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# ON THE SECOND ORDER PHASE TRANSITION OF A REISSNER NORDSTORM BLACK HOLE

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# On the Second Order Phase Transition of a Reissner Nordstorm Black Hole

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**Abstract**

The thermodynamics of a Reissner-Nordstorm (RN) black hole is studied with particular attention paid to a possible second order phase transition.It is shown that, at the thermally stable phase at which the heat capacity at constant charge is positive, the mean square fluctutation of mass and entropy of a massive RN black hole are divergent as a certain critical temperature is approached, similiar to other thermally stable thermodynamical systems which undergo second order phase transition. In the thermally unstable phase at which the heat capacity at constant charge is negative, the fluctutation probability diverges exponentially as the critical temperature is approached. An order parameter is then introduced to describe the transition between the thermally stable and thermally unstable phase phenomenologically. Within this phenomenological framework, the fluctutation dissipation properties of the black hole near the critical temperature is studied. Critical exponents of relevant thermodynamical quantites are also computed



## 1 Introduction

In black hole thermodynamics, it is known that a black hole may have negative heat capacity when regarded as a thermodynamical object. However, it was discovered by Davies[1] and Hut[2] that a black hole may also have positive heat capacity, depending on the charge or angular momentum of the black hole. The transition from positive to negative heat capacity or vice versa takes place at a certain critical temperature at which the certain thermodynamic potential has first continuous partial derivative with respect to temperature but the second partial derivative with respect to temperature is discontinuous. This characteristic is shared by many other thermodynamic systems which exhibit second order phase transition. In view of this similiarity, it is suggested [1] that a second order phase transition also occurs for a black hole with charge or angular momentum. So far very little is known about this possible phase transition. Some [3,4] even express scepticism about whether there is really any phase transition taking place at all.

The aim of this work is to investigate this possible phase transition phenomenon further in the case of a RN black hole. The content of this work is arranged as follows: The necessary background material is presented in section two. In section three, the critical exponent of the heat capacity at constant charge is first determined. In section four, the thermodynamical fluctutation near the critical temperature is studied. An order parameter is introduced in section five to provide a phenomenological description of the transition between the thermally stable and unstable phase. The fluctutation dissipation properties of the black hole near the critical temperature are then

discussed in section six. Section seven consists of a few concluding remarks of this work.

## 2 Thermodynamic Preliminaries

We first consider the case of a RN black hole first. To facilitate subsequent discussions, the necessary background material is reviewed and some notations are fixed in this section. Throughout, we shall set $G = \hbar = c = 1, k = \frac{1}{8\pi}$ where all symbols have their usual meanings.

For a RN black hole with mass $M$ and charge $e$ such that $e^2 \leq M^2$, the entropy is given by

$$S = \frac{1}{4}M^2 - \frac{1}{8}e^2 + \frac{1}{4}M^2\left(1 - \frac{e^2}{M^2}\right)^{\frac{1}{2}} \tag{1}$$

From (1),

$$T = \left(\frac{\partial M}{\partial S}\right)_e = \frac{4(M^2 - e^2)^{\frac{1}{2}}}{2M(M + (M^2 - e^2)^{\frac{1}{2}}) - e^2} \tag{2}$$

From a thermodynamical perspective, (2) may be looked on as the equation of state of the black hole. It gives a functional relation among the thermodynamical variables similiar to, for example, that of an ideal gas.

Let $e^2 = \beta m^2$ where $0 \leq \beta \leq 1$. Define

$$a = (1 - \beta)^{\frac{1}{2}} \tag{3}$$

where the positive root is taken in (3). With (3), (1) and (2) may be rewritten respectively as

$$S = \frac{1}{8}M^2(1 + a)^2 \tag{4}$$



$$T = \frac{4a}{M(1 + a)^2} \tag{5}$$

Define the Helmoltz free energy as

$$F = M - TS \tag{6}$$

The heat capacity $C_e$ with constant charge is then given as

$$C_e = T\left(\frac{\partial S}{\partial T}\right)_e = -T\left(\frac{\partial^2 F}{\partial^2 T}\right)_e \tag{7}$$

