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Quantum Hall Effect under Non-uniform External Fields

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ABSTRACT

In this Letter, we predict that the quantum Hall effects, including both integral and fractional cases, can be observed when the magnetic field and the applied electric field are not uniform. In the present model, the non-uniformity breaks the higher Landau levels into energy bands while it may not affect the degeneracy of the lowest Landau level.

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The integral and fractional quantum Hall effects (IQHE and FQHE) were discovered in 1980 [1] and 1982[2]. Many investigations in this field have been provided.[3] In most of the experimental and theoretical proposals, one considers the effect under the uniform external fields because the uniformity of the external fields makes the simplity both of experiment and theory. However, one can ask if the quantum Hall effect can be observed when the magnetic field and the applied electric field are not uniform. In this letter, we discuss this subject. We find that when the uniform magnetic field is perturbed the ground state wave function of the single-electron is exactly soluble if there is a suitable electric field. Thus, the variational ground state wave function of the N-electrons may be constructed by means of a similar way to the construction in the common FQHE. The filling factor of this state is not uniform. But, the non-uniformity of the filling factor is same as that of the magnetic field in our case such that the Hall coefficient is still fractional. Under such external fields, however, the excited states of the single electron are not exactly soluble. To the first order of the non-uniformity, we find that the degeneracy of the Landau levels is lifted completely. Therefore, if the non-uniformity is regarded as some kind of imperfections, the IQHE should be observed in such case.

Consider a free electron moving on a two-dimensional plane. Assume there are non-uniform magnetic and electric fields. For specification[4], we investigate following two cases:

$$\mathbf{B}_1 = -B_0(1+8g_1r^2)\hat{k}, \quad \mathbf{E}_1 = \frac{8g_1\hbar B_0}{m_e c} r\hat{r}; \tag{1a}$$

$$\mathbf{B}_2 = -B_0(1-\frac{g_2}{2r})\hat{k}, \quad \mathbf{E}_2 = \frac{g_2\hbar B_0}{2m_e c r^2}\hat{r}, \tag{1b}$$

where g_i, $i = 1, 2$, are the parameters with dimensions L and L^{-2} respec-

tively; B_0 are constant and $\hat{r}$ is the unit vector in the radius direction. The fields (1a) is related to the normal matrix model with the potential $V(M) = (1/4)Tr(M^\dagger M + (g_1/2)Tr(M^\dagger M)^2$ (M is an $N \times N$ normal matrix).[5] The fields (1b) may be prefered by experimentists. For example, a ring sample called Corbino disk is put closely between two sets of solenoids. In each set, the solenoids are ring upon ring. Then, by adjusting the electric currents through these solenoids, the magnetic field will have a reduced distribution along the radius. The electric field $\mathbf{E}_2$ may be made by putting a point charge on the center of the ring, whose charge, of course, should coincide with eq.(1b). In symmetric gauge, the vector potentials and the scalar potentials of eq.(1) are $\mathbf{A}^{(1)} = \frac{cB_0}{2e}[y(1+4g_1r^2)\hat{i} - x(1+4g_1r^2)\hat{j}], \varphi^{(1)} = \frac{4g_1\hbar B_0}{m_e c}r^2$ and $\mathbf{A}^{(2)} = \frac{cB_0}{2e}[y(1-g_2/r)\hat{i} - x(1-g_2/r)\hat{j}], \varphi^{(2)} = -\frac{g_2\hbar B_0}{2m_e cr}$, the single-electron Hamiltonians are

$$H_{S_i} = \frac{1}{2m_e}[(\frac{\hbar}{i}\frac{\partial}{\partial x} + \frac{e}{c}A_x^{(i)})^2 + (\frac{\hbar}{i}\frac{\partial}{\partial y} + \frac{e}{c}A_y^{(i)})^2] - e\varphi^{(i)}. \tag{2}$$

It is easy to see that the Schrodinger's equations $H_{S_i}\phi^{(i)} = \mathcal{E}\phi^{(i)}$ have the exact ground state wave functions $\phi^{(1)} = z^m exp\{-(1/4l^2)r^2 - (g_1/2l^2)r^4\}$ and $\phi^{(2)} = z^m exp\{-(1/4l^2)r^2 - (g_2/2l^2)r\}$ ($l = (\hbar c/eB_0)^{1/2}$) and the energy of the groud states is $\mathcal{E}_0 = (1/2)\hbar\omega_c^0$, where $z = x + iy$ and $\omega_c^0 = eB_0/m_e c$ is the cyclotron frequency associated with the uniform field B_0.

