AS-ITP-93-44

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AS-ITP-93-44
July 1993

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Gauge Field Theories with Nonlinear Connections

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Massless N-component free complex Klein-Gordon and Dirac field equations possess global affine symmetry. Localizing this symmetry leads to gauge models with nonlinear connections. In a special gauge in which the original fields disappear, these models contain only connection fields. Generally new gauge fields can be decomposed into several massive fields with different spins and masses. The translation gauge fixed theories look like the usual $SU(N)$ gauge models with several matter fields.

Local gauge invariance as a guiding principle to find possible interaction theories, suggested by Yang and Mills [1], has been successfully applied to the electroweak unified theory [2] and quantum chromodynamics of strong interaction [3]. It is well known that the mathematical structure of above gauge field theories is the principal fibre bundle theory with linear connections. In this paper, starting from the global affine invariance of equations of motion of free massless fields and following Yang and Mills's idea, we try to establish the local affine invariant interaction theories, i.e. the nonlinear gauge theories with local affine symmetry, examine their mathematical structure and physical implications.

Let us look at the equation of motion of the massless N-component complex free scalar field first:

$$\partial^\mu \partial_\mu \phi^i(x) = 0 \ , \quad i = 1, 2, ..., N. \tag{1}$$

Obviously this equation is invariant under the global affine transformations which are defined to be

$$\phi'^i(x) = U^i_j(\alpha)\phi^j(x) + \sigma^i \tag{2}$$

where $U^i_j(\alpha)$ is the SU(N) group element with parameter α^m $(m = 1, 2, ...r)$, and σ^i is the "affine translation" parameter. Our aim is to construct an interaction theory which is invariant under the local affine transformations

$$\phi'^i(x') = U^i_j(\alpha(x))\phi^j(x) + \sigma^i(x). \tag{3}$$

Since the transformations (2) and (3) are not linear with respect to ϕ^i, we can not use ordinary linear connection to construct the gauge theory. In fact, the set $\{\phi\}$ here does not form a linear space, it is only a differential manifold. So instead of usual vector and principle bundle theory which is frequently used in the physics literature, we need to use a more general differential bundle theory. That is, it is necessary to use the nonlinear affine connections [4][5] which will be briefly reviewed in the following. The formulation used here comes mainly from ref.[5], since we think, the coordinate description there is more convenient for the physicists.

The nonlinear connection fields $B^i_\mu(x, \phi)$ are vector fields which transform as

$$B'^i_\mu(x, \phi') = U^i_j(x)B^j_\mu(x, \phi) - \partial_\mu U^i_j(x)\phi^j(x) - \partial_\mu \sigma^i(x) \tag{4}$$

under (3). These $B^i_\mu(x, \phi)$ can be expressed in term of group generators as follows

$$B^i_\mu(x, \phi) = gA^a\mu(x)(T_a)^i_j\phi^j(x) + im\Theta^i_\mu(x) \tag{5}$$

where $(T_a)'s$ are anti-Hermitian representation matrices of $SU(N)$ generators. $A^a_\mu(x)'s$ are usual $SU(N)$ gauge fields and $\Theta^i_\mu(x)'s$ are affine translation connection. Under transformation (3), $A^a_\mu(x)$ and $\Theta^i_\mu(x)$ transform as

$$\begin{aligned} A'_\mu(x) &= UA_\mu(x)U^{-1} - \tfrac{1}{g}(\partial_\mu U)U^{-1} \\ \Theta'_\mu(x) &= U\Theta_\mu(x) + \tfrac{i}{m}\partial_\mu\sigma(x) + \tfrac{i}{m}[U(gA_\mu)U^{-1} - (\partial_\mu U)U^{-1}]\sigma(x). \end{aligned} \tag{6}$$

