ASITP

INSTITUTE OF THEORETICAL PHYSICS ACADEMIA SINICA

AS–ITP–93–19
Apr. 1993

NEW THEORETICAL RESULTS OF $2\upsilon\beta\beta$ DECAY WITH THE OPERATOR EXPANSION METHOD

X. R. WU M. Hirsch

A. Staudt H. V. Klapdor–Kleingrothaus

Cheng–Rui CHING Tso–Hsiu HO

P.O.Box 2735, Beijing 100080, The People's Republic of China

Telefax : (086)-1-2562587 Telephone : 2568348

Telex : 22040 BAOAS CN Cable : 6158

New theoretical results of $2\nu\beta\beta$ decay with the Operator Expansion Method

X.R. Wu, M. Hirsch, A. Staudt and H.V. Klapdor-Kleingrothaus

Max-Planck-Institut für Kernphysik, D-6900 Heidelberg, Germany

and

Ching Cheng-rui * and Ho Tso-hsiu *

Centre for Theoretical Physics, CCAST (World Laboratory) and

Institute of Theoretical Physics, Academia Sinica, Beijing, China

Abstract The half-lives for $2\nu\beta\beta$ decay for all potential $\beta\beta$-emitters with A$\geq$ 70 are calculated by the operator expansion method. Compared with the directly measured half-lives of ^{76}Ge, ^{82}Se, ^{100}Mo and ^{238}U the theoretical values are in excellent agreement with experimental ones.

Recently, more than half a century after Goeppert-Mayer first discussed $\beta\beta$ decay under Wigner's suggestion in 1935[1], direct counter experiments reported the observation of $2\nu\beta\beta$ decay for the nuclei ^{76}Ge [2,3], ^{82}Se [4], ^{100}Mo [5,6] and ^{238}U [7]. These measurements now supplement the earlier geochemical experiments for the $\beta\beta$ candidates ^{82}Se, ^{128}Te and ^{130}Te [8-11].

For the more interesting $0\nu\beta\beta$ decay mode, however, no evidence has been found so far, and only lower limits for the half-lives have been quoted in the literature [2-7]. At present new experiments like that of the Heidelberg-Moscow collaboration have been started[12,13], in a new attempt to look for $0\nu\beta\beta$ decay with increased sensitivity.

The amount of information which can be extracted from such experiments depends in a decisive way on the reliability of the theoretical estimates for the $0\nu\beta\beta$ decay matrix

* Project supported by the Natural Science Foundation of China

elements. The calculation of $2\nu\beta\beta$ decay is a valuable test for the accuracy which can be expected for the $0\nu\beta\beta$ transition matrix elements, since both calculations involve essentially the same nuclear physics. Here, we would like to report the calculation of $2\nu\beta\beta$ decay half-lives for all potential $\beta\beta$-emitters with $A \geq 70$, within a new model, the operator expansion method.[14-15]

The calculation of the $\beta\beta$ nuclear transition matrix elements between the initial even-even parent nucleus (A, Z) and the final state in the daughter nucleus $(A, Z+2)$ includes a sum of - in principle - infinite number of intermediate states in the adjacent odd-odd nucleus $(A, Z+1)$. The determination of these intermediate states is a difficult task and their treatment in shell model calculations[16,17] as well as calculations within the usual quasiparticle random-phase approximation (QRPA)[18-20] shows some weakness. As is well known, in shell model calculations the energies of the intermediate states are replaced by an average value (closure approximation) which has been shown to be doubtable for $2\nu\beta\beta$-decay. In QRPA the matrix elements for $2\nu\beta\beta$ decay are very sensitive to the particle-particle interaction parameter g_{pp} when g_{pp} is near its physical value of $g_{pp} = 1$. Therefore, recently an alternative method – the Operator Expansion Method (OEM), which can treat the sum over the infinite intermediate states in a more elegant way, has been proposed by Ching and Ho and applied to the calculation of ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo and ^{130}Te with rather promising results[14-15]. Encouraged by these successes and as a further application of OEM, we have calculated the $2\nu\beta\beta$-decays half-lives for all possible $\beta\beta$-emitters with A$\geq$ 70.