With the help of (1), further elucidation of (7) [1] gives

$$C_e = \frac{8MS^3T}{\frac{1}{4}e^4 - 8S^3T^2}$$

With (4) and (5), it may be further be expressed as

$$C_e = \frac{32S^3T}{M^3(1 + a)^2(1 - 2a)} \tag{8}$$

Thus $C_e$ has a singularity at $a = \frac{1}{2}(e^2 = \frac{3}{4}M^2)$. $C_e \to \pm\infty$ as $a \to \frac{1}{2}^{\mp}$. At the same time, both $F$ and $S = -\left(\frac{\partial F}{\partial T}\right)_e$ are continuous at $a = \frac{1}{2}$. Thus a RN black hole exhibits the characteristic of a second order phase transition at $a = \frac{1}{2}$

## 3 Critical Exponent of $C_e$

The theme of this section is to work out the critical exponent of $C_e$ which sheds light on the asymptotic behaviour of $C_e$ nears $T_c$. In doing so, the temperature dependence of $F$ and $S$ near $T_c$ will also be determined.

To this end, we need to express $F$ as a function of $T$ and $e$. This is feasible by virtue of the equation of state in (2). In general, to express $M$ in terms

of $T$ and $e$, it is required to solve the following cubic equation derived from (2)

$$M^3 - \frac{1}{T}M^2 - Me^2 + \frac{e^2}{T} + \frac{1}{16}Te^4 = 0$$

This cubic equation, though in principle solvable, yields very complicated solutions which are not particularly illuminating for our purpose. Since we are only interested in the physics near the critical temperature $T_c$ at which $a = \frac{1}{2}$, let

$$\begin{aligned} M &= M_c(1+\delta) \\ T &= T_c(1+\epsilon) \end{aligned} \tag{9}$$

where $|\delta|, |\epsilon| \ll 1$, $M_c$ is the mass of the black hole at $T_c$ with $e$ constant. (9) describes $M$ and $T$ of the black hole near $T_c$.

For $\delta$ sufficiently small, Taylor expansion of (3) in terms of $\delta$ yields

$$a(\delta) = \frac{1}{2}\left(1 + 3\delta - 9\delta^2\right) \tag{10}$$

where terms of order higher than $\delta^2$ are neglected. Eventually, for $\delta$ sufficiently small, we only consider the first order terms of $\delta$. For reasons becoming apparent later, we shall include $\delta^2$ term in (10). Putting (10) back into (8), for $\delta$ sufficiently small, we have

$$C_e = -\frac{M_c}{12T_c}\frac{1}{\delta} \tag{11}$$

which is the asymptotic form of $C_e$ near $T_c$ in terms of $\delta$. To further express (11) in terms of $\epsilon$, with the help of (9) and (10), (5) may be written as

$$1 + \epsilon = (1 + 3\delta - 9\delta^2)(1+\delta)^{-1}(1 + \delta - 3\delta^2)^{-2} \tag{12}$$



Taylor expanding $(1+\delta)^{-1}$ and $(1+\delta-3\delta^2)^{-2}$ and keeping only up to second order terms of $\delta$, (12) becomes

$$\begin{aligned} 1+\epsilon &= (1+3\delta-9\delta^2)(1-3\delta+12\delta^2) \\ &= 1-6\delta^2 \end{aligned}$$

As a result,

$$\epsilon = -6\delta^2 \tag{13}$$

As expected, the first order term of $\delta$ vanishes in (13) otherwise, by virtue of (11) and (7), $S$ would be singular at $T_c$ which is a contradiction. At first sight, (13) may seem to be counter intuitive since, at $T_c$, both increase or decrease in mass of the black hole will lead to a decrease in the temperature. However, a moment of thought reveals that this is actually the correct description of the physical situtation peculiar to a RN black hole.

Consider first $\delta > 0$. In this case $C_e < 0$. A RN black hole behaves similiarly to a Schwarzchild black hole in the sense that increase in mass will lead to a decrease in temperature of the black hole, in accordance with (13). In the case $\delta < 0$, increase in $\delta^2$ means that $\delta$ becomes more negative, i.e. the mass of the black hole decreases. Since $C_e > 0$ for $\delta < 0$, a RN black hole behaves like a thermally stable system. Hence a decrease in mass leads to a decrease in temperature as described in (13). From this, we also see that, at constant $e$, $T_c$ is the temperature ceiling of the black hole.

Incidentially, we note that (13) together with (9) give

$$C_e = \left(\frac{\partial M}{\partial T}\right)_e = \frac{M_c}{T_c}\frac{\partial\delta}{\partial\epsilon} = -\frac{1}{12}\frac{M_c}{T_c}\frac{1}{\delta}$$

near $T_c$. It coincides with (11) which is obtained without using (13). This provides a consistency check that (13) is a good approximation near $T_c$.