The covariant derivative of fields $\phi^i(x)$ is defined as

$$\begin{aligned} \nabla_\mu\phi^i(x) &= \partial_\mu\phi^i(x) + B^i_\mu(x, \phi) \\ &= \partial_\mu\phi^i(x) + gA_\mu{}^i_j\phi^j(x) + im\Theta^i_\mu(x) \equiv [D_\mu\phi(x)]^i + im\Theta^i_\mu(x) \end{aligned} \tag{7}$$

which transforms as

$$\nabla'_\mu\phi'^i(x) = U^i_j(x)\nabla_\mu\phi^j(x) \tag{8}$$

under (3). The curvature tensor has the following form

$$\begin{aligned} R^i_{\mu\nu} &= R^{(1)i}_{\ j\mu\nu}\phi^j + R^{(2)i}_{\ \mu\nu} \\ R^{(1)i}_{\ j\mu\nu} &= (F_{\mu\nu})^i_j = (F^a_{\mu\nu}T_a)^i_j = (\partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu])^i_j \\ R^{(2)i}_{\ \mu\nu} &= im(\partial_\mu\Theta^i_\nu - \partial_\nu\Theta^i_\mu + gA^i_{\mu j}\Theta^j_\nu - gA^i_{\nu j}\Theta^j_\mu). \end{aligned} \tag{9}$$

They transform as

$$\begin{aligned} &R'^i_{\mu\nu} = U^i_j(x)R^j_{\mu\nu} \ , \qquad R'^{(1)i}_{\ j\mu\nu} = U^i_k R^{(1)k}_{\ l\mu\nu}(U^{-1})^l_j, \\ &R'^{(2)i}_{\ \mu\nu} = U^i_j(x)[R^{(2)j}_{\ \mu\nu} - R^{(1)j}_{\ l\mu\nu}(U^{-1})^l_k\sigma^k] \end{aligned} \tag{10}$$

In general nonlinear connection theory one needs to introduce a new connection to define the covariant derivative of $\nabla_\mu\phi$, but for affine case it is enough to define second covariant derivative by use of linear connection $A_\mu(x)$:

$$\begin{aligned} (\nabla_\mu\phi^i)_{\|\nu} &\equiv \partial_\nu(\nabla_\mu\phi^i) + g(A_\nu)^i_j(\nabla_\mu\phi^j) \\ &= \partial_\nu\partial_\mu\phi^i + g\partial_\nu(A_\mu)^i_j\phi^j + g(A_\mu)^i_j\partial_\nu\phi^j + g(A_\nu)^i_j\partial_\mu\phi^j \\ &\quad + g^2(A_\nu)^i_j(A_\mu)^j_k\phi^k + im[\partial\nu\Theta^i_\mu + g(A_\nu)^i_j\Theta^j_\mu]. \end{aligned} \tag{11}$$

Also the second curvature tensor is nothing but $R^{(1)i}_{\ j\mu\nu}$. Their transformations under (3) are

$$(\nabla'_\mu\phi'^i(x))_{\|\nu} = U^i_j(x)(\nabla_\mu\phi^j(x))_{\|\nu} \tag{12}$$

and (10) respectively.

Above mathematical formulas have been obtained by mathematicians. What we do is to write them for particular case of our affine gauge group.

Now we are in a position to construct the physical model invariant under the local affine transformation. With the requirement of no higher derivative of fields and renormaliziability of the final action, we can write down a Lagrangian

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5$$

$$
\begin{aligned}
\mathcal{L}_1 &= -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}, & \mathcal{L}_2 &= g_2(R^i_{\mu\nu})^* R^{i\mu\nu}, \\
\mathcal{L}_3 &= g_3(\nabla_\mu\phi^i)^*(\nabla^\mu\phi^i), & \mathcal{L}_4 &= g_4(\nabla_\mu\phi^i)^*_{\|\nu}(\nabla^\mu\phi^i)^{\|\nu}, \\
\mathcal{L}_5 &= g_5(\nabla_\mu\phi^i)^{\|\mu *}(\nabla_\nu\phi^i)^{\|\nu}.
\end{aligned}
\tag{13}
$$

