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DIFFERENCES IN MOMENTS OF INERTIA BETWEEN IDENTICAL SUPERDEFORMED BANDS

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J. Y. ZENG

Y. A. LEI

C. S. WU

P.O.Box 2735, Beijing 100080, The People's Republic of China

Telefax : (086)-1-2562587 Telephone : 2568348

Telex : 22040 BAOAS CN Cable : 6158

Differences in moments of inertia between identical superdeformed bands

J. Y. Zeng[1,2], Y. A. Lei[1] and C. S. Wu[1,2]

[1]Department of Physics, Peking University, Beijing 100871, China

[2]Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100080, China

Abstract

To account for the near equality of transition energies between "identical" superdeformed (SD) bands, a phenomenological analysis is presented. It is demonstrated that though the "identical" SD bands usually have different bandhead moments of inertia, $\delta\mathcal{J}_0/\mathcal{J}_0 \geq 10^{-2}$, under certain conditions different increasing (or decreasing) paces of the moments of inertia with ω may result in an apparent equality of $\mathcal{J}^{(2)}$'s or E_γ's ($|\delta E_\gamma/E_\gamma| = |\delta\mathcal{J}^{(2)}/\mathcal{J}^{(2)}| \sim 10^{-3}$) in certain frequency range. No pseudospin alignment is involved in this analysis and it seems to be consistent with the microscopic calculations.

PACS numbers: 21.10.Re. 21.60.Ev, 27.70.+q, 27.80.+w

One of the most amazing properties of superdeformed (SD) rotational bands is that the observed cascade transition energies for bands in some adjacent nuclei are almost identical ($\delta E_\gamma/E_\gamma \sim 10^{-3}$), much similar than expected [1, 2]. It was argued that [2, 3] the very similar SD bands can be characterized as having virtually identical moments of inertia and alignments that differ from each other by quantized amounts and to understand this property pseudospin alignment was introduced [2-4]. However, some comments were presented, e.g. in Refs. [5, 6]. The questions surrounding the occurence of identical SD bands remain an area of debate. *Does the near equality of transition energies observed actually imply a near equality of moments of inertia of the "identical" SD bands?* In other words, *does the "identical" SD bands really have identical moments of inertia (bandhead moment of inertia, kinematic and dynamic moments of inertia, and their variation with ω, etc.)?* To account for the near equality of transition energies, this letter presents a phenomenological analysis, which seems to be consistent with the available microscopic calculations [7-10].

In order to extract kinematic moment of inertia and alignment one needs to know the actual angular momenta of the band levels. However, in no case have the angular momenta in an SD band been determined by direct experiment. To determine the spins of an SD band, a method was proposed [11] on the basis of the Harris parametrization of the dynamic moment of inertia $\mathcal{J}^{(2)}$, extracted by the observed transition energy differences $\Delta E_\gamma(I) \equiv E_\gamma(I+2) - E_\gamma(I)$, $E_\gamma(I) \equiv E(I) - E(I-2)$. In Ref. [12] an more effective method was presented to determine the spins of an SD band directly from the transition energies themselves, i.e. the observed transition energies are least squares fit to the analytic expression deduced from the closed two

parameter expression for rotational spectra [13]

$$E(I) = a\left[\sqrt{1 + bI(I+1)} - 1\right], \tag{1}$$

which may be derived from the Bohr Hamiltonian for a well deformed nucleus with small axial asymmetry. The spins of all the SD bands observed in the $A \sim 190$ and 150 regions have been determined by this approach [14, 15].

In this paper, to improve further the calculation, the transition energies are least squares fit to a three-parameter expression [16] (hereafter called the *abc* fit)

$$E(I) = a\left[\sqrt{1 + bI(I+1)} - 1\right] + cI(I+1), \tag{2}$$

in which the anharmonic effect has been taken into account. In Table 1 is given the analysis for the four SD bands in ^{192}Hg and ^{194}Hg, whose spins have been assigned consistently by different methods [2, 11, 14, 17, 18]. It is seen that *without exception all the sixty-nine E_γ's observed in these bands can be reproduced incredibly well* (deviation ≤ 0.5keV) by the *abc* fit. The microscopic calculations [8–10] reveal the appearance of SD minima already at $I = 0$ for Hg-Pb nuclei. Therefore, it is expected that the predicted E_γ's for $I < 10$ and $I > 40$ are meaningful. From eq. (2), the kinematic and dynamic moments of inertia can be expressed as

$$\frac{\hbar^2}{\mathcal{J}^{(1)}} = \frac{ab}{[1 + bI(I+1)]^{1/2}} + 2c, \qquad \frac{\hbar^2}{\mathcal{J}^{(2)}} = \frac{ab}{[1 + bI(I+1)]^{3/2}} + 2c. \tag{3}$$

