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Inflation in Generalized Jordan-Brans-Dicke Theory [1]

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Abstract

New inflation in generalized Jordan-Brans-Dicke theory is investigated. We estimate the rollover time which gives a limit on the coupling constant in the generalized JBD theory.

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Recently the reinterpretation of Jordan-Brans-Dicke (JBD) theory [1] was proposed by Cho [2], in which the Pauli metric is identified as physical space-time metric. The reason is that only the Pauli metric can represent the massless spin-two graviton and thus can correctly describe Einstein theory of gravitation. We refer this theory as generalized JBD theory. The generalized JBD theory is a new version of scalar-tensor theory of gravitation and hence the coupling constant in it should be a new parameter to be determined by re-analyses of observations. It is shown [3] that the linearized field equation predicts the same effects with the linearized Einstein's field equation. Although motion of a test particle deviates from geodesic motion because of existence of an additional force exerted by the dilaton field, however, there is no way to distinguish the new force from that exerted by metric field in Newtonian limit. This implies that the effects of strong fields and post-Newtonian terms should be considered in order to determine the coupling constant in generalized JBD theory. In fact, some authors [3,4] has used the Pauli metric to construct inflation models which seems to resolve cosmological puzzles. However some ambiguities exist in the literature as shown in [5]. In this paper we consider an inflation model in the generalized JBD theory and provide a limit on the coupling constant in the theory.

In original JBD theory Lagrangian is given by

$$\mathcal{L}_{BD} = -\sqrt{-\gamma}\left[\Phi\tilde{R} - \omega\gamma^{\mu\nu}\frac{\partial_\mu\Phi\partial_\nu\Phi}{\Phi} + \frac{1}{2}\gamma^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right] \tag{1}$$

where $\gamma_{\mu\nu}$ is the Jordan metric, $\gamma \equiv \det(\gamma_{\mu\nu})$, $\tilde{R} \equiv \tilde{R}(\gamma)$, Φ is a scalar field and ω is the JBD parameter, φ is a matter (scalar) field.

Introduce the Brans-Dicke dilaton field σ and conformal transformation (i.e. the Weyl scaling of metric),

$$\Phi = \frac{1}{2\kappa^2}e^{2\sigma}, \tag{2}$$

$$g_{\mu\nu} = e^{2\sigma}\gamma_{\mu\nu} \tag{3}$$

with $a^2 = 2\kappa^2/(2\omega+3)$, the Lagrangian (1) becomes

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}e^{-a\sigma}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - e^{-2a\sigma}V(\varphi)\right] \quad (4)$$

where $R = R(g)$, $g_{\mu\nu}$ is the Pauli metric. It is shown by Cho [2] that only the Pauli metric can represent the massless spin-two, and thus identify it as physical. Lagrangian (4) yields the following field equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa^2(T_{(\sigma)\mu\nu} + e^{-a\sigma}T_{(m)\mu\nu}), \quad (5)$$

$$\Box\sigma = \frac{1}{2}ae^{-a\sigma}T_{(m)}, \quad (6)$$

$$\Box\varphi = ag^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + e^{-a\sigma}V'(\varphi) \quad (7)$$

where

$$T_{(\sigma)\mu\nu} = \partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma. \quad (8)$$

$$T_{(m)\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - g_{\mu\nu}e^{-a\sigma}V(\varphi), \quad (9)$$

$$T_{(m)} = g^{\mu\nu}T_{(m)\mu\nu} = -g^{\mu\nu}\partial_\mu\varphi\partial_\beta\varphi - 4e^{-a\sigma}V(\varphi) \quad (10)$$

and $\Box \equiv \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$, $V'(\varphi) = dV(\varphi)/d\varphi$. Eq. (7) may be obtained by making use of Bianchi identity and Eqs. (5) and (6). For a perfect fluid

$$T_{(m)\mu\nu} = pg_{\mu\nu} + (\rho + p)U_\mu U_\nu \quad (11)$$

with $g^{\mu\nu}U_\mu U_\nu = -1$, ρ is energy density and p is pressure. For this case Eq. (7) should be replaced by covariant conservation law,

$$(T^{\mu\nu}_{(\sigma)} + e^{-a\sigma}T^{\mu\nu}_{(m)})_{;\nu} = 0. \quad (12)$$