This Lagrangian looks quite complicated, moreover $\mathcal{L}_4$ and $\mathcal{L}_5$ look nonrenormalizable. But since this gauge system is a highly constrained one, we can use the transformation (3) to eliminate some nonphysical degree of freedom. As the first step of the gauge fixing, we can use the local affine translation transformation to eliminate ϕ^i fields:

$$
\phi'^i(x) = \phi^i(x) + \sigma^i(x) = 0. \tag{14}
$$

Thus we have

$$
\begin{aligned}
\mathcal{L}_2 + \mathcal{L}_3 &= \tilde{g}_2(D_\mu\Theta^i_\nu - D_\nu\Theta^i_\mu)^*(D^\mu\Theta^{i\nu} - D^\nu\Theta^{i\mu}) + \tilde{g}_3\Theta^{i*}_\mu\Theta^{i\mu} \\
\mathcal{L}_4 + \mathcal{L}_5 &= \tilde{g}_4(D_\mu\Theta^i_\nu)^*(D^\mu\Theta^{i\nu}) + \tilde{g}_5(D_\mu\Theta^{i\mu})^*(D^\nu\Theta^{i\nu}).
\end{aligned}
\tag{15}
$$

Here $\tilde{g}_k = g_k m^2$, $k = 2,3,4,5$, and the definition of D_μ see eq.(7). We can find that $\mathcal{L}_4$ and $\mathcal{L}_5$ become renormalizable in the gauge (14). The situation is similar to the supergauge field theory of superfields, where the Lagrangian becomes renormalizable in the Wess-Zumino gauge [6]. Notice that now $\mathcal{L}_5$ in (15) can be expressed as a linear combination of $\mathcal{L}_2$ and $\mathcal{L}_4$ so we simply omit it in the following. In order to make clear the physical contents let us write (15) as

$$
\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\Theta + \mathcal{L}_{\Theta-A} \tag{16}
$$

where

$$
\mathcal{L}_\Theta = 2\tilde{g}_2(\partial_\mu\Theta^i_\nu)^*(\partial^\mu\Theta^{i\nu} - \partial^\nu\Theta^{i\mu}) + \tilde{g}_3\Theta^{i*}_\mu\Theta^{i\mu} + \tilde{g}_4(\partial_\nu\Theta^i_\mu)^*(\partial^\nu\Theta^{i\mu}) \tag{17}
$$

is the quadratic part of Θ^i_μ field. The variation of (17) with respect to Θ^{i*}_μ produces the free equation of motion of Θ^i_μ :

$$
-2\tilde{g}_2\partial_\mu(\partial^\mu\Theta^{i\nu} - \partial^\nu\Theta^{i\mu}) + \tilde{g}_3\Theta^{i\nu} - \tilde{g}_4\partial_\mu(\partial^\mu\Theta^{i\nu}) = 0. \tag{18}
$$

Taking derivative ∂_ν of (18) we get

$$
\tilde{g}_4\partial_\mu\partial^\mu(\partial_\nu\Theta^\nu) - \tilde{g}_3(\partial_\nu\Theta^\nu) = 0. \tag{19}
$$

It shows that $(\partial_\nu\Theta^\nu)$ is a scalar field with mass $m_s^2 = -\dfrac{\tilde{g}_3}{\tilde{g}_4}$. Furthermore we define that

$$
\Theta_\mu = \Theta^T_\mu - \frac{1}{m_s^2}\partial_\mu(\partial_\nu\Theta^\nu). \tag{20}
$$

Substituting it into (18) and using (19) we obtain that

$$
\partial_\mu\partial^\mu(\Theta^T_\nu) - \frac{\tilde{g}_3}{2\tilde{g}_2 + \tilde{g}_4}\Theta^T_\nu = 0. \tag{21}
$$

It shows that Θ^T_ν is a vector particle with mass $m_V^2 = -\dfrac{\tilde{g}_3}{2\tilde{g}_2 + \tilde{g}_4}$. We would like to comment that we have not found any basic principle to fix the values of m_s and m_V. Some special choice of parameters, e.g., $\tilde{g}_4 = -2\tilde{g}_2$, seems possible so that m_V becomes infinite and spin 1 particle becomes ineffective. But it seems for us the natural way is both of scalar and vector particles are active.