In view of the astouding coincidence between the calculated and observed E_γ's, the moments of inertia [$\mathcal{J}^{(1)}$, $\mathcal{J}^{(2)}$, and bandhead moment of inertia $\mathcal{J}_0 = \hbar^2/(ab + 2c)$] thus extracted are reliable. In fact, the moments of inertia calculated by eq. (3) agree quite well with the usually adopted results extracted by the difference quotients, $\mathcal{J}^{(1)}(I-1) = \hbar^2(2I-1)/E_\gamma(I)$ and $\mathcal{J}^{(2)}(I) = 4\hbar^2/\Delta E_\gamma(I)$, as expected.

The moments of inertia of the SD bands in ^{192}Hg and ^{194}Hg are shown in Fig. 1. Microscopically, the smooth increase of the moments of inertia may be attributed to the gradual alignment of high-N orbitals [8].

Now, let us address the bandhead moments of inertia $\mathcal{J}_0$. For comparison, firstly we investigate the $\mathcal{J}_0$'s of the signature partner SD bands. To date nine signature pairs have been observed. The extracted $\mathcal{J}_0$'s are listed in Table 2. It is seen that each signature pair have almost identical $\mathcal{J}_0$ and the accuracy of the coincidence is of order $\delta\mathcal{J}_0/\mathcal{J}_0 \sim 10^{-3}$. Therefore, the signature partner SD bands, apart from a possible relative shift in energy, can be considered as *truly identical*. This is understandable because they are characterized by the same intrinsic (configuration) structure. However, the situation is quite different for the so-called "identical" SD bands. In Table 2 are also given the extracted $\mathcal{J}_0$'s of several "identical" SD bands. The differences in $\mathcal{J}_0$'s between the "identical" SD bands are usually $\delta\mathcal{J}_0/\mathcal{J}_0 \geq 10^{-2}$, an order of magnitude larger than those of the signature partners, but somewhat similar to the situation often encountered in normally deformed nuclei.

It is interesting to note that the difference in $\mathcal{J}_0$'s between the SD bands ^{192}Hg and ^{194}Hg(1), though usually *not* considered as identical, $\delta\mathcal{J}_0/\mathcal{J}_0 \sim 1.6\%$, is much smaller than that between the "identical" SD bands, ^{192}Hg and ^{194}Hg(2,3), $\delta\mathcal{J}_0/\mathcal{J}_0 \sim 7\%$. This result seems to be consistent with the microscopic calculations [7–10]. Microscopic calculation [7] shows that the feature of an SD band is mainly influenced by the high-N configuration. In fact, the high-N configurations of the yrast SD bands in ^{192}Hg and ^{194}Hg (i.e. ^{194}Hg(1)) were reasonably assigned [8–10] as $\pi 6^4 \nu 7^4$, which may account for their similar moments of inertia. The high-N configuration of ^{194}Hg(2,3) were also assigned as $\pi 6^4 \nu 7^4$, but with two unpaired valence neutrons

occupying the high-K orbitals [512 5/2]⊗[624 9/2] strongly coupled to the core, which may be responsible for the difference in $\mathcal{J}_0$'s between the "identical" SD bands ^{192}Hg and ^{194}Hg(2,3).