Now, we consider the N-electron system described by the Hamiltonian

$$H^{(i)} = \sum_{j=1}^{N}[\frac{1}{2m_e}(\frac{\hbar}{i}\nabla_j + \frac{e}{c}\mathbf{A}_j^{(i)})^2 - e\varphi^{(i)}(r_j)] + U, \tag{3}$$

where U includes the potential generated by a neutralizing background and the Coulomb interactions among the electrons. And the interactions are weak

i.e. $|U| \ll \hbar\omega_c^0$. The variational ground state wave functions, thus, can be constructed by the Laughlin-Jastraw's form[6]

$$\begin{aligned}\psi_m^{(1)}(z_1,...,z_N) &= \prod_{i<j}^{N}(z_i - z_j)^m exp\{-\frac{1}{4}\sum_i |z_i|^2 - \frac{g_1}{2}\sum_i |z_i|^4\},\\ \psi_m^{(2)}(z_1,...,z_N) &= \prod_{i<j}^{N}(z_i - z_j)^m exp\{-\frac{1}{4}\sum_i |z_i|^2 - \frac{g_2}{2}\sum_i |z_i|\},\end{aligned} \tag{4}$$

where the magnetic length $l = (\hbar c/eB_0)^{1/2}$ is set equal to unit and m is an odd integer.

Along the Laughlin's formulation in the discussions of the common FQHE,[6] one can write the square of the modulus of $\psi_m^{(i)}$ as a classical Boltzmann distribution

$$|\psi_m^{(i)}(z_1,...,z_N)|^2 = exp\{-\beta\Phi_{eff}^{(i)}(z_1,..,z_N)\}, \tag{5}$$

where $1/\beta = m$ is the fictitious temperature and the effective potentials

$$\begin{aligned}\Phi_{eff}^{(1)} &= -2m^2\sum_{i<j} ln|z_i - z_j| + m\sum_k[\frac{|z_k|^2}{2} + g_1|z_k|^4],\\ \Phi_{eff}^{(2)} &= -2m^2\sum_{i<j} ln|z_i - z_j| + m\sum_k[\frac{|z_k|^2}{2} + g_2|z_k|].\end{aligned} \tag{6}$$

These effective potentials describe such a system: the first term in the potential represents the repulsion between particles of charge m via two dimensional Coulomb interaction; the second term is the attraction of these particles to the origin due to a neutralizing background on the same ring of charge densities $\rho_0^{(1)} = (1+8g_1r^2)/(2\pi l^2)$ and $\rho_0^{(2)} = (1-g_2/2r)/(2\pi l^2)$, i.e.

$$\Phi_{eff}^{(i)} = -2m^2\sum_{i<j} ln|z_i - z_j| + m\sum_k\int dz^2\rho_0^{(i)}(|z|)ln|z - z_k|. \tag{7}$$

The neutrality of the system tells us that the electron density in states $\psi_m^{(i)}$ is equal to

$$\rho_m^{(i)} = \frac{\rho_0^{(i)}}{m} = \frac{1}{m}\frac{eB_i(r)}{hc}. \tag{8}$$

This results in the filling factors of these states have a radius distribution, $\nu^{(1)} = (1/m)(1 + 8g_1r^2)$ and $\nu^{(2)} = (1/m)(1 - g_2/2r)$. Rigorously, we should prove that the perturbation to the incompressible quantum fluid theory do not change too much properties of the unperturbed theory. But if the perturbation is sufficient small we believe that the present system remains the main properties of the two-dimensional one-component plasma. For instance, in the small $\Gamma = 2m$, the system is a liquid. We shall give a detailed discussion for this matter in a separate proposal.

To understand the physical implication of the above result, we consider the Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m_e} \cdot \frac{\partial f}{\partial \mathbf{v}} = (\frac{\partial f}{\partial t})_{coll.}, \tag{9}$$

where $\mathbf{F} = e\mathbf{E}^{tot} + (e/c)\mathbf{v} \times \mathbf{B}$ with $\mathbf{E}^{tot}$ being the sum of the $\mathbf{E}_i$ and a uniform applied electric field $\mathbf{E}_0$. By definition, $\rho_m^{(i)} = \int f(\mathbf{r}, \mathbf{v})d\mathbf{v}$ so that $\partial f/\partial \mathbf{r}$ is of the g_i-order. The velocity $\mathbf{v}$ in the lowest order is proportional to the electric field. Thus, in the weak field approximation, the second term of the left-hand side of eq.(9) may be ignored because it is proportional to $g_i|\mathbf{E}^{tot}|$. By using the relaxation time approximation, therefore, the conductivity tensor still has the familiar form $\sigma_{xx} = \sigma_0/[1 + (\omega_c\tau)2], \sigma_{xy} = \rho_m^{(i)}ce/B_i - \sigma_{xx}/\omega_c\tau$ where $\sigma_0 = \rho_m^{(i)}e^2\tau/m$ and ω_c is the cyclotron frequency associated with B_i. When the relaxation time $\tau \to \infty$, we find that the Hall conductivity $\sigma_{xy} = \rho_m^{(i)}ce/B_i = \frac{1}{m}e^2/h$ is fractional while the longitude conductivity $\sigma_{xx} = 0$. As a result, one expects that the FQHE can be observed when the external magnetic an electric fields are not uniform.