From (11) and (13), we then have near $T_c$,

$$C_e = \begin{cases} -A(-\epsilon)^{-\frac{1}{2}} & \delta > 0 \\ A(-\epsilon)^{-\frac{1}{2}} & \delta < 0 \end{cases} \tag{14}$$

where $A = \frac{M_c}{24\sqrt{3}T_c}$ which is a positive function of $e$. Thus, both above and below $M_c$, $C_e$ has critical exponent $-\frac{1}{2}$. From (7) and (14), we further have, near $T_c$,

$$S = \begin{cases} \frac{A}{2}(-\epsilon)^{\frac{1}{2}} + S_c & \delta > 0 \\ -\frac{A}{2}(-\epsilon)^{\frac{1}{2}} + S_c & \delta < 0 \end{cases} \tag{15}$$

$$F = \begin{cases} \frac{1}{3}A(-\epsilon)^{\frac{3}{2}} - S_c + F_c & \delta > 0 \\ -\frac{1}{3}A(-\epsilon)^{\frac{3}{2}} - S_c + F_c & \delta < 0 \end{cases} \tag{16}$$

where the subscript $c$ denotes the relevant quantity evaluated at $T_c$.

## 4 Thermodynamic Critical Fluctutation

Having determined the temperature dependence of $F$ and $S$ near $T_c$ in previous section, we shall go on to investigate the thermodynamical fluctutation near $T_c$. Our experience with systems known to have a second order phase transition show that, apart from the discontinuity of $\left(\frac{\partial^2 F}{\partial^2 T}\right)_e$ at $T_c$, the thermodynamical fluctutation also behaves anomalously near the critical temperature for these systems. The aim of this section is to see whether there is any similiar anomalous behaviour for the thermodynamical fluctutation for a RN black hole near $T_c$.

Consider a RN black hole in equilibrium with a constant temperature heat bath consisting of black body radiation. The temperature of the heat bath is taken to be $T_c(1+\epsilon)$ where $\epsilon < 0$ and $|\epsilon| \ll 1$. For $\delta > 0$, to ensure thermal stability, we need to assume the heat bath consists of a box made



of perfect reflecting walls whose volume is sufficiently small[1] with the black hole immersed in it. No such restriction is necessary for $\delta < 0$. Further, we also assume the mass of the black hole is sufficiently massive so that its charge may be regarded as constant[5].

Suppose the black hole deviates from the equilibrium of the heat bath with temperature deviation given by $\Delta\epsilon$ where $\Delta\epsilon$ is sufficiently small so that the equilibrium of the heat bath is not disturbed. Let $p(\epsilon, e)$ denote the fluctutation probability of the black hole at constant $\epsilon$ and $e$. Then [6]

$$p(\epsilon, e) \propto exp\left\{\frac{-8\pi\Delta F'}{T_c(1+\epsilon)}\right\} \tag{17}$$

Here the $8\pi$ factor comes in because we have taken $k = \frac{1}{8\pi}$, $\propto$ denotes proportional to and

$$F' = M - T_c(1+\epsilon)S \tag{18}$$

Note that $F'$ is not the Helmoltz free energy of the black hole because the temperature of the black hole is now $T_c(1+\epsilon+\Delta\epsilon)$ and the Helmoltz free energy of the black hole is given as

$$F = M - T_c(1+\epsilon+\Delta\epsilon)S \tag{19}$$

Using (18) and (19), we may deduce from (17) that, for $|\epsilon|, |\Delta\epsilon| \ll 1$

$$\begin{aligned} p(\epsilon, e) &\propto exp\left\{-8\pi\left(\frac{\Delta F}{T_c} + \Delta S\Delta\epsilon\right)\right\} \\ &= exp\{-8\pi(\frac{\Delta F}{T_c} + C_e(\Delta\epsilon)^2)\} \end{aligned} \tag{20}$$

where the second line follows from (7) and $\Delta\epsilon$ being sufficiently small. From (16), we have

$$\Delta F = \begin{cases} \left(-\frac{A}{2}(-\epsilon)^{\frac{1}{2}} + S_c\right)\Delta\epsilon & \delta > 0 \\ \left(\frac{A}{2}(-\epsilon)^{\frac{1}{2}} + S_c\right)\Delta\epsilon & \delta < 0 \end{cases} \tag{21}$$