But how can we understand the near equality of transition energies observed in the "identical" SD bands? The essential points are: (a) This near equality is established only in a limited range of rotational frequencies [2]. (b) With increasing frequency ω the moments of inertia of "identical" SD bands increase (decrease) with different paces due to their different configuration structures. (c) $\mathcal{J}^{(2)}$ changes with ω much faster than $\mathcal{J}^{(1)}$ [12, 14]. Therefore, though the $\mathcal{J}_0$'s of the two "identical" bands are different, the near equality of the consecutive E_γ's ($\delta E_\gamma/E_\gamma \sim 10^{-3}$) or $\mathcal{J}^{(2)}$'s may occur within certain frequency range. A beautiful example is displayed in Fig. 1. The near equality of $\mathcal{J}^{(2)}$ between ^{192}Hg and ^{194}Hg(2, 3) occurs in the range $\hbar\omega \sim 0.2$—0.4 MeV. In fact, as a rough estimate, from the usual expression for rotational spectra, $E(I) = \hbar^2 I(I+1)/2\mathcal{J}$ (with I-dependent $\mathcal{J}$), we have

$$\frac{\delta E_\gamma}{E_\gamma} = \frac{\delta I}{I} - \frac{\delta \mathcal{J}}{\mathcal{J}} \tag{4}$$

For "identical" SD bands $\delta I/I$ is of order 10^{-2} [19]. If δI and $\delta\mathcal{J}$ are of the *same* sign and $\delta\mathcal{J}/\mathcal{J} \sim 10^{-2}$ maintains in certain frequency range, the two terms on the righthand side of eq. (4) may cancel with each other and result in $|\delta E_\gamma/E_\gamma| = |\delta\mathcal{J}^{(2)}/\mathcal{J}^{(2)}| \sim 10^{-3}$. These requirements are met in the case of the "identical" pair ^{192}Hg and ^{194}Hg(2,,3). Similar situation occurs for the other "identical" SD bands.

From Table 1 it is seen that both for the observed and calculated E_γ's,

$$E_\gamma(I,\ ^{192}\mathrm{Hg}) > E_\gamma(I,\ ^{194}\mathrm{Hg}(1)) \gg E_\gamma(I,\ ^{194}\mathrm{Hg}(2))$$
$$\approx \frac{1}{2}\left[E_\gamma(I+1,\ ^{194}\mathrm{Hg}(3)) + E_\gamma(I-1,\ ^{194}\mathrm{Hg}(3))\right]$$

which is consistent with the analysis for the moments of inertia. However, if we compare E_γ's with different I values in ^{194}Hg(3) and ^{192}Hg (note: different signature values), it is found that $E_\gamma(I+1,\ ^{194}\mathrm{Hg}(3)) = E_\gamma(I,\ ^{192}\mathrm{Hg})$ indeed holds well in the spin range $I \sim 18$—42. However, just as *already observed* in the range $I \sim 10$—20, the calculated $E_\gamma(I)$'s in ^{192}Hg for $I < 20$ ($\hbar\omega < 0.2$ MeV) gradually become smaller than the corresponding $E_\gamma(I+1)$'s in ^{194}Hg(3), and for $I > 40$ ($\hbar\omega > 0.4$ MeV), $E_\gamma^{\mathrm{cal}}(I,\ ^{192}\mathrm{Hg}) > E_\gamma^{\mathrm{cal}}(I+1,\ ^{194}\mathrm{Hg}(3))$.

Finally, the alignment difference, $i = I_x(\omega,\ ^{194}\mathrm{Hg}) - I_x(\omega,\ ^{192}\mathrm{Hg})$, is displayed in Fig. 2. It is seen that for ^{194}Hg(2, 3) and ^{192}Hg, $i \approx 1$ indeed holds well [2] in the frequency range $\hbar\omega \sim 0.2$—0.4MeV. However, in the ranges $\hbar\omega \sim 0.1$—0.2MeV (*observed already*), $\hbar\omega \leq 0.1$MeV, and $\hbar\omega > 0.4$MeV (not yet observed), there seems to exist a systematic deviation from $i \approx 1$. For ^{194}Hg(1) and ^{192}Hg, though they are quite similar both from the phenomenological and microscopic points of view, no quantized alignment difference is found.

In summary, the above analysis shows that: (1) Just as seen in the normally deformed nuclei, the $\mathcal{J}_0$'s of the so-called "identical" SD bands in neighboring nuclei usually are different, $\delta\mathcal{J}_0/\mathcal{J}_0 \geq 10^{-2}$. (2) Because with increasing ω the moments of inertia of "identical" SD bands increase (decrease) with different paces, the near equality of $\mathcal{J}^{(2)}$ between the "identical" bands may occur in certian frequency range. Careful analysis shows that all the "identical" SD bands observed can be accounted for consistently in this phenomenological scheme, which seems to be consistent with the available cranked shell model calculations, in which no pseudospin symmetry is assumed. Of course, the analysis given above was made on the basis of the observed data now available. More experimental data on the SD bands in the lower and

higher spin regions and more careful microscopic calculations are needed to judge the validity of the statement given above. The preliminary calculation using the particle-number-conserving treatment [20, 21] for the cranked Nilsson model showes that the pairing interaction and particularly the blocking effect play an important role in the differences of moments of inertia between the "identical" SD bands ^{192}Hg and ^{194}Hg(2, 3). Details will be published elsewhere.