If one assumes that the sum of the energies for each pair of emitted electron and neutrino can be replaced by the average value of $\Delta = (E_I - E_F)/2 = \frac{1}{2}Q_{\beta\beta} + m_e$, (for a discussion see for example ref.[21]) then, as is well-known, the half-life for $0^+ \rightarrow 0^+$ $2\nu\beta\beta$ decay can be expressed in a factorized form as

$$\left[T^{2\nu}_{1/2}\right]^{-1} = F^{2\nu}\left|M_{GT}\right|^2 \tag{1}$$

where $F^{2\nu}$ is a lepton phase-space integral. The matrix element is given by

$$M^{2\nu}_{GT} = \sum_N \frac{\langle 0^+_F|A^\alpha|1^+_N\rangle\langle 1^+_N|A^\alpha|0^+_I\rangle}{\Delta + (E_N - E_I)}. \tag{2}$$

$M_{GT}^{2\nu}$ of eq.(2) is a special case for $0^+ \to 0^+$ transition. The basic idea of OEM now is to transfer the energy denominator to the numerator by the following mathematical procedure. We take instead of $M_{GT}^{2\nu}$ the following matrix element

$$M_{GT}(Z) = \frac{1}{2}\sum_N \left\{ \frac{\langle F|A^\alpha|N\rangle\langle N|A^\beta|I\rangle}{\Delta + (E_N - E_I)Z} - \frac{\langle F|A^\beta|N\rangle\langle N|A^\alpha|I\rangle}{\Delta + (E_F - E_N)Z} \right\}, \tag{3}$$

where $|I\rangle$, $|N\rangle$ and $|F\rangle$ denote the wave functions of the initial, intermediate and final nuclear states, respectively and E_I, E_N, and E_F are the corresponding energies and Z is a complex variable. Mathematically $M_{GT}(Z)$ is a single-valued, regular function of Z in an open, connected region containing the origin and Z = 1, except some possible poles along the real axis. For small Z $M_{GT}(Z)$ can be expanded using the binomial theorem:

$$\frac{1}{\Delta + (E_N - E_I)Z} = \frac{1}{\Delta}\left\{1 - \frac{Z(E_N - E_I)}{\Delta} + \frac{Z^2(E_N - E_I)^2}{\Delta^2} - \cdots\right\}. \tag{4}$$

Then, after introducing the nuclear Hamiltonian H_S the following $M_{GT}(Z)$ is obtained

$$M_{GT} = \frac{1}{2\Delta}\left\langle F\left|\left\{[A^\alpha, A^\beta] - \frac{Z}{\Delta}[A^\alpha, [H_S, A^\beta]] + \frac{Z^2}{\Delta^2}\left[A^\alpha, \left[H_S, [H_S, A^\beta]\right]\right] - \cdots\right\}\right|I\right\rangle, \tag{5}$$

where the summation over the intermediate states has been carried out. It is important to note that eq.(5) is an exactly equivalent formulation of the matrix element of eq.(3). On the other hand, mathematically eq.(5) is a divergent series for $2\nu\beta\beta$ with Z = 1 and thus one has to sum up all terms up to infinity for the calculation of the matrix element (see for example G.M. Hardy, in: Divergent Series [22]).

For the summation of the infinite series, however, we have to assume a specific form of the nuclear Hamiltonian, and we take

$$H_S = \langle H_0\rangle + V_S \tag{6}$$

where

$$V_S = \frac{1}{2}\sum_{i\neq j}\Big\{v_0(r_{ij}) + v_\tau(r_{ij})\tau_i\cdot\tau_j + v_\sigma(r_{ij})\sigma_i\cdot\sigma_j + v_{\sigma\tau}(r_{ij})\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j\Big\} \tag{7}$$

which is the most general central static force. $v_0(r_{ij})$, $v_\sigma(r_{ij})$, $v_\tau(r_{ij})$ and $v_{\sigma\tau}(r_{ij})$ denote the radial parts of the corresponding interactions, respectively, and $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ stands for the relative coordinates of the nucleons.