Compare $-\frac{\Delta F}{T_c}$ with $-C_e(\Delta\epsilon)^2$ in (20) using (14) and (21). We see that near $T_c$, $-C_e(\Delta\epsilon)^2$ dominates over $-\frac{\Delta F}{T_c}$. Hence, for $|\epsilon| \ll 1$, (20) becomes

$$p(\epsilon, e) \propto exp\{-8\pi C_e(\Delta\epsilon)^2\} \tag{22}$$

Consider first the case $\delta < 0$. Followed from (7), (14) and $|\Delta\epsilon| \ll 1$,

$$(\Delta M)^2 = A^2(-\epsilon)^{-1}(\Delta\epsilon)^2 \tag{23}$$

Using (23), (22) then becomes

$$p(\epsilon, e) \propto exp\{-\frac{8\pi}{C_e}(\Delta M)^2\} \tag{24}$$

For $\delta < 0$, $C_e > 0$. $p(\epsilon, e)$ is then a Gaussian distribution in terms of the variable $\Delta M$. Denote the mean square fluctutation of $M$ by $< (\Delta M)^2 >$. $< (\Delta M)^2 >$ is then given by the variance of $p(\epsilon, e)$ and from (24), we have

$$< (\Delta M)^2 > \propto C_e$$

which diverges to infinity as $\frac{1}{(-\epsilon)^{\frac{1}{2}}}$ when $\epsilon \to 0$. Similiarly, we may also show $< (\Delta S)^2 > \to \infty$ as $\frac{1}{(-\epsilon)^{\frac{1}{2}}}$ when $\epsilon \to 0$. Thus, the thermodynamical fluctutation of a RN black hole (at least a sufficiently massive one) in its thermally stable phase behaves similiarly to other thermally stable thermodynamical systems which undergo second order phase transition.

For $\delta > 0$, $C_e < 0$. From (14) and (22),

$$p(\epsilon, e) \propto exp\left\{\frac{A(\Delta\epsilon)^2}{(-\epsilon)^{\frac{1}{2}}}\right\}$$

Since $A(\Delta\epsilon)^2 > 0$, $p(\epsilon, e)$ diverges exponentially as $\epsilon \to 0$.



If we envisage a large number of copies of RN black holes prepared in an identically way such that their average behaviour is described as being in equilibrium with constant temperature heat bath consisting of black body radiation and $C_e < 0$. As $T \to T_c$, number of systems in the ensemble which deviate from the average behaviour is infinite. This indicates a break down of the description of a RN black hole by means of canonical ensemble near $T_c$ in the thermally unstable phase, even though we enclose a RN black hole in a box to ensure thermal stability.

The divergences obtained in both $\delta < 0$ and $\delta > 0$ cases seem to contradict the result of Pavon et. al.[4] whose derivation is based on the hydrodynamic approximation to a black hole, no use is made of the equation of state (2). However, one assumption behind this approximation scheme is that the thermodynamical system under consideration may be subdivided into subsystems. The fluctutation in each subsystem is independent of each other and uncorrelated. It is questionable whether this assumption would be upheld for a black hole within which gravitational force is so intense, not to mention the fact that gravitational interaction is also long range.

## 5 The Introduction of an Order Parameter

Due to our lack of understanding of the statistical mechanics of a black hole, there is no hope of going further in our search for a detail, microscopic understanding of the transition between the thermally stable and unstable phase of a RN black hole. However, in analogy to other thermodynamical systems which exhibit a second order phase transition, we may introduce

an order parameter to describe the transition phenomenologically, without necessarily knowing the underlying microscopic dynamics.

Let $\eta$ be a real parameter such that

$$\begin{array}{ll} \eta = 0 & \delta > 0 \\ \eta \neq 0 & \delta < 0 \end{array} \tag{25}$$

Assume for $\delta < 0$ and $|\delta|, |\epsilon| \ll 1$, $F$ is of the form

$$F = f_1(e,\eta)(-\epsilon)^{\frac{3}{2}} + f_2(e,\eta)(-\epsilon) + F_c \tag{26}$$

where $f_1, f_2$ are smooth functions of $e$ and $\eta$. Thus the singularity of $\left(\frac{\partial^2 F}{\partial T^2}\right)_e$ at $\epsilon = 0$ is entirely due to the term $(-\epsilon)^{\frac{3}{2}}$. Taylor expanding $f_1, f_2$ in terms of $\eta$, (26) becomes

$$F = (-\frac{1}{3}A + b_1(e)\eta + b_2\eta^2 + \cdots)(-\epsilon)^{\frac{3}{2}} + (-S_c + d_1(e)\eta + \cdots)(-\epsilon) + F_c \tag{27}$$

where $b_i(e), d_i(e), i = 1, 2 \cdots$ are functions of $e$. We then recover (16) as the zero order term of $\eta$ in (27).