The dynamical system (5)-(7) is called the generalized JBD theory. In this theory [5] the coupling constant $a = \beta\kappa$, $\kappa^2 = 8\pi G$, G is Newtonian gravitational constant and β is an arbitrary constant to be independent of the JBD parameter ω. Thus we should make different analyses of observations in order to determine β. In this paper we discuss the new inflation and give a limit to

the constant β.

Let us consider the Robertson-Walker universe with the line element

$$ds^2 = -dt^2 + R(t)^2\left[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2\right]. \quad (13)$$

Eqs. (5)-(7) become

$$H^2 + \frac{k^2}{R^2} = \frac{\kappa^2}{3}(\frac{1}{2}\dot{\sigma}^2 + e^{-a\sigma}\rho) \quad (14)$$

$$\ddot{\sigma} + 3H\dot{\sigma} = \frac{1}{2}ae^{-a\sigma}(\rho - 3p) \quad (15)$$

$$\dot{\rho} + 3H(\rho + p) = \frac{1}{2}a\dot{\sigma}(\rho + 3p) \quad (16)$$

where the overdot $\equiv d/dt$. $H = \dot{R}/R$ is Hubble "constant", and

$$\rho = \frac{1}{2}\dot{\varphi}^2 + e^{-a\sigma}V(\varphi), \quad p = \frac{1}{2}\dot{\varphi}^2 - e^{-a\sigma}V(\varphi). \quad (17)$$

Inflationary epoch of universe is the potential-dominated inflationary phase [6], i.e. $p \approx -\rho$, and Eqs. (14)-(16) reduce to

$$H^2 + \frac{k^2}{R^2} = \frac{\kappa^2}{3}(\frac{1}{2}\dot{\sigma}^2 + e^{-a\sigma}\rho), \quad (18)$$

$$\ddot{\sigma} + 3H\dot{\sigma} = 2ae^{-a\sigma}\rho, \quad (19)$$

$$\dot{\rho} = -a\dot{\sigma}\rho. \quad (20)$$

Solution to Eq. (20) is given by

$$\rho = V_0e^{-a\sigma}. \quad (21)$$

Comparing Eq. (21) to (17) in case of $\rho = -p$, we know that V_0 is just the potential $V(\varphi) =$ constant. for a flat universe ($k = 0$), Eqs. (18) and (19) with (21) have the following power-law solution

$$e^{a\sigma(t)} = [2\beta^2\kappa^2V_0/(\frac{3}{2\beta^2} - 1)]^{\frac{1}{2}}t, \quad (22)$$

$$R(t) = (t/t_0)^{\frac{1}{2\beta^2}} \tag{23}$$

where t_0 is an arbitrary constant.

We consider the new inflation model [7] in which the standard $SU(5)$ model with Coleman–Weinberg (CW) effective potential [8] is taken as the matter sector. We take the inflation potential to be

$$V(\varphi) = V_0 - \frac{1}{4}\lambda\varphi^4 \tag{24}$$

where the adjoint Higgs field is re–expressed as $\phi = \varphi \mathrm{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2})$, $\lambda \sim \frac{1}{2}$, $V_0^{1/4} \sim 2 \times 10^{14}$Gev, and the finite temperature corrections to the effective potential is neglected. This is a good approximate for WC potential [7].

