$

The relation between $t$ and $t_{0}$ follows from Eqs.(3.4) and (3.5)

$$
\begin{equation*}
t=t_{0}+x\left(\frac{1}{c_{x}}-\frac{1}{c}\right)+y\left(\frac{1}{c_{y}}-\frac{1}{c}\right)+z\left(\frac{1}{c_{z}}-\frac{1}{c}\right) . \tag{3.6a}
\end{equation*}
$$

Similarly in frame $S^{\prime}$, we have

$$
\begin{equation*}
t^{\prime}=t_{0}^{\prime}+x^{\prime}\left(\frac{1}{c_{x^{\prime}}}-\frac{1}{c}\right)+y^{\prime}\left(\frac{1}{c_{y^{\prime}}}-\frac{1}{c}\right)+z^{\prime}\left(\frac{1}{c_{z^{\prime}}}-\frac{1}{c}\right) . \tag{3.6b}
\end{equation*}
$$

For Edwards clock synchronization, using Eq.(3.1), Eq.(3.6) leads to

$$
\begin{equation*}
t=t_{0}-q \frac{x}{c}, \quad t^{\prime}=t_{0}^{\prime}-q^{\prime} \frac{x^{\prime}}{c} . \tag{3.7}
\end{equation*}
$$

Using Eq.(3.7) we obtain relations between the velocities ( $u_{x}=d x / d t, u_{x^{\prime}}=d x^{\prime} / d t^{\prime}, \cdot$. .) corresponding to Edwards simultaneity and the ones $โ\left(u_{x}\right)_{0}=d x / d t_{0}$,
$\left.\left(u_{x^{\prime}}\right)_{0}=d x^{\prime} / d t_{0}^{\prime}, \cdots\right)$ corresponding to Einstein simultaneity:

$$
\begin{equation*}
u_{x}=\frac{\left(u_{x}\right)_{0}}{1-q\left(u_{x}\right)_{0}}, \quad u_{x^{\prime}}=\frac{\left(u_{x^{\prime}}\right)_{0}}{1-q^{\prime}\left(u_{x^{\prime}}\right)_{0}} . \tag{3.8a}
\end{equation*}
$$

In Eq.(3.2),$v=d x^{\prime} / d t^{\prime}$, and hence we have from Eq.(3.8a)

$$
\begin{equation*}
v=\frac{v_{0}}{1-\frac{v_{0}}{c} q^{q^{\prime}}} . \tag{3.8b}
\end{equation*}
$$

Using Eqs.(3.7) and (3.8b), the Edwards transformation, Eq.(3.2), reduces to the standard form

$$
\begin{gather*}
t_{0}=\frac{1}{\sqrt{1-\frac{v_{0}^{2}}{c^{2}}}}\left(t_{0}^{\prime}-\frac{v_{0}}{c^{2}}\right), \\
x=\frac{1}{\sqrt{1-\frac{v_{0}^{2}}{c^{2}}}}\left(x^{\prime}-v_{0} t_{0}^{\prime}\right) \\
y=y^{\prime} \\
z=z^{\prime} . \tag{3.9}
\end{gather*}
$$

This result shows that the difference between the Edwards transformation and the Lorentz transformation is just their different simultaneities, and that the Edwards transformation predicts the same observable effects as the Lorentz transformation. In other words, the directional parameters $q$ and $q^{\prime}$, and hence the one-way speed of light, cannot be tested in any physical experiment. Let us give an example in illustration of this fact. The Doppler effect can be easily obtained (18]

$$
\begin{equation*}
\nu^{\prime}=\frac{\nu \sqrt{\left(1+\frac{v}{c} q^{\prime}\right)^{2}-\frac{v^{2}}{c^{2}}}}{1+\frac{v}{c} q^{\prime}-\frac{v}{c} \cos \alpha}, \tag{3.10}
\end{equation*}
$$

where $\nu$ is the frequency of light emitted by source moving at velocity v relative to an observer, $\nu^{\prime}$ is the corresponding frequency measured by the observer, $\alpha$ is the angle between the propagating direction of light and the velocity $\mathbf{v}$. It is stressed

$$
\begin{equation*}
c_{x}=c_{-x}=\bar{c}_{\|}, \quad c_{y}=c_{-y}=c_{z}=c_{-z}=\bar{c}_{1} . \tag{4.76}
\end{equation*}
$$