In all thermodynamic quantites describing a RN black hole, only $e^2$ terms appear. As a result, the thermodynamics of a RN black hole is invariant under charge conjugation. Now the presence of charge is responsible for the change of phase. Like in superconductivity, $\eta$ is a measure of density of Cooper pairs of electron. $\eta$ is presumably a measure of electron density for a particular kind of electrons whose behaviour is responsible for the phase transition of a RN black hole. Hence charge conjugation symmetry implies that only terms of order $\eta^{2n}$ where $n$ is a positive integer will appear in the series expansion in (26). As a result, we have from (27)

$$F = (-\frac{1}{3}A + b_2(e)\eta^2 + b_4\eta^4 + \cdots)(-\epsilon)^{\frac{3}{2}} + (-S_c + d_2(e)\eta^2 + d_4(e)\eta^4 + \cdots)(-\epsilon) + F_c \tag{28}$$



(28) looks similiar to the expansion of $F$ in Landau theory [7] in the sense that only even powers of $\eta$ appear in the series. However, unlike Landau theory in which the singularity part of $F$ is encoded in the series, the singular part of $F$ in the present context is in the $(-\epsilon)^{\frac{3}{2}}$ term. The series expansion in terms of $\eta$ is merely Taylor expansion. Further, at constant $e$ and $\epsilon$, $F$ is a local minimum. Hence

$$\left(\frac{\partial F}{\partial \eta}\right)_{e,\epsilon} \tag{29}$$

(28) and (29) together with $\eta = 0$ at $\epsilon = 0$ imply $d_2(e) = 0$. (28) then becomes

$$F = (-\frac{1}{3}A + b_2(e)\eta^2 + b_4\eta^4 + \cdots)(-\epsilon)^{\frac{3}{2}} + (-S_c + d_4(e)\eta^4 + \cdots)(-\epsilon) + F_c \tag{30}$$

From (29) and (30), we also get, for sufficiently small $-\epsilon$,

$$\eta^2 = -\frac{b_2}{d_4}(-\epsilon)^{\frac{1}{2}} \tag{31}$$

By our assumption, $\eta$ is real. Thus, $b_2$ and $d_4$ are of opposite sign in (30). Further, (25) together with (31) give, for $|\epsilon| \ll 1$,

$$\eta = \begin{cases} 0 & \delta > 0 \\ B(-\epsilon)^{\frac{1}{4}} & \delta < 0 \end{cases} \tag{32}$$

where $B$ is a real valued function of $e$. $\frac{1}{4}$ is then the critical exponent of $\eta$ when $\delta < 0$. For $\delta < 0$ and constant $e$, (32) implies

$$\Delta\eta = B(-\epsilon)^{-\frac{3}{4}}\Delta\epsilon \tag{33}$$

Using (33) and arguments similiar to that in last section, we also have $\lim_{\epsilon \to 0} < (\Delta\eta)^2 > \to \infty$ as $\frac{1}{(-\epsilon)}$ in the thermally stable phase.

## 6 Fluctutation Dissipation Relation

To enhance our understanding ot the physics of the black hole near $T_c$. let us introduce a weak perturbation of unspecified nature to the black hole and investigate the response of the black hole near $T_c$ to this external perturbation within the framework described in last section. The relationship between the weak perturbation and the response of the black hole to it is usually termed the fluctutation dissipation relation. Note that within the context of linear irreversible thermodynamics of a black hole. there are two senses we may talk about the fluctutation dissipation relationship [8]: (i) Fluctutation dissipation relation for an accelerated quantum field or a quantum field on a black hole spacetime. (ii) Fluctutation dissipation relation for a black hole itself by regarding a black hole as a thermodynamical system in equilibrium.

Obviously it is the second sense we refer to here. Moreover, the fluctutation dissipation relation we are going to discuss is not entirely equivalent to that governed by the fluctutation dissipation theorem. The distinction will be made clear in what follows.