The assumption that the unperturbed nuclear Hamiltonian can be replaced by its average value $\langle H_0\rangle$, which is constant and thus does not contribute to the commutator series (5), is the main approximation of the present OEM. The accuracy of this approximation has still to be checked. However, one can argue [23] that for the case of a small model space H_0 is nearly degenerate, i.e. $\langle H_0\rangle - H_0$ is small compared with V_S and thus the approximation should be reasonable.

Adopting this approximation, and after a fair amount of algebraic operations, one can sum up the infinite series (5) and simplify the $0^+ \to 0^+$ transition matrix element (2) into

$$M_{GT} \equiv \left\langle 0_F^+\left|\sum_{i\neq j}\mathcal{M}_{ij} + \sum_{i\neq j\neq k}\mathcal{M}_{ijk} + \cdots\right|0_I^+\right\rangle, \tag{8}$$

where there are totally $(A-1)$ operators from 2-body operator $\mathcal{M}_{ij}$ to A-body operator.

Finally for the two-body operator OEM gives the matrix element M_{GT} as the analytical continuation $M_{GT} = M_{GT}(Z)|_{Z=1}$,

$$\mathcal{M}_{ij}(Z) = \frac{12(v_\sigma(r) - v_\tau(r))Z}{\Delta^2 - 16Z^2(v_\sigma(r) - v_\tau(r))^2}\,\Omega_0(ij) + \frac{4Z(2v_{\sigma\tau}(r) - v_\sigma(r) - v_\tau(r))}{\Delta^2 - 16Z^2(2v_{\sigma\tau}(r) - v_\sigma(r) - v_\tau(r))^2}\,\Omega_1(ij) \tag{9}$$

where we have introduced the spin singlet and spin triplet operators

$$\Omega_1(ij) = \frac{3 + \sigma_i\cdot\sigma_j}{4}, \qquad \Omega_0(ij) = \frac{1 - \sigma_i\cdot\sigma_j}{4}. \tag{10}$$

The mathematical procedure just outlined is the so-called Euler's method of summation. We note that Simkovic and Gmitro [24] starting from the time-ordered product of two operators rederived the OEM result for the two-body operator (9). They obtained exactly the same expression for $\mathcal{M}_{ij}$.

In principle, all $A-1$ terms can be given a compact form, but due to the technical difficulty to calculate the matrix elements of the many-body operators. only the two-body terms are taken into account. This is our second approximation. We expect this approximation to

be reasonable, since for $\beta\beta$ decay the transition operator is of two-body character. However, we plan to investigate the influence of the three-body terms in a future publication.

We note that if $v_\sigma = v_\tau = v_{\sigma\tau}$ in the effective interaction (7), the Hamiltonian (7) would be of [0] irreducible tensor form of Wigner's SU(4) multiplet, and the two-body operator $\mathcal{M}_{ij}$ in eq (9) is exactly zero. The nonzero contributions come from differences of the interaction strengths v_σ, v_τ and $v_{\sigma\tau}$. We note that in ref.[25], a $H' = V(\tau_i \cdot \tau_j - \sigma_i \cdot \sigma_j)$ SU(4) symmetry-breaking Hamiltonian was taken into account and the M_{GT} given there by use of the SU(4) algebra perhaps corresponds to the contribution of the $\Omega_0(ij)$ term (the first term) in equation (9). But the Hamiltonian used in the present work is more realistic than those used in the SU(4) algebra techniques and thus more effects of SU(4) symmetry-breaking are picked up here.

To calculate the $2\nu\beta\beta$ matrix element (8) we only need to know the effective interactions for evaluating $\mathcal{M}_{ij}$ and the ground state wave functions of the initial and final states. This is an delighting advantage of the present approach.

