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QUANTUM HOLONOMY AND COVARIANT QUANTIZATION OF THREE-DIMMENSIONAL FIELD THEORY

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Quantum Holonomy
and
Covariant Quantization of Three-dimmensional Field Theory

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Abstract

By means of covariant quantization of Faddeev-Popov technique for three dimensional general covariantnon-Abelian quantum field theory with an anomaly and symmetry breaking free, we consider the expectation value of quantum holonomy defined on a path α and reproduce the interest result obtained in reference [1], and overcome the shortcoming in reference [1] that no covariant gauge fixing is introduced. The reasonableness of the conclusions is analysed in term of group reresentation theory and the case that quantum holonomy defined on a many-component link is investigated.

It is well known that during the past several years, a great number of people have been devoted to the study of low dimensional general covariant quantum field theories due to a series of papers written by Witten[2-5], in which the expectation values some obserbles can identiacl to topological invariants of the manifolds on which field theories are defined such as Donaldson polynomials etc. One of the most important discoveries is that Witten, taking the canonical quantization of three-dimensional Chern-Simons field theory and basing on the assumption that the quantum Hilbert space of C-S field theory is identical to that of the conformal block of WZW model, successfully reproduced the celebrated Jones polynomial in knot theory in term of the expectation value of Wilson loop[2].

Recently, two of our authors Lee and Zhu communicated a interesting result[1], they thought that, in Wilson loop $W[\gamma] = TrPexp\oint_\gamma A$, namely the trace of holonomy operator $Pexp\oint_\gamma A$ with γ a contour in three dimensional space-time manifold M, $A = A^a_\mu T^a dx^\mu$ the connection 1-form of gauge group principle bundle valued in some representation of gauge group $\mathcal{G}$ and P denoting the path ordering, the act of taking the trace of the matrix valued quantum holonomy must have masked some nontrivial algebratic property that the matrix should possess. Starting from this viewpoint, they directly considered the expectation value of holonomy operator $f(A,\alpha) = Pexp\int_\alpha A$ in term of path integral formalism with α denoting a path, but during the process of their deriving the interesting result, their functional integral evaluating the quantum holonomy is not very well-defined, for they didn't adopt the explicite covariant formalism, that is, they didn't introduce nonphysical ghost fields to eliminate the degeneracy caused by the gauge invariance possessed by the classical action, their operation is in essential taken on the orbit space $\tilde{\mathcal{A}} = \frac{\mathcal{A}}{\mathcal{G}}$ with $\mathcal{A}$ the connection space and $\mathcal{G}$ the gauge group.

In this letter, we shall overcome the shortcomings stated as above, and perform

the covariant quantization by means of Faddeev-Popov technique for three dimensional general covariant field theory and arrive at the same result as reference [1]. Furthermore the reasonableness of the result is analysed from the viewpoint of group representation theory.

Let $S[A]$ be the classical action of three dimensional field theory with $A = A^a_\mu T^a dx^\mu$ being the connection 1-form valued in some represention of Lie algebra of gauge group $\mathcal{G}$. we assume that $S[A]$ is a diffeomorphism and gauge transformation inaviant, and is independent of the metric of the manifold on which field theory is defined. That is ,$S[A]$ satisfies

$$\frac{\delta S[A]}{\delta g_{\mu\nu}} = 0 \tag{1}$$

and

$$\mathcal{L}_V S[A] = 0 \tag{2}$$

where $\mathcal{L}_V$ is the Lie derivative along a integral curve of some vector field V on the manifold M. The gauge invariance means that $S[A]$ is invariant under the local gauge transformation

$$A(x) \longrightarrow A(x)^{g(x)} = \Omega_{g^{-1}}(x)A(x)\Omega_g(x) + \Omega_{g^{-1}}(x)d\Omega_g(x) \tag{3}$$

Of course, it is also a invariant under all the global gauge transformation

$$A(x) \longrightarrow A^g(x) = \Omega_{g^{-1}}A(x)\Omega_g \tag{4}$$

for the set of all global gauge transformations is a subset of that of all local ones. As for the diffeomorphism invariance of the theory, the diffeomorphism transformation of A is given by the Lie derivative

$$\mathcal{L}_V A_\mu = V^\rho \partial_\rho A_\mu + A_\rho \partial_\mu V^\rho \tag{5}$$

It can be proved that the needed constraints caused by diffeomorphism invariance

is completely identical to those caused by gauge symmetries. so we only concentrate to eliminate the degeneracy caused by gauge transformation symmetries.