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Table Captions

Table 1. Calculation of the transition energies (in units of keV) of the SD bands in ^{192}Hg and ^{194}Hg.

Table 2. Bandhead moments of inertia of the signature partner SD bands and the "identical" SD bands.

Table 1

	^{192}Hg, $\alpha = 0$ $E_\gamma(I)$		^{194}Hg(1), $\alpha = 0$ $E_\gamma(I)$		^{194}Hg(2), $\alpha = 0$ $E_\gamma(I)$		^{194}Hg(3), $\alpha = 1$ $E_\gamma(I+1)$	
I	expt[a]	calc[b]	expt[c]	calc[d]	expt[e]	calc[f]	expt[e]	calc[g]
48		881.7		868.1		863.8		879.5
46		852.7	841.0	840.7		835.8		851.3
44		823.3	812.9	812.7	(807)	807.0		822.4
42	793.4	793.4	783.9	784.1	777.7	777.6	(793)	793.0
40	762.8	763.0	754.6	754.9	747.6	747.5	762.7	762.8
38	732.1	732.1	725.4	725.0	716.7	716.6	732.2	732.0
36	700.6	700.6	693.8	594.3	684.5	684.9	700.4	700.4
34	668.6	668.5	662.4	662.9	652.2	652.4	668.0	668.1
32	635.8	635.7	630.5	630.6	619.3	619.2	635.1	635.0
30	602.3	602.1	597.3	597.4	585.2	585.1	600.9	601.2
28	567.9	567.7	563.6	563.3	550.3	550.1	566.4	566.6
26	532.4	532.4	528.3	528.2	514.3	514.4	531.6	531.2
24	496.3	496.3	492.3	492.2	477.7	477.8	494.6	495.0
22	459.1	459.1	455.2	455.1	440.7	440.5	458.3	458.1
20	420.8	420.9	417.1	416.9	402.1	402.3	420.4	420.3
18	381.6	381.7	377.8	377.8	363.7	363.4	382.1	381.8
16	341.1	341.4	337.7	337.6	323.8	323.8	342.8	342.6
14	299.9	300.0	296.2	296.4	283.3	283.5	302.5	302.8
12	257.7	257.7	254.3	254.3	242.7	242.7	262.3	262.3
10	214.6	214.4		211.4	201.3	201.3		221.3
8		170.3		167.8		159.4		179.8
6		125.5		123.6		117.2		137.9
4		80.1		78.8		74.7		95.7
2		34.4		33.8		32.1		53.2

[a]Reference 11.

[b]a=6597.3 keV, b=8.168×10^{-4}, c=3.043 keV.

[c]Reference 17.

[d]a=10855 keV, b=5.796×10^{-4}, c=2.498 keV.

[e]Reference 18.

[f]a=22052 keV, b=3.102×10^{-4}, c=1.923 keV.

[g]a=19241 keV, b=3.297×10^{-4}, c=2.153 keV.

Table 2

signature pairs	$\mathcal{J}_0$ ($\hbar^2$MeV^{-1})	$\frac{\delta\mathcal{J}_0}{\mathcal{J}_0} \times 10^3$	identical bands	$\mathcal{J}_0$ ($\hbar^2$MeV^{-1})	$\frac{\delta\mathcal{J}_0}{\mathcal{J}_0} \times 10^2$
^{153}Dy(2,3)	86.20, 86.07	1.51	^{152}Dy–^{151}Tb*	88.88, 90.51	1.8
^{191}Hg(2,3)	94.45, 94.34	1.16	^{151}Tb–^{150}Gd*	108.85, 106.91	1.8
^{193}Hg(2,3)	93.09, 92.73	3.88	^{150}Tb–^{149}Gd*	82.70, 81.13	1.9
^{194}Hg(2,3)	93.57, 93.89	3.42	^{153}Dy(2,3)–^{152}Dy	86.14^{a}, 88.88	3.2
^{193}Tl(−,+)	95.53, 95.73	2.09	^{191}Hg(2,3)–^{192}Hg	94.40^{a}, 87.15	8.0
^{194}Tl(1a,1b)	99.69, 99.71	2.40	^{193}Hg(2,3)–^{192}Hg	92.91^{a}, 87.15	6.4
^{194}Tl(2a,2b)	95.16, 94.93	2.42	^{194}Hg(2,3)–^{192}Hg	93.73^{a}, 87.15	7.3
^{194}Tl(3a,3b)	100.53, 100.16	3.69			
^{195}Tl(−,+)	94.92, 94.66	2.74			