The same consideration can be applied to the fermion case. The Dirac equation of N-internal component free massless fermion field is

$$
i\gamma_\mu\partial^\mu\psi^i(x) = 0 \quad , \quad i = 1,2,...,N. \tag{22}
$$

which is invariant under the following affine transformations

$$
\psi'^i_\beta(x) = U^i_j(\alpha)\psi^j_\beta(x) + \chi^i_\beta \tag{23}
$$

where $\beta = (1,2,3,4)$ is the spinor component index and χ^i_β is the spinor parameter. The local affine transformation is defined as

$$
\psi'^i_\beta(x) = U^i_j(\alpha(x))\psi^j_\beta(x) + \chi^i_\beta(x). \tag{24}
$$

The nonlinear spinor-vector connection can be written as

$$
B^i_{\mu\beta}(x) = g(A^a_\mu T_a)^i_j\psi^j_\beta(x) + im\Theta^i_{\mu\beta}(x), \tag{25}
$$

which transforms under (24) as

$$
B'^i_{\mu\beta}(x) = U^i_j(x)B^j_{\mu\beta}(x) - \partial_\mu U^i_j(x)\psi^j_\beta(x) - \partial_\mu\chi^i_\beta(x). \tag{26}
$$

The first and the second covariant derivatives of $\psi^i_\beta(x)$ are

$$
\begin{aligned}
\nabla_\mu\psi^i_\beta(x) &= [D_\mu\psi_\beta(x)]^i + im\Theta^i_{\mu\beta}(x) \\
&= \partial_\mu\psi^i_\beta(x) + g(A^a_\mu T^a)^i_j\psi^j_\beta(x) + im\Theta^i_{\mu\beta}(x), \\
(\nabla_\mu\psi^i_\beta(x))_{\|\nu} &= [(\partial_\nu + gA_\nu)\nabla_\mu\psi_\beta(x)]^i.
\end{aligned}
\tag{27}
$$

The total curvature tensor is

$$
R^i_{\beta\mu\nu} = R^{(1)i}{}_{j\mu\nu}\psi^j_\beta(x) + R^{(2)i}{}_{\beta\mu\nu}. \tag{28}
$$

where $R^{(1)i}{}_{j\mu\nu}$ is the same as in eq.(9) and

$$
R^{(2)i}{}_{\beta\mu\nu} = im(D_\mu\Theta_{\nu\beta} - D_\nu\Theta_{\mu\beta})^i. \tag{29}
$$

The second curvature tensor is the same with $R^{(1)i}{}_{j\mu\nu}$. Under the transformation (24) they transform respectively as

$$
\begin{aligned}
&\nabla'_\mu\psi'^i_\beta(x) = U^i_j(x)\nabla_\mu\psi^j_\beta(x), \quad (\nabla'_\mu\psi'^i_\beta(x))_{\|\nu} = U^i_j(x)(\nabla_\mu\psi^j_\beta(x))_{\|\nu}, \\
&R'^i_{\beta\mu\nu} = U^i_j(x)R^j_{\beta\mu\nu}, \qquad R'^{(1)i}{}_{j\mu\nu} = U^i_k R^{(1)k}{}_{l\mu\nu}(U^{-1})^l_j.
\end{aligned}
\tag{30}
$$