The quasiparticles of the present model can also be discussed by the similar

way to describe the quasiparticles in the common FQHE.[6] From eq.(7), we see that the only difference between our model and the two-dimensional one-component plasma is the density of the state. The charge of the particles is not changed. This, and the fractional Hall coefficient, imply that the quasiparticles in our model should have the fractional charge. In concluding our description to the FQHE, we world like to mention that the general fractional quantum Hall state in our model may be constructed by the Haldane's hierarchical scheme.[7] We do not give the details here.

We turn to understand the excited states of the single-electron Schrodinger's equation. It is found that the excited states of the Schrodinger's equation are not exactly soluble. Up to the first order of perturbation, the higher Landau levels are broken into the energy bands. The standard perturbative theory in the course of quantum mechanecis may be employed. However, it is too complicated to solve our problem because the Landau levels are high degeneracy. Fortunately, the ground state of the single-electron is exactly known, which may be taken as a starting point to do the perturbative theory. Take the Hamiltonian H_{S_1} as an example, we present the results of the first order perturbation. Also consider the symmetric gauge, the wave functions are denoted by $\phi_{n,m}$ where n is the index of energy band and m is corresponding to the wave vector. The ground states $\phi_{0,m} = z^m e^{-f}$ $(f = (1/4)|z|^2 + (g_1/2)|z|^4)$ are still degenerate as we have mentioned. Then, it may be check that the lowest state in each energy band, up to the first order of g_1,

$$\phi_{n,0} = L^n e^{-f}, \quad L = (1 - 4g_1|z|^2)\bar{z}, \tag{10}$$

and the eigenvalue of the first perturbation is (in the unit of l)

$$\mathcal{E}_{n,0} = [(1 + 16g_1)n + 1/2]\hbar\omega_c^0. \tag{11}$$

This implies that the lowest energy in each band is higher the original Landau energy. The other states may be constructed in the following way. For example, assuming the sought wave function $\phi_{n,1}$ is

$$\phi_{n,1} = L^{n-1}[a_n(\partial + \bar{z}) + L]ze^{-f}, \tag{12}$$

then a consistent coefficient a_n and eigen-energy are

$$a_n = -\frac{2n}{3 + (n+1)32g_1}, \quad \mathcal{E}_{n,1} = [(1 + (1+1)16g_1)n + \frac{1}{2}]\hbar\omega_c^0. \tag{13}$$

The state

$$\phi_{1,m} = [a_{1,m}(\partial + \bar{z}) + L]z^m e^{-f} \tag{14}$$

is, for a suitable $a_{1,m}$, corresponding to

$$\mathcal{E}_{1,m} = [(1 + (m+1)16g_1) + \frac{1}{2}]\hbar\omega_c^0. \tag{15}$$

Repeat the previous process, one can obtain the state $\phi_{n,m}$ and the corresponding energy $\mathcal{E}_{n,m} = [(1 + (m+1)16g_1)n + \frac{1}{2}]\hbar\omega_c^0$. The conclusion is the higher Landau levels are completely separated into energy bands. The theory of the IQHE is essentially based on the non-interaction two-dimensional electron gas with weak impurity or imperfection. In our case, the non-uniform parts of the external fields can be regarded as a type of weak imperfection. This imperfection breaks completely the Landau levels into energy bands. Furthermore, the existence of the electric field leads to that the extended states could exist. Thus, one expects that the IQHE can be observed in the present case.

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[3] For a review book, see *The Quantum Hall Effect*, ed. by R. E. Prange and S. M. Girvin (Springer, New York, 1987).

[4] In general, the magnetic field may be taken as $\mathbf{B} = \nabla \times \mathbf{A}$, $\mathbf{A} = eB_0/2c[y(1 + f(r)) - x(1 + f(r))]$ and the electric field $\mathbf{E} = \nabla\varphi$, $\varphi = \hbar B_0/m_e c(r^2\partial f/\partial r^2 + f)$.

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