*The $\mathcal{J}_0$ value averaged over the signature partners.

Figure Captions

Fig. 1. The moments of inertia of the SD bands in ^{192}Hg and ^{194}Hg. The open circle and triangle denote the $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ of ^{192}Hg extracted by the difference quotients $\mathcal{J}^{(1)}(I-1) = \hbar^2(2I-1)/E_\gamma(I \to I-2)$ and $\mathcal{J}^{(2)}(I) = 4\hbar^2/\Delta E_\gamma(I)$, while the full circle and triangle represent those for the SD bands in ^{194}Hg. The dashed and solid curves are the results calculated by eq. (3).

Fig. 2. The angular momentum alignments ($I_x = \mathcal{J}^{(1)}\omega$) of the SD bands in ^{194}Hg relative to ^{192}Hg, $i = I_x(\omega,\ ^{194}\mathrm{Hg}) - I_x(\omega,\ ^{192}\mathrm{Hg})$. The triangle, full circle, and open circle denote the experimental values for ^{194}Hg band 1, 2, and 3, respectively, while the curves stand for the results calculated by the *abc* fit.

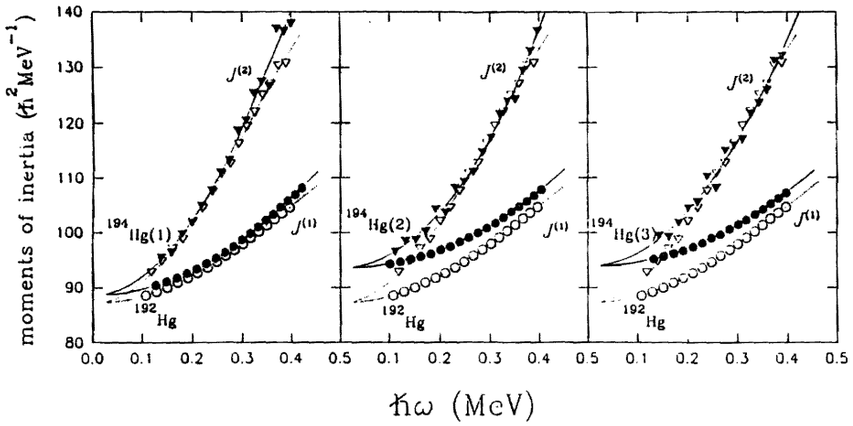
moments of inertia ($\hbar^2$ MeV^{-1})
140
130
120
110
100
90
80
$\mathcal{J}^{(2)}$
$\mathcal{J}^{(1)}$
^{194}Hg(1)
^{192}Hg
^{194}Hg(2)
^{194}Hg(3)
0.0 0.1 0.2 0.3 0.4 0.5
$\hbar\omega$ (MeV)

Fig 1

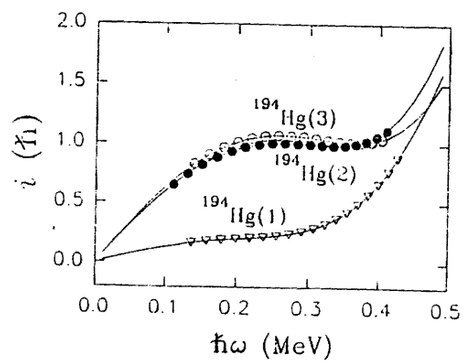
i ($\hbar$)
2.0
1.5
1.0
0.5
0.0
^{194}Hg(3)
^{194}Hg(2)
^{194}Hg(1)
0.0 0.1 0.2 0.3 0.4 0.5
$\hbar\omega$ (MeV)

Fig. 2