Now we construct the model invariant under transformation (24). We also require the Lorentz and C.P.T invariance. Since $\Theta^i_{\mu\beta}$ is a fermion field, we also omit terms with the second and higher order derivative of $\Theta^i_{\mu\beta}$. The general form satisfying above requirements is

$$
\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6
$$

$$
\begin{aligned}
&\mathcal{L}_1 = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}, && \mathcal{L}_2 = \tfrac{\lambda_2}{m}\overline{\nabla_\mu\psi}\nabla^\mu\psi, \\
&\mathcal{L}_3 = \tfrac{\lambda_3}{m}\overline{\nabla_\mu\psi}\gamma^\mu\gamma^\nu\nabla_\nu\psi, && \mathcal{L}_4 = i\tfrac{\lambda'_4}{m^2}\overline{\nabla_\mu\psi}\gamma^\nu(\nabla^\mu\psi)_{\|\nu} + h.c., \\
&\mathcal{L}_5 = i\tfrac{\lambda'_5}{m^2}\overline{\nabla_\mu\psi}\gamma^\nu(\nabla_\nu\psi)^{\|\mu} + h.c., && \mathcal{L}_6 = i\tfrac{\lambda'_6}{m^2}\overline{\nabla_\mu\psi}\gamma_\nu R^{\mu\nu} + h.c.
\end{aligned}
\tag{31}
$$

Again we can choose the gauge

$$
\psi'^i_\beta(x) = \psi^i_\beta(x) + \chi^i_\beta(x) = 0. \tag{32}
$$

The free $\Theta^i_{\mu\beta}$ part of the Lagrangian is

$$
\begin{aligned}
\mathcal{L}_\Theta = \; & \lambda_2 m\overline{\Theta}_\mu\Theta^\mu + \lambda_3 m\overline{\Theta}_\mu\gamma^\mu\gamma_\nu\Theta^\nu + i\lambda_4\overline{\Theta}_\mu\gamma_\nu\partial^\mu\Theta^\nu \\
& -i\lambda_4\overline{\partial^\mu\Theta}^\nu\gamma_\nu\Theta_\mu + i\lambda_5\overline{\Theta}_\mu\gamma_\nu\partial^\nu\Theta^\mu - i\lambda_5\overline{\partial^\nu\Theta}^\mu\gamma_\nu\Theta_\mu
\end{aligned}
\tag{33}
$$

where $\lambda_4 = \lambda'_4 + \lambda'_6$ and $\lambda_5 = \lambda'_5 - \lambda'_6$. So the motion equation of free $\Theta^i_{\mu\beta}$ field is

$$
\lambda_2 m\Theta^\mu + \lambda_3 m\gamma^\mu\gamma_\nu\Theta^\nu + i\lambda_4\partial^\mu(\gamma_\nu\Theta^\nu) + i\lambda_4\gamma^\mu(\partial_\nu\Theta^\nu) + 2i\lambda_5\hat{\partial}\Theta^\mu = 0. \tag{34}
$$

In order to clear the physical property of $\Theta^i_{\mu\beta}$ fields, we define two spin $\frac{1}{2}$ fields

$$\Psi^i \equiv \begin{pmatrix} S^i \\ Q^i \end{pmatrix} = \begin{pmatrix} \gamma^\mu \Theta^i_\mu \\ \frac{i}{m_o}\partial^\mu \Theta^i_\mu \end{pmatrix} \tag{35}$$

where m_o is a parameter determined later. After tedious calculation we obtain that

$$i\hat{\partial}\Psi - M\Psi = 0\,, \qquad M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}. \tag{36}$$

where

$$M_{11} = \frac{\lambda_2 + 4\lambda_3}{\lambda_4 - 2\lambda_5} m\,, \qquad M_{12} = M_{21} = \frac{4(\lambda_4 + \lambda_5)}{\lambda_4 - 2\lambda_5} m_o\,,$$

$$M_{22} = \frac{\lambda_2 + \frac{4(\lambda_4+\lambda_5)}{\lambda_4-2\lambda_5}(\lambda_4 \frac{\lambda_2+4\lambda_3}{\lambda_4-2\lambda_5} - \lambda_3)}{\lambda_4 + 2\lambda_5 - \lambda_4 \frac{4(\lambda_4+\lambda_5)}{\lambda_4-2\lambda_5}} m\,,$$

$$m_o = \left\{ \frac{(\lambda_2 + 4\lambda_3)(\lambda_4 \frac{\lambda_2+4\lambda_3}{\lambda_4-2\lambda_5} - \lambda_3)}{4(\lambda_4 + \lambda_5)(\lambda_4 + 2\lambda_5 - \lambda_4 \frac{4(\lambda_4+\lambda_5)}{\lambda_4-2\lambda_5})} \right\}^{\frac{1}{2}} m\,.$$