The operator $\mathcal{M}_{ij}$ can be determined by the effective interaction, for which we take the Paris-potential [26]. In principle, one should take the eigenfunctions of H_S with the correct eigenvalues E_I and E_F, respectively. However, since most of the potential $\beta\beta$-emitters are located far away from closed shells, it is impossible at present to use shell model wave functions for the initial and final states. Thus, for the determination of $|0_I^+\rangle$ and $|0_F^+\rangle$ QRPA wave functions are used in the present work. For a description of the QRPA we refer to the original literature for brevity[18-20], for the combination of OEM and QRPA wavefunctions to [15].

We describe the initial $|0_I^+\rangle$ and final state $|0_F^+\rangle$ by the QRPA vacuum based on BCS states. Then we insert two sets of complete and orthogonal mathematical functions $|a\rangle$ and $|b\rangle$ into eq.(9). We construct these two sets of mathematical functions by the phonon operators acting on the QRPA vacuum, these phonons consisting of the quasiparticle proton-neutron-pair. Then we can directly use the QRPA techniques to calculate the transition matrix element M_{GT}.

The main advantage of the present approach lies in the fact that the calculated matrix element M_{GT} is only weakly sensitive to the choice of the particle-particle interaction parameter g_{pp}. This was demonstrated in fig.1 for the example of ref.[15] for ^{100}Mo. The similar behaviour of the matrix element holds also for ^{238}U. The OEM leads to matrix elements which are always smaller than the QRPA calculation for $g_{pp} = 0$, but do not exhibit a strong dependence on g_{pp}.

It is a natural feature of the present approach that M_{GT} is insensitive to g_{pp}. Recall that the two-body operator $\mathcal{M}_{ij}$ in eq.(9) is not explicitly dependent on the intermediate energy spectrum and that the wave functions of the intermediate states are only used as a complete and orthogonal set of mathematical functions. The numerical results of M_{GT} should only care of whether these two sets of functions are complete and orthogonal, and thus $M_{GT}^{2\nu}$ should be constant no matter how g_{pp} changes. On the other hand, our present calculation uses the QRPA vacuum for the description of the initial $|0_I^+\rangle$ and final state $|0_F^+\rangle$ and these depend on g_{pp}. However, the QRPA ground state is

$$|QRPA\rangle = |BCS\rangle + YX^{-1}|BCS\rangle + \cdots. \tag{11}$$

Clearly, the $|BCS\rangle$ state does not depend on g_{pp} and since in QRPA the coefficient necessarily fulfills the condition $YX^{-1} << 1$, in the first order, the QRPA vacuum and thus our calculated M_{GT} should only weakly depend on g_{pp}. On the other hand, in usual QRPA calculations the wave functions of the intermediate states should be the real wave functions of the odd-odd nucleus. The energy eigenvalues of these intermediate states depend on g_{pp}, and consequently in usual QRPA the results are more sensitive to g_{pp}.

We think that the strong dependence of the $2\nu\beta\beta$ decay half-lives on g_{pp} is a major disadvantage of QRPA calculations, as we would like to discuss for the special case of ^{238}U. As can be seen from fig.1 the $2\nu\beta\beta$ matrix element crosses zero for a certain value of g_{pp}, translating into an infinite half-life. For the parameter choice of [20] one obtains $T_{1/2}^{2\nu} = 1.5 \cdot 10^{23}$y, which is a factor of 100 larger than the experimental result of [7] but could easily be adapted to the experimental value by a small change of g_{pp}. On the other hand, the OEM result of $T_{1/2}^{2\nu} = 0.9 \cdot 10^{21}y$ is relatively stable against variations on g_{pp}, and also in good agreement with the experimental data.