We consider the expectation value of quantum holonomy operator $f(A,\alpha)$=$Pexp\int_\alpha A$ with α a path in M

$$Z[\alpha]\equiv V^{-1}\int \mathcal{D}A exp iS[A] f(A,\alpha) \tag{6}$$

where $V=\int\mathcal{D}g$ is the gauge group volume, $\mathcal{D}g=\Pi_x d\Omega_g(x)$ and $\mathcal{D}A$ are respectively gauge group measure and gauge field variables functional intrgral measure. Obviousely $\mathcal{D}g$ is a gauge invariant for that

$$\begin{aligned}\mathcal{D}g(x) &= \Pi_x d\Omega_g(x)=\Pi_x d\Omega_{g''}(x)\\ &= \Pi_x d\Omega_{gg'}(x)=\mathcal{D}(gg')\end{aligned} \tag{7}$$

Under assumption that the field theory has no anomaly and broken symmetry. $\mathcal{D}A$ is invariant under gauge transformation.

$$\mathcal{D}A=\mathcal{D}A^g \tag{8}$$

The fact that the gauge invariance of classical action $S[A]$ means that the action takes constant value in the orbit formed by gauge transformation eq.(3) and that the functional integral is performed in all the connection space makes the functional integral proportionate to the group volume and hence be a infinity. Faddeev and Popov proposed the following technique to deal with this kind of case, they defined a hypersurface in the connection space by gauge conditions

$$F[A]=0 \tag{9}$$

and made the hypersurface intersect with gauge orbits only one time, (proper boundary conditions must be chosen otherwise this cannot be done in general for the appearance of Gribov ambigurity.) and then defined the functional $\Delta_F[A]$ by

$$1=\Delta_F[A]\int\mathcal{D}g\Pi_x\delta(F[A^g]) \tag{10}$$

It is well known that $\Delta_F[A]$ is a gauge invariant. namely

$$(\Delta_F[A^g])=(\Delta_F[A]) \tag{11}$$

In order to remove the degeneracy caused by gauge invariance, inserting eq.(10) into eq.(6), we have following formula for the expectation value of $f(A,\alpha)$

$$\begin{aligned}Z(\alpha) &= V_{-1}\int\mathcal{D}A\Delta_F[A]\int\mathcal{D}g(x)\Pi_x\delta(F[A^{g(x)}])exp(iS[A])f(A,\alpha)\\ &= V^{-1}\int\mathcal{D}g\mathcal{D}A\Delta_F[A]\Pi_x\delta(F[A^{g(x)}])exp(iS[A])f(A,\alpha)\end{aligned} \tag{12}$$

Make a maneuver in eq.(12): denoting $A^{g(x)}$ as A, then the original A is replaced by $A^{g^{-1}(x)}$. eq.(12) becomes the following form

$$\begin{aligned}Z(\alpha) &= V^{-1}\int\mathcal{D}g\mathcal{D}A^{g^{-1}}\Delta_F[A^{g^{-1}}]\Pi_x\delta(F[A])expiS[A^{g^{-1}}]f(A^{g^{-1}},\alpha)\\ &= V^{-1}\int\mathcal{D}g\mathcal{D}A\Delta_F[A]\Pi_x\delta(F[A])exp(iS[A])f(A^{g^{-1}},\alpha)\end{aligned} \tag{13}$$

The δ function in eq.(13) is very inconvienient to operate, since the real physical contents are independent of gauge choice, as usual,we take the following operation: choosing the other gauge condition

$$\delta(F^a[A])\longrightarrow\delta(F^a[A(x)]-f^a(x)) \tag{14}$$

then averaging the functional integral eq.(13) with weight after gauge conditions $F^a[A]$=0 are replaced by those $F^a[A(x)]-f^a(x)$=0

$$Z(\alpha)\longrightarrow\int\Pi_{a,x}\mathcal{D}f^a(x)exp(-\frac{i}{2\xi}\int(dx)[f^a(x)]^2Z(\alpha) \tag{15}$$

where ξ is a gauge choice parameter.Finally, we obtain that

$$Z(\alpha)=V^{-1}\int\mathcal{D}g\mathcal{D}Aexp(iS[A])exp-\frac{i}{2\xi}(F^a[A])^2\Delta_F[A]f(A^{g^{-1}},\alpha) \tag{16}$$