Diagonalizing M matrix we get

$$m_\pm = \frac{1}{2}[(M_{11} + M_{22}) \pm \sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}]. \tag{37}$$

They correspond following two states respectively:

$$F_+ = S - \frac{M_{11} - m_+}{M_{12}} Q\,, \qquad F_- = Q - \frac{M_{22} - m_-}{M_{12}} S. \tag{38}$$

Decomposing Θ_μ into three parts:

$$\Theta_\mu = \Theta^T_\mu + (a_1\gamma_\mu + \frac{ia_2}{m_+}\partial_\mu)F_+ + (b_1\gamma_\mu + \frac{ib_2}{m_-}\partial_\mu)F_- \tag{39}$$

where

$$\begin{aligned} a_1 &= \tfrac{1}{m_+ - m_-}(\tfrac{4m_o(M_{11}-m_-)}{m_-} - M_{12})\,, \quad a_2 = \tfrac{1}{m_+ - m_-}(M_{11} - m_- - \tfrac{m_o}{m_-}M_{12}) \\ b_1 &= \tfrac{1}{m_+ - m_-}(M_{12} - \tfrac{m_o(M_{11}-m_-)}{m_-})\,, \quad b_2 = \tfrac{1}{m_+ - m_-}(\tfrac{4m_o}{m_+}M_{12} - M_{11} + m_-), \end{aligned} \tag{40}$$

we can prove that

$$\gamma^\mu \Theta^T_\mu = 0\,, \qquad \partial^\mu \Theta^T_\mu = 0\,, \qquad (i\hat{\partial} - \frac{-\lambda_2 m}{2\lambda_5})\Theta^T_\mu = 0. \tag{41}$$

These equations show that Θ^T_μ is a spin $\frac{3}{2}$ Rarita-Schwinger field with mass $|\frac{-\lambda_2 m}{2\lambda_5}|$.

So far by using nonlinear connections we have established the local affine invariant interaction theories for both scalar and spinor cases. In the gauge choices (14) and (32), the theories appear to be renormalizable. The matter fields have been absorbed into the connection fields. Thus, generaly speaking, except ordinary gauge fields, one massive scalar and one massive vector fields appear in the bosonic theory and two spin $\frac{1}{2}$ and one spin $\frac{3}{2}$ massive fields in fermionic theory. We have not found any basic principle to restrict their mass values. It seems possible that some particular choices of parameters make some particles (e.g. spin 1 and spin $\frac{3}{2}$ particles) become infinite heavy and disappear in the effective theories. We think the quantization in the interaction

picture can be carried out in terms of the methods for Stueckelberg field [7] and Rarita-Schwinger field [8], based on the complete sets of solutions of equations of free motion. We are trying to work out it. As a final comment the formalism used here can be used in $SU(3)$ and $SU(2)\times U(1)$ theories. Thus the standard model would be generalized to include some new particles. We think it is interesting to search for these new particles experimentally while seach for heavy particles in other theories (such as SUSY etc.).

In summary, we have constructed several local affine gauge models by using nonlinear connections. After special gauge fixing of local translations both of matter and gauge fields in usual gauge theory become components of nonlinear connection fields. We predict the existence of some new particles if nature choose above mentioned local affine invariance. We think that further study of these local affine symmetries and their consequence for field theory behavior is interesting.

We are indebted very much to prof. Q.K.Lu for explaining us the nonlinear connection theory. This work is supported by the NSF of China and the Grant LWTZ-1298 of Chinese Academia of Science.

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