Since the results of the present approach are not sensitive to a specific choice of the parameters g_{ph} and g_{pp} we can use the physical values $g_{pp} = g_{ph} = 1$ directly. However, in

some cases (as indicated in table 1) the QRPA equation collapses for values below $g_{pp} = 1$. For these isotopes the g_{pp} value before the collapse points are used. The calculated matrix elements M_{GT} and corresponding half-lives for all potential $\beta\beta$-emitters are given in table 1. The half-lives given are for $g_A/g_V = -1.254$ and the phase-space factors of ref [21]. Note, however, that for the heaviest isotopes the phase space factor of [21] differs from the one used in the QRPA calculation of [20] by a factor of ~ 3. On the other hand, for medium heavy isotopes, such as ^{76}Ge, the phase factors of [21] and [20] are essentially equal.

For a comparison we listed also the experimental data available to us. For all nuclides which have been measured by direct counter experiments (^{76}Ge, ^{82}Se, ^{100}Mo and ^{238}U), the theoretical half-lives are in quite well agreement with the experimental ones. However for the Te isotopes our calculation gives a larger decay rate than the geochemical experiments.

To summarize, we have calculated $2\nu\beta\beta$ decay half-lives for all potential $\beta\beta$ emitters with $A \geq 70$. Our results are not sensitive to a specific choice for the particle-particle interaction parameter g_{pp}. The present approach thus overcomes problems of earlier QRPA calculations.

Acknowledgement

We thank Dr. K. Muto for many helpful discussions and for his QRPA code.*

*The present calculation was completed while one of the authors, Wu X.R. was visiting M.P.I. in the beginning of 1992. Resently we have learned that in a paper appeared in Phys.Rev.C46 (1992)R2153 the authors J.Engel, W.C.Haxton and P.Vogel criticized strongly our approach in double Bata decay calculation. First, they criticized the "power series expansion beyond the radius of convergence"; second is the neglect of kinetic energy operator T in the commutators; and third, the neglect of many-body scattering terms. Since the improvement of the OEM calculation is still under way, and a detailed analysis on the questions raised by these authors will be given elsewhere. Here we only brief our main arguments by pointing out 1/,the first criticism is groundless, and this question is also explained in the prsent paper; 2/, the neglect of operator T in the commutators certainly brings error in the matrix element, but it is tolerable for a realistic nuclear force according to the standard of nuclear physics, moreover, the claim that this approximation always leads

REFERENCES

[1] M. Goeppert-Mayer, Phys. Rev. 48, 512 (1935)

[2] H.S. Miley, F.T. Avignone, III, R.L. Brodzinski, J.I. Collar and J.H. Reeves, *Phys. Rev. Lett.* **65** (1990) 3092;

[3] F.T. Avignone III et al., *Phys. Lett.* **B256** (1991)559

[4] S.R. Elliott, A.A. Hahn, and M.K. Moe, *Phys. Rev. Lett.* **59**, (1987)2020.

[5] H. Ejiri et al., *Phys. Lett.***B258** (1991)17.

[6] K. Okada et al, Nucl Phys B (Proc. Suppl.) 19(1991)

[7] A.L. Turkevich, T.E. Economou and G.A. Cowan, *Phys. Rev. Lett.* **67** (1991)3211

[8] T. Kirsten, W. Gentner and O.A. Schaeffer, Z. Phys. 202 (1967) 273.

[9] T. Kirsten, Proc. Int. Symp on Nuclear Beta Decays and Neutrino, June 1986, Osaka, eds. T. Kotani, H.Ejiri and E. Takasugi (World Scientific, Singapore, 1986) p.81

[10] O.K. Manuel, as ref.[9] but p.103

[11] W.J. Lin et al., Nucl. Phys. A481 (1988) 477

[12] H.V. Klapdor-Kleingrothaus, in: Proc. Second Int. Workshop on Theoretical and Phenomenological Aspects of Underground Physics (TAUP 91), Sep. 9-13 1991, Tolédo Spain, in press.

[13] M. Beck, J. Bockholt, J. Echternach, G. Heusser, H.V. Klapdor-Kleingrothaus, B. Maier, F.Petry, A. Piepke, U. Schmidt-Rohr, H. Strecker, R. Zuber, A. Balysh, S.T. Belyaev, A. Demehin, A. Gurov, I. Kondratenko, V.I. Lebedev, submitted to Phys. Lett. B.