One can see from Eq.(4.7) that in the Robertson test theory the one-way speed of ligh in a given direction is equal to the one in its opposite direction, but the two-way speed of light, in general, depends upon $v^{2}$ and is anisotropic. Now we consider the problem of how to compare the Robertson transformation (4.3) with physical experiments. It needs to be emphasized that contrary to MacArthur [10], Robertson simultaneity is different from Einstein simultaneity because of the anisotropy of the two-way speed of light. So that we still need a relation between Robertson clock synchronization and Einstein clock synchronization. The general relation is given by Eq.(3.6a). In the present case (i.e., $c_{x}=\bar{c}_{\|}, c_{y}=c_{z}=\bar{c}_{\perp}$ ), Eq.(3.6a) becomes

$$
\begin{equation*}
t=t_{0}+x\left(\frac{1}{\bar{c}_{\|}}-\frac{1}{c}\right)+(y+z)\left(\frac{1}{\bar{c}_{\perp}}-\frac{1}{c}\right) . \tag{4.8}
\end{equation*}
$$

Using Eq.(4.8) in Eq.(4.3b), the Robertson transformation becomes

$$
\begin{gather*}
t_{0}=(d) \frac{c}{\bar{c}_{\perp}}\left\{\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}\left\{\left[1+\frac{v\left(c-\bar{c}_{\|}\right)}{c^{2}}\right] T-\frac{v+\left(c-\bar{c}_{\|}\right)}{c^{2}} X\right\}-\frac{c-\bar{c}_{\perp}}{c^{2}}(Y+Z)\right\} \\
x=(d) \\
\bar{c}_{\| I} \frac{1}{\bar{c}_{\perp}} \frac{\sqrt{1-\frac{v^{2}}{c^{2}}}}{\sqrt{2}}(X-v T) \\
y=(d) Y  \tag{4.9}\\
z=(d) Z
\end{gather*}
$$

One should compare Eq.(4.9) with physical experiments. The Robertson transformation differs from the Lorentz transformation by the anisotropy of the two-way speed of light, $\bar{c}_{\|}$and $\bar{c}_{1}$, and the parameter $d$. The difference is of the second order in $v / c$.

## 5 The Mansouri-Sexl Transformation

Mansouri and Sexl (1977) [8] proposed a more general linear transformation as

$$
t=a T+\epsilon \cdot \mathbf{x},
$$

$$
\begin{gather*}
x=b(X-v T), \\
y=(d) Y, \\
z=(d) Z, \tag{5.1}
\end{gather*}
$$

where the frame $\Sigma(T X Y Z)$ is a preferred inertial reference system in which the oneway speed of light is isotropic. The frame $S(t x y z)$ is moving at velocity $v$ in the positive $x$ direction with respect to $\Sigma$, the parameters $a, b, d$, and $\epsilon$ are functions of $v$. Let us introduce the following new vector parameter $\mathrm{q}=\left(q_{x}, q_{v}, q_{z}\right)$ in place of the old vector parameter $\epsilon=\left(\epsilon_{x}, \epsilon_{y}, \epsilon_{z}\right)$