In the original JBD theory the energy density $\tilde{\rho}$ and pressure $\tilde{p}$ for φ are given by [9]

$$\tilde{\rho} = \frac{15}{4}\dot{\varphi}^2 + V(\varphi), \quad \tilde{p} = \frac{15}{4}\dot{\varphi}^2 - V(\varphi) \tag{25}$$

where the loss of energy through radiation is ignored, and the overdot is the derivative with respect to the time coordinate $\tilde{t}$ in Jordan frame. Using the transformation, $d/d\tilde{t} = e^{\frac{1}{2}a\sigma} d/dt$, to change the "Jordan time" $\tilde{t}$ to the Einstein conformal time t, one arrives at

$$\rho = e^{-a\sigma}\tilde{\rho} = \frac{15}{4}\dot{\varphi}^2 + e^{-a\sigma}V(\varphi), \quad p = e^{-a\sigma}\tilde{p} = \frac{15}{4}\dot{\varphi}^2 - e^{-a\sigma}V(\varphi) \tag{26}$$

where the overdot corresponds to the Einstein conformal time, i.e. the physical time.

During the inflationary epoch,

$$p = -\rho = -e^{-a\sigma}V(\varphi) = -e^{a\sigma}V_0. \tag{27}$$

Now we consider the time evolution of the Higgs field φ. Substituting (26) into (16) we obtain the evolution equation of the Higgs field

$$\frac{15}{2}\ddot{\varphi} + 3H(\frac{15}{2}\dot{\varphi}) - a\dot{\sigma}(\frac{15}{2}\dot{\varphi}) + e^{-a\sigma}V'(\varphi) = 0. \tag{28}$$

We shall estimate the time required after the tunnelling event for the Higgs field to roll down from $\varphi_0 = \varphi(0) \sim 3 \times 10^8 Gev$ to the global minimum at $\varphi = \varphi(t_r) \approx 4 \times 10^{14} Gev$. In this range, $\ddot{\varphi}$ can be ignored since the potential is very flat. Using Eqs. (22), (23) and (24), Eq. (28) becomes

$$\varphi^{-3}\dot{\varphi} \approx \frac{2\lambda}{15}[(3 - 2\beta^2)\kappa^2 V_0]^{-1/2}. \tag{29}$$

Solution to Eq. (29) is

$$\varphi = \left[\frac{1}{\varphi_0^2} - \frac{4\lambda t}{15\sqrt{(3 - 2\beta^2)\kappa^2 V_0}}\right]^{-1/2}. \tag{30}$$

The rollover time is

$$t_r \approx \left(\frac{1}{\varphi_0^2} - \frac{1}{\varphi(t_r)^2}\right)\frac{15\sqrt{(3 - 2\beta^2)\kappa^2 V_0}}{4\lambda} \approx \frac{15}{4\lambda\varphi_0^2}\sqrt{3\kappa^2 V_0} \tag{31}$$

for small β^2 and $\varphi_0 << \varphi(t_r)$. Therefore the amount of inflation of the universe after the formation of a bubble from Eq. (23) is

$$Z = \frac{R(t_r)}{R(t_0)} = (t_r/t_0)^{1/2\beta^2}. \tag{32}$$

In order to resolve the cosmological puzzles, we should have $Z > 10^{28}$. This gives a limit for the coupling constant β as follows

$$(t_r/t_0)^{1/2\beta^2} > 10^{28}. \tag{33}$$

Substituting Eq. (31) into (33) one arrives at

$$\beta^2 < ln\left[\frac{15\sqrt{6\pi G V_0}}{2\lambda\varphi_0^2 t_0}\right]/56 ln 10. \tag{34}$$

Using the GUT's values for the parameters: $\varphi_0 \sim 3 \times 10^8 Gev$, $t_0 \sim 10^{-13}$ Gev^{-1}, $V_0^{1/4} \sim 2 \times 10^{14} Gev$ and $\lambda \sim 0.5$, we obtain $\beta^2 < 0.3$. Eq. (34) shows that the value of β^2 is not sensitive to the values of the parameters.

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