According to F-P techinique, introducing two nonphysical fields $C(x)$ and $\bar{C}(x)$, which are Lie algebra valued and in the adjoint representation of gauge group, that is,

$$C^{ag}(x)=\Omega_g^{-1}C^a(x)\Omega_g,\bar{C}^{ag}=\Omega_g^{-1}\bar{C}^a(x)\Omega_g \tag{17}$$

and with aid of the property of Gauss functional integral over Grassman variables, we have

$$\Delta_F[A^a_\mu] = detM_F \sim \int \Pi_a \mathcal{D}C^a \mathcal{D}\bar{C}^a expi \int (dx)(dy)\bar{C}^a(x)M^{ab}(x,y)C^b(y) \qquad (18)$$

with M_F being a matrix with respect to gauge group indices, space-time indices and space-time coordinates and its matrix elements taking the following form

$$M_F^{ad}(x,y) = \frac{\delta F^a[A^b_\mu(x)]}{\delta A^c_\nu(y)} D^{cd}_\nu(y) \qquad (19)$$

Accordingly, eq.(16) becomes that

$$\begin{aligned} Z(\alpha) &= V^{-1}\int \mathcal{D}g\mathcal{D}A\mathcal{D}C\mathcal{D}\bar{C} exp[iS[A] - \tfrac{i}{2\xi}\int (F^a[A])^2(dx) + \\ & \quad i\int (dx)(dy)\bar{C}^a(x)M^{ab}(x,y)C^b(y)]f(A^{g^{-1}},\alpha) \end{aligned} \qquad (20)$$

Now we perform the same maneuver as that Lee and Zhu did in reference [1] ,which reveals the transformation property of $Z(\alpha)$. Let g' be a global gauge transformation, write A in eq.(20) as $A^{g'^{-1}g'}$ and rename $A^{g'^{-1}}$ as A, and hence the original A is replaced by $A^{g'}$, but from above gauge fixing process, we can see that ghost fields $\bar{C}(x)$, $C(x)$ are in essential originated from the gauge condition $F[A]$=0, hence if we take some operation on A, correspondingly C and $\bar{C}$ must have some changes, so under above maneuver $A \longrightarrow A^{g'}$ with g' a global gauge element, for linear Lorentz covariant gauge conditions, we have

$$\begin{aligned} M_F = \tfrac{\delta F[A]}{\delta A} \longrightarrow M_F{}^{g'} &= \tfrac{\delta F[A^{g'}]}{\delta A} \\ &= \Omega^{-1}_{g'}\tfrac{\delta F[A]}{\delta A}\Omega_{g'} \\ &= \Omega^{-1}_{g'}M_F\Omega_{g'} \end{aligned} \qquad (21)$$

and hence

$$\begin{aligned} \Delta_F[A^{g'}] &= \int \mathcal{D}C^{g'}\mathcal{D}\bar{C}^{g'} expi\int (dx)Tr[\bar{C}^{g'}(x)M^{g'}_F(x)C^{g'}(x)] \\ &= detM^{g'}_F = detM_F = \Delta_F[A] \\ &= \int \mathcal{D}C\mathcal{D}\bar{C} expi\int (dx)Tr\bar{C}(x)M_F(x)C(x) \end{aligned} \qquad (22)$$

This coincides with the fact that ghost fields belong to adjoint representation of gauge group (eq.(17)). After the maneuver stated as above, eq.(20) can be rewritten as following

$$\begin{aligned} Z(\alpha) &= V^{-1}\int \mathcal{D}g\mathcal{D}A\mathcal{D}C\mathcal{D}\bar{C} exp[iS[A] - \tfrac{i}{2\xi}\int (dx)Tr(F[A])^2 + \\ & \quad i\int (dx)\bar{C}(x)M_F(x)C(x)]f((A^{g^{-1}})^{g'},\alpha) \\ &= V^{-1}\int \mathcal{D}g\mathcal{D}A\mathcal{D}\bar{C}\mathcal{D}C exp[iS[A] - i\tfrac{1}{2\xi}\int (dx)Tr(F[A])^2 + \\ & \quad i\int (dx)Tr(\bar{C}M_FC)]\Omega^{-1}_{g'}f(A^{g^{-1}},\alpha)\Omega_{g'} \\ &= \Omega^{-1}_{g'}Z(\alpha)\Omega_{g'} \end{aligned} \qquad (23)$$