$$
\begin{gather*}
\epsilon_{x}=-\frac{a}{c b\left(1-\frac{v^{2}}{c^{2}}\right)}\left(\frac{v}{c}+q_{x}\right)=-\frac{c}{\bar{c}_{\|}}\left(\frac{v}{c}+q_{x}\right), \\
\epsilon_{y}=-\frac{a}{c d \sqrt{1-\frac{v^{2}}{c^{2}}}} q_{y}=-\frac{c}{\bar{c}_{\perp}} q_{v}, \\
\epsilon_{z}=-\frac{a}{c d \sqrt{1-\frac{v^{2}}{c^{2}}}} q_{z}=-\frac{c}{\bar{c}_{\perp}} q_{z}, \tag{5.2}
\end{gather*}
$$

where the constant $c$ is the speed of light in $\Sigma$, and $\bar{q}_{\|}$and $\bar{q}_{\|}$are given by Eq.(4.4). Putting Eq.(5.2) in Eq.(5.1), the Mansouri-Sexl transformation can be expressed as

$$
\begin{gather*}
t=(d) \frac{c}{\bar{c}_{\perp}}\left\{\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}\left[\left(1+\frac{v}{c} q_{x}\right) T-\left(\frac{v}{c}+q_{x}\right) \frac{X}{c}\right]-q_{v} \frac{Y}{c}-q_{z} \frac{Z}{c}\right\} \\
x=(d) \frac{\overline{c_{\|}}}{\bar{c}_{\perp}} \frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}(X-v T) \\
y=(d) Y \\
z=(d) Z \tag{5.3}
\end{gather*}
$$

Next we shall prove that the new parameters $\left(q_{x}, q_{y}, q_{z}\right)$ are just the directional parameters defined by Eqs.(2.3) and (2.4). For this reason, we first calculate the speed of
light in $S$. Substituting the Mansouri-Sexl transformation (5.3) into (4.5), we get the equation satisfied by the one-way speed of light in frame $S$ :

$$
\begin{equation*}
c_{r}^{2}\left[\frac{1}{\bar{c}_{r}^{2}} q_{r}^{2}-\frac{1}{\bar{c}_{\perp}^{2}}-\left(\frac{1}{\bar{c}_{\|}^{2}}-\frac{1}{\bar{c}_{\perp}^{2}}\right) \cos ^{2} \alpha\right]+c_{r}\left(\frac{1}{\bar{c}_{r}} q_{r}\right)+1=0, \tag{5.4a}
\end{equation*}
$$

where $c_{r}=r / t, x / t=c_{r} \cos \alpha, y / t=c_{r} \cos \beta, z / t=c_{r} \cos \gamma, \cos ^{2} \alpha+\cos ^{2} \beta+\cos ^{2} \gamma=1, \bar{c}_{\perp}$ and $\bar{c}_{\|}$are given by Eq.(4.4) which we will show are just the two-way speed of light parallel and perpendicular to v , and $q_{r} / c_{T}$ is defined by

$$
\begin{equation*}
\frac{q_{r}}{\bar{c}_{r}}=\frac{q_{z}}{\bar{c}_{\|}} \cos \alpha+\frac{q_{y}}{\bar{c}_{\perp}} \cos \beta+\frac{q_{z}}{\bar{c}_{\perp}} \cos \gamma . \tag{5.4b}
\end{equation*}
$$

Solutions to Eq.(5.4) for $c_{r}$ and $c_{-r}$ are given by

$$
\begin{align*}
& c_{r}=\frac{1}{\sqrt{\frac{1}{\bar{c}_{\perp}^{2}}+\left(\frac{1}{\bar{c}_{\|}^{2}}-\frac{1}{\bar{c}_{\perp}^{2}}\right) \cos ^{2} \alpha}-\frac{q_{r}}{\bar{c}_{r}}}  \tag{5.5a}\\
& c_{-r}=\frac{1}{\sqrt{\frac{1}{\bar{c}_{\perp}^{2}}+\left(\frac{1}{\bar{c}_{\|}^{2}}-\frac{1}{\bar{c}_{\perp}^{2}}\right) \cos ^{2} \alpha+\frac{q_{r}}{\bar{c}_{r}}}} . \tag{5.5b}
\end{align*}
$$