Let $h$ be a weak perturbation of unspecified nature and $\Delta\eta$ be the deviation of $\eta$ from equilibrium due to the presence of $h$. Assume $h$ is sufficiently weak so that the following linear relationship holds:

$$\Delta\eta = \chi h \tag{34}$$

where. at constant $e$ and $\epsilon$, $\chi$ is a constant called generalised susceptibility. From (34), we also have

$$\chi = \left(\frac{\partial\eta}{\partial h}\right)_{e,\epsilon,h=0} \tag{35}$$



Assume that near $T_c$, for $\delta < 0$, $F$ assumes the form as in (26) with $f_1, f_2$ now smooth functions of $e, \eta$ as well as $h$. Then $F$ acquires the form

$$F = (-\frac{1}{3}A + b_2(e)\eta^2 + b(e)h + \cdots)(-\epsilon)^{\frac{3}{2}} + (-S_c + d(e)h + \cdots)(-\epsilon) + F_c$$

$\left(\frac{\partial F}{\partial \eta}\right)_{e,\epsilon,h} = 0$ then implies

$$(2\eta b_2(e) + b(e)h)(-\epsilon)^{\frac{1}{2}} + d(e)h = 0 \tag{36}$$

From (35), (36) and $|\epsilon| \ll 1$,

$$\chi = D(-\epsilon)^{-\frac{1}{2}} \tag{37}$$

where $D = -\frac{d(e)}{2b_2(e)}$ is a functioa Reissner-Nordstorm n of $e$. n the presence of $h$, $\eta \neq 0$ for $\delta > 0$. Similiarly, we may show, for $\delta > 0$,

$$\chi = D'(-\epsilon)^{-\frac{1}{2}} \tag{38}$$

where $D'$ is some function of $e$. From (37) and (38), the critical exponent of $\chi$ is then $-\frac{1}{2}$ for both $\delta > 0$ and $\delta < 0$. Since $\chi \to \infty$ as $\epsilon \to 0$, a weak perturbation will entail a large response from the black hole in terms of $\Delta\eta$ near $T_c$. Again within the phenomenologcal framework described here. the fluctutation dissipation relation of a RN black hole near $T_c$ is similiar to other thermodymamical systems undergoing phase transition.

To end this section. it is worth pointing out that $\Delta\eta$ discussed in this section is different from that discussed in section 4. In linear irreversible thermodynamics, no distinction is usually made between these two due to the Onsager regression hypothesis. However, the assumption of Taylor expansion of the entropy function near the equilibrium state behind the hypothesis

breaks down in the present context near $T_c$ as may be seen from (15). Thus, there is no reason to identify the spontaneous fluctutation of $\eta$ and the fluctutation of $\eta$ induced by external perturbation. We are then not able to go a step further and assert the connection between $\chi$ and $< (\Delta\eta)^2 >$ in section 4, as we usually do in the context of fluctutation dissipation theorem.

## 7 Concluding Remarks

We have shown that the thermodynamical fluctutation and the fluctutation dissipation relation of a RN black hole bear some resemblance to other thermodynamical systems known to undergo second order phase transition. However, despite all these similiarities, the existence of a thermally unstable phase for a RN and Kerr black holes have no analog in any known thermodynamical systems outside the realm of gravitation. Probably, we are faced with a new kind of critical phenomenon peculiar to gravitation here. The relevant critical exponents were also calculated here. Our experience in mean field theory suggests to us that the critical exponents of $\eta$ and $\chi$ may not be so reliable. In future, microscopic or other method are in need to calculate these critical exponents and compare with the results presented here. However, the critical exponent of $C_e$ should be quite reliable since the calculation of which use only the equation of state for a black hole and series expansion.

Since Kerr and Kerr-Newman black holes in general also have a singularity in the second partial derivative of the free energy with respect to temperature, it is also possible to extend present investigation to those cases. Preliminary investigation in the Kerr case show that the critical exponent of the heat



capacity at constant angular momentum is identical to that of $C_e$. Moreover, modulo some constants, the temperature dependence of the free energy and entropy are also identical to that of a RN black hole. Thus, despite all the apparent differences between a RN and Kerr black hole, at constant angular momentum, the thermodynamical behaviour of a Kerr black hole near its critical temperature is strikingly similiar to that of a RN black hole.It remains to see whether a generic Kerr-Newman black hole also behaves similiarly. It could be the case that all Kerr-Newman black holes with non zero charge or angular momentum form a universality class of thermodynamical systems in the sense that they share the set of critical exponents and hence have identical thermodynamics near critical temperature. Further work remains to be done to elaborate on the Kerr and Kerr-Newman cases.

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