[14] C.R. Ching and T.H. Ho, Commun. Theor. Phys. 11, (1989)433
C.R. Ching, T.H. Ho and X.R. Wu, Phys. Rev. C40, (1989)304; Commun. Theor.

to smaller matrix element (comparing to the exact one) is wrong. It is potential-dependent; 3/, the neglect of many-body scattering terms is a real problem in OEM. However, part of this effect has been absorbed in the wave functions. And the validity can be evaluated by including the three-body terms, which can be shown to be summable as in the case of two-body case. The results presented in this paper are interesting not only because they are not so easily obtained using method different from OEM, but also the OEM+QRPA wave functions is no less better than other results which are inevitably based upon a certain approximation.

some cases (as indicated in table 1) the QRPA equation collapses for values below $g_{pp} = 1$. For these isotopes the g_{pp} value before the collapse points are used. The calculated matrix elements M_{GT} and corresponding half-lives for all potential $\beta\beta$-emitters are given in table 1. The half-lives given are for $g_A/g_V = -1.254$ and the phase-space factors of ref [21]. Note, however, that for the heaviest isotopes the phase space factor of [21] differs from the one used in the QRPA calculation of [20] by a factor of ~ 3. On the other hand, for medium heavy isotopes, such as ^{76}Ge, the phase factors of [21] and [20] are essentially equal.

For a comparison we listed also the experimental data available to us. For all nuclides which have been measured by direct counter experiments (^{76}Ge, ^{82}Se, ^{100}Mo and ^{238}U), the theoretical half-lives are in quite well agreement with the experimental ones. However for the Te isotopes our calculation gives a larger decay rate than the geochemical experiments.

To summarize, we have calculated $2\nu\beta\beta$ decay half-lives for all potential $\beta\beta$ emitters with $A \geq 70$. Our results are not sensitive to a specific choice for the particle-particle interaction parameter g_{pp}. The present approach thus overcomes problems of earlier QRPA calculations.

Acknowledgement

We thank Dr. K. Muto for many helpful discussions and for his QRPA code.*

*The present calculation was completed while one of the authors, Wu X.R. was visiting M.P.I. in the beginning of 1992. Resently we have learned that in a paper appeared in Phys.Rev.C46 (1992)R2153 the authors J.Engel, W.C.Haxton and P.Vogel criticized strongly our approach in double Bata decay calculation. First, they criticized the "power series expansion beyond the radius of convergence"; second is the neglect of kinetic energy operator T in the commutators; and third, the neglect of many-body scattering terms. Since the improvement of the OEM calculation is still under way, and a detailed analysis on the questions raised by these authors will be given elsewhere. Here we only brief our main arguments by pointing out 1/,the first criticism is groundless, and this question is also explained in the prsent paper; 2/, the neglect of operator T in the commutators certainly brings error in the matrix element, but it is tolerable for a realistic nuclear force according to the standard of nuclear physics, moreover, the claim that this approximation always leads

REFERENCES

[1] M. Goeppert-Mayer, Phys. Rev. 48, 512 (1935)

[2] H.S. Miley, F.T. Avignone, III, R.L. Brodzinski, J.I. Collar and J.H. Reeves, *Phys. Rev. Lett.* **65** (1990) 3092;

[3] F.T. Avignone III et al., *Phys. Lett.* **B256** (1991)559

[4] S.R. Elliott, A.A. Hahn, and M.K. Moe, *Phys. Rev. Lett.* **59**, (1987)2020.

[5] H. Ejiri et al., *Phys. Lett.***B258** (1991)17.

[6] K. Okada et al, Nucl Phys B (Proc. Suppl.) 19(1991)

[7] A.L. Turkevich, T.E. Economou and G.A. Cowan, *Phys. Rev. Lett.* **67** (1991)3211

[8] T. Kirsten, W. Gentner and O.A. Schaeffer, Z. Phys. 202 (1967) 273.