In particular, the one-way speed of light along the $i$-axis can be found from Eqs.(5.5) and (5.4b)

$$
\begin{equation*}
c_{i}=\frac{\bar{c}_{i}}{1-q_{i}}, \quad c_{-i}=\frac{\bar{c}_{i}}{1+q_{i}}, \quad i=x, y, z, \tag{5.5c}
\end{equation*}
$$

where $\bar{c}_{x}=\bar{c}_{\mid}$and $\bar{c}_{y}=\bar{c}_{x}=\bar{c}_{1}$. The result shows that $\bar{c}_{\| \mid}$and $\bar{c}_{\perp}$ are just the two-way speed of light along $x$ and $y$-axis (or $z$-axis), respectively, and the new parameters ( $q_{x}, q_{y}, q_{z}$ ) defined by Eq.(5.2) have the same meaning as the directional parameters given in Eq.(2.3), and hence $q_{r} / \vec{c}_{r}$ defined by Eq.(5.4b) is the same as the one given in Eq.(2.9). Consequently, from (5.5), the speed of light in the frame $S$ reduces to

$$
\begin{align*}
& c_{r}=\frac{\bar{c}_{r}}{1-q_{r}}, \quad c_{-r}=\frac{\bar{c}_{r}}{1+q_{r}},  \tag{5.5d}\\
& \bar{c}_{r}=\frac{\bar{c}_{\|} \bar{c}_{\perp}}{\sqrt{\bar{c}_{\|}^{2}+\left(\bar{c}_{\perp}^{2}-\bar{c}_{\|}^{2}\right) \cos ^{2} \alpha}} \tag{5.5e}
\end{align*}
$$

We can see that the two-way speed of light, Eq.(5.5e), is the same as the one, Eq.(4.7), in the Robertson test theory. Again a relation between $t$ in the MansouriSexl transformation (5.3) and $t_{0}$ corresponding to Einstein simultaneity is needed, in
order to compare the test theory with physical experiments. This relation is given by Eq.(3.6). Putting Eq.(3.6a) in Eq.(5.3) and using Eq.(5.5c), we obtain

$$
\begin{gather*}
t_{0}=(d) \frac{c}{\bar{c}_{\perp}}\left\{\frac{1}{\left.\sqrt{1-\frac{v^{2}}{c^{2}}}\left\{\left[1+\frac{v\left(c-\bar{c}_{\|}\right)}{c^{2}}\right] T-\frac{v+\left(c-\bar{c}_{\|}\right)}{c^{2}} X\right\}-\frac{c-\bar{c}_{1}}{c^{2}}(Y+Z)\right\}}\right. \\
x=(d) \frac{\bar{c}_{\|}}{\bar{c}_{\perp}} \frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}(X-v T) \\
y=(d) Y \\
z=(d) Z . \tag{5.6}
\end{gather*}
$$

For the same reason as in Sec. 3, we should directly compare Eq.(5.6) with physical experiments. Eq.(5.6) is the same with Eq.(4.9), as expected. This implies that the directional parameter $q$ in the Mansouri-Sexl transformation also cannot be tested in any physical experiment.

## 6 Discussion and Conclusions

The following relations can be shown from comparing the Lorentz transformation, the Edwards transformation (3.3), the Robertson transformation (4.3), and the Mansouri-Sexl transformation (5.3):

| Lorentz | $\leftarrow \mathbf{q}=0 \longleftarrow$ | Edwards |
| :---: | :---: | :---: |
| $\dagger$ | $\dagger$ |  |
| $\bar{c}_{\\|}=\bar{c}_{1}=c$ |  |  |
| $\dagger$ |  | $\bar{c}_{\\|}=\bar{c}_{1}=c$ |
| Robertson | $\longmapsto \mathbf{q}=0 \longmapsto$ | Mansouri-Sexl |