[9] T. Kirsten, Proc. Int. Symp on Nuclear Beta Decays and Neutrino, June 1986, Osaka, eds. T. Kotani, H.Ejiri and E. Takasugi (World Scientific, Singapore, 1986) p.81

[10] O.K. Manuel. as ref.[9] but p.103

[11] W.J. Lin et al., Nucl. Phys. A481 (1988) 477

[12] H.V. Klapdor-Kleingrothaus, in: Proc. Second Int. Workshop on Theoretical and Phenomenological Aspects of Underground Physics (TAUP 91), Sep. 9-13 1991, Tolédo Spain, in press.

[13] M. Beck, J. Bockholt, J. Echternach, G. Heusser, H.V. Klapdor-Kleingrothaus, B. Maier, F.Petry, A. Piepke, U. Schmidt-Rohr, H. Strecker, R. Zuber, A. Balysh, S.T. Belyaev, A. Demehin, A. Gurov, I. Kondratenko, V.I. Lebedev, submitted to Phys. Lett. B.

[14] C.R. Ching and T.H. Ho, Commun. Theor. Phys. 11, (1989)433
C.R. Ching, T.H. Ho and X.R. Wu, Phys. Rev. C40, (1989)304; Commun. Theor.

to smaller matrix element (comparing to the exact one) is wrong. It is potential-dependent; 3/, the neglect of many-body scattering terms is a real problem in OEM. However, part of this effect has been absorbed in the wave functions. And the validity can be evaluated by including the three-body terms, which can be shown to be summable as in the case of two-body case. The results presented in this paper are interesting not only because they are not so easily obtained using method different from OEM, but also the OEM+QRPA wave functions is no less better than other results which are inevitably based upon a certain approximation.

Phys. 12, (1989)167; in Proceeding of the BIMP Symposium on Heavy Flavor Physics, Eds. Chao Kuangta et al, World Scientific, Singapore, 1989, p359

[15] X.R. Wu, S. Staudt, H.V. Klapdor-Kleingrothaus, C.R. Ching and T.H. Ho, Phys. Lett. **B272** (1991) 169

[16] W.C. Haxton and G.J. Stephenson, Jr., Progress in Particle and Nuclear Physics 12, (1984)409.

[17] J.D. Vergados, Phys. Rep. 133 (1986)1

[18] P. Vogel and M.R. Zirnbauer, Phys. Rev. Lett. 57, (1986)3148.
O. Civitarese, A. Faessler and T. Tomoda, Phys. Lett. B194, (1987)11.
K. Muto and H.V. Klapdor, Phys. Lett. B201, (1988)420
T. Tomoda, Rep. Prog. Phys. 54, (1991)53.

[19] H.V. Klapdor and K. Grotz, Phys. Lett. B142 (1984)323 ; K. Grotz and H.V. Klapdor, Phys. Lett. B153 (1985)1, Phys. Lett. B157 (1985)242 and Nucl. Phys. A460 (1986)395

[20] A. Staudt, K. Muto and H.V. Klapdor-Kleingrothaus, Europhys. Lett. 13 (1990)31
A. Staudt, T.T.S. Kuo and H.V. Klapdor-Kleingrothaus, Phys. Lett. B242 (1990)17
K. Muto, E. Bender and H.V. Klapdor, Z. Phys. A334 (1989)177 and 187

[21] M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. 83, (1985)1

[22] G.M. Hardy, Divergent Series, Clarendon, Oxford, 1949

[23] X.R. Wu, S. Staudt, T.T.S. Kuo and H.V. Klapdor-Kleingrothaus, Phys. Lett. (in press)

[24] F. Simkovic, JINR Rapid Commun. 39 (1989)21 and M. Gmitro and F. Simkovic, Izv. AN SSSR 54 (1990)1780.

[25] J. Bernabeu et al., Z. Phys. C46 (1990) 792.
K.T. Hecht, S.C. Pang, J. Math. Phys. 8 (1969) 1571

[26] M. Lacombe et al. Phys. Rev. C21, (1980) 861.