When the different clock synchronizations are taken into account, the Edwards transformation Eq.(3.9) is the same as the Lorentz transformation, while the MansouriSexl transformation Eq.(5.6) is also the same as the Robertson transformation Eq.(4.9). So that we come to the following conclusions:
(i) the Mansouri-Sexl transformation predicts the same observable effects as the Robertson transformation, just as the Edwards transformation does with the Lorentz transformation.
(ii) In other words, the directional parameter q cannot be observed in any physical experiment. This is to say that its modulus can be taken as any value in the range $(-1,+1)$, or to say that the definition of simultaneity can be chosen arbitrarily. Einstein simultaneity is the simplest one among the theories in which the two-way speed of light is isotropic; while Robertson simultaneity is the simplest one among the theories where the two-way speed of light is anisotropic.
(iii) the Robertson transformation differs from the Lorentz transformation by the anisotropy of the two-way speed of light and another parameter $d$. So they predict different observable effects. It is same for the Mansouri-Sexl transformation and the Edwards transformation. Thus for comparing with physical experiments, it is better to use the Robertson transformation, one does not need to employ the physically equivalent the Mansouri-Sexl transformation.
(iv) Therefore, a test of the Mansouri-Sexl transformation is just a test of anisotropy of the two-way speed of light (and a test of the parameter $d$ ), but not a test of anisotropy of the one-way speed of light. For instance, the constancy of values obtained by measuring the two-way speed of light in physical experiments performed before may yield a limit on the two parameters $\bar{q}_{\|}$and $\bar{c}_{1}$; and then the second-order Doppler effects may give a limit on the third parameter $d$.

Finally we suggest that in order to construct a covariant dynamics with anisotropy of the two-way speed of light, it is better to start from Eq.(5.6) where the Einstein simultaneity is used.

## ACKNOWLEDGMENT:

One of authors (Y. Z. ZHANG) would like to thank Prof. Wei-Tou Ni, Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, for some helpful discussions.

## References

1 A. Einstein, Ann. Phys. (Leipzig) 17, 891 (1905).
2 H. P. Robertson, Rev. Mod. Phys. 21, 378 (1949)
3 H. Reichenbach, The Philosophy of Space and Time (Dover Publications, Inc., New

York, 1958), p. 142.
4 A. Grunbaum, "Logical and Philosophical Foundations of the Special Theory of Relativity", in Philosophy of Science, ed. by A. Danto and S. Morgenbesser (Meridian Books, New York, 1960), Sec. 2.
5 M Ruderfer, Proc. IRE 48, 1661 (1960).
6 W. F. Edwards, Am. J. Phys. 31, 482 (1963).
7 J. A. Winnie, Phil. Sci. 37, 81 (1970); 37, 223 (1970).
8 R. Mansouri and R. U. Sexl, Gen. Relativ. Gravit. 8, 497 (1977); 8, 515 (1977); 8, 809 (1977).
9 B. Bertotti, Radio Sci. 14, 621 (1979).
10 D. W. MacArthur, Phys. Rev. A 33, 1 (1986).
11 M. P. Haugan and C. M. Will, Phys. Today 40(5), 69 (1987).
12 G. Abolghasem, M. R. H. Khajehpour, and R. Mansouri, Phys. Lett. A 132, 310 (1988).

13 E. Riis et al., Phys. Rev. Lett. 60, 81 (1988); 62, 842 (1989)
14 Z. Bay and J. A. White, Phys. Rev. Lett. 62, 841 (1989).
15 M. D. Gabriel and M. P. Haugan, Phys. Rev. D 41, 2943 (1990).
16 T. P. Krisher et al., Phys. Rev. D 42, 731 (1990).
17 C. M. Will, Phys. Rev. D 45, 403 (1992)
18 Y. Z. Zhang, "Experimental Foundations of Special Relativity", Science Press, Beijing, 1979, p. 33.