[27] A.S. Barabash, V.V. Kuzminov et al, Sov. J. Nucl. Phys. 51 (1990)1

[28] E. Bellotti et al, Phys. Lett 266B (1989)209

[29] A.A. Klimenko et al, Nucl. Instrum. and Meth. B16 (1986) 446

Table Caption

Table 1 The calculated Gamow-Teller matrix elements and the corresponding half-lives of all potential $\beta\beta$-emitters with $A \geq 70$. The experimental half-lives for some nuclei are included. For all isotopes, except those marked by †, $g_{pp} = 1$ is taken. For isotopes marked by † the QRPA equation collapses for $g_{pp} = 1$, and thus g_{pp} before the collapse point is used. [a] ref [2],[b] ref [3],[c] ref [4], [d] ref [9],[e] ref [5],[f] ref [6],[g] ref [10], [h] ref [11],[i] ref [27],[j] ref [28],[k] ref [29] and [l] ref [7].

Table 1

Nuclei	M_{GT}	Half life (yrs)	Experimental half life (years)
^{70}Zn	0.0918†	1.44×10^{24}	
^{76}Ge	0.3351	2.61×10^{20}	$(1.1^{+0.6}_{-0.3}) \times 10^{21}$ [a], $(9.2^{+0.7}_{-0.4}) \times 10^{20}$ [b]
^{80}Se	0.3424	2.68×10^{29}	
^{82}Se	0.1019	0.848×10^{20}	$(1.1^{+0.8}_{-0.3}) \times 10^{20}$ [c], $(1.30 \pm 0.05) \times 10^{20}$ [d]
^{86}Kr	0.0580	3.42×10^{23}	
^{94}Zr	0.0117†	1.68×10^{24}	
^{96}Zr	0.0314†	2.02×10^{20}	
^{98}Mo	0.0800	6.16×10^{30}	
^{100}Mo	0.1065†	3.58×10^{19}	$(1.15^{+0.3}_{-0.2}) \times 10^{19}$ [e], $(1.16^{+0.34}_{-0.22}) \times 10^{19}$ [f]
^{104}Ru	0.1162†	3.09×10^{22}	
^{110}Pd	0.0879†	1.24×10^{21}	
^{114}Cd	0.1642†	9.84×10^{24}	
^{116}Cd	0.0171†	1.57×10^{22}	
^{122}Sn	0.1712	1.25×10^{26}	
^{124}Sn	0.0391	1.49×10^{21}	
^{128}Te	0.1462	2.11×10^{23}	$> 5 \times 10^{24}$ [g], $(1.8 \pm 0.7) \times 10^{24}$ [h]
^{130}Te	0.1006	0.787×10^{20}	$(1.5 - 2.8) \times 10^{21}$ [d], $(7.5 \pm 0.3) \times 10^{20}$ [h]
^{134}Xe	0.1286	2.69×10^{23}	
^{136}Xe	0.0280	1.01×10^{21}	$> 8.4 \times 10^{19}$ [i], $> 1.6 \times 10^{20}$ [j]
^{142}Ce	0.0400	3.30×10^{22}	
^{146}Nd	0.3285†	7.31×10^{30}	
^{148}Nd	0.0548†	1.19×10^{21}	
^{150}Nd	0.0441	1.66×10^{19}	$\geq 1.8 \times 10^{19}$ [k]
^{154}Sm	0.0793	1.49×10^{22}	
^{160}Gd	0.0454	2.81×10^{21}	
^{170}Er	0.1685	2.46×10^{23}	
^{176}Yb	0.1593†	4.92×10^{21}	
^{186}W	0.1506†	1.30×10^{24}	
^{192}Os	0.1777	2.40×10^{24}	
^{198}Pt	0.0741	1.14×10^{22}	
^{204}Hg	0.0510	1.81×10^{25}	
^{232}Th	0.1263	4.03×10^{21}	
^{238}U	0.0785	0.914×10^{21}	$(2.0 \pm 0.6) \times 10^{21}$ [l]