Since this is true for every global gauge transformation, and Ω is supposed to be some irreducible representation of gauge group $\mathcal{G}$ that can be realized in a infinte dimensional representation space for the assumption that the classical action is general covariant, according to Schur's lemma in group representation theory, we obtain the conclusion

$$Z(\alpha) = C(\alpha)1_\rho \qquad (24)$$

where ρ denotes some irreducible representation of gauge group $\mathcal{G}$ and $C(\alpha)$ is a scalar function depending on the path α. The same as that Lee and Zhu's analylising the consequence of changing gauge field A to $A^{g(x)}$ by a local gauge transformation shows that[1]

$$C(\alpha) = \begin{cases} 0, & when\ \alpha\ is\ a\ open\ path \\ \neq 0, & when\ \alpha\ is\ a\ contour. \end{cases} \qquad (25)$$

Eqs.(24) and (25) are just the interesting result in reference[1]

In essential, from the viewpoint group representation theory , the result eq.(24) and (25) are very evident, let's see this in detail. Since the classical action $S[A] = \int_M (dx)\mathcal{L}[A]$ is a gauge invariant, of course it is invariant under global

gauge transformation

$$\begin{aligned} A \longrightarrow A^{g'} &= \Omega_{g'}^{-1} A(x) \Omega_{g'} \\ &= exp(-i\theta^a T^a) A(x) exp(i\theta^b T^b) \end{aligned} \tag{26}$$

where θ^a, θ^b are parameters independent of space-time manifold and T^a, T^b (a,b=1,2... dimG) are generators of gauge group $\mathcal{G}$, which satisfy

$$[T^a, T^b] = i f^{abc} T^c \tag{27}$$

then there exist Noether current

$$j_\mu^a = -iTr(\frac{\partial \mathcal{L}}{\partial \partial_\mu A_\nu}[T^a, A_\nu]) \tag{28}$$

which is conserved for the gauge invariance

$$\partial^\mu j_\mu^a = 0 \tag{29}$$

and the corresponding covariant conserved charge

$$Q^a = \int j_\mu^a d\Sigma^\mu \tag{30}$$

where

$$d\Sigma^\mu = (-1)^\mu \wedge_{\nu=0}^{2'} \tag{31}$$

with the prime denoting the omission of dx^ν from $\wedge_{\nu=0}^2$. We, the same as Witten did[3], assume that for a general space-time manifold M, there still exist the following classical Possion bracket

$$\begin{aligned} [Q^a, A_\mu]_{P.B} &= i[T^a, A_\mu] \\ [Q^a, Q^b]_{P.B} &= i f^{abc} T^c \end{aligned} \tag{32}$$

(For a flat space-time manifold, the evolution time is well defined, correpondingly so are the conserved charge and canonical momenta π_μ conjugate to A_μ. With aid of the well-defined basic Possion bracket between A_μ and π_ν, the above Possion bracket can be proved by a direct calculation)

After quantization, the classical field functions A_μ are replaced by operators $\hat{A}_\mu(x)$ (their concrete forms depends on the polarization choice) and the classical possion brackets by Lie brackets. With above assumption that there exsits no anomaly, the quantum Nother current conservative equation

$$\partial^\mu < \hat{j}_\mu^a >= 0 \tag{33}$$

is still satisfied. With aid of another assumption that gauge symmetry is not broken, that is, the vaccum state is invariant under gauge transformation

$$\Omega_{g'}|0>=|0> \tag{34}$$

or

$$\hat{Q}^a|0>=0 \tag{35}$$

Hence the conservative operators $\hat{Q}^a$ are still well defined and preserve hermicity. They satisfy the following commutative relation correponding to the classical Possion brackets eq.(32)

$$\begin{aligned} [\hat{Q}^a, \hat{A}_\mu(x)] &= -i[T^a, \hat{A}_\mu(x)] \\ [\hat{Q}^a, \hat{Q}^b] &= i f^{abc} \hat{Q}^c \end{aligned} \tag{36}$$

From eqs.(26),(27) and (36), one can $\hat{Q}^a$ constitute the representation for the Lie algebra of gauge group $\mathcal{G}$ and the Hilbert space of the field theory is just the representation space for the charge operators act on the Hilbert space. For an arbitrary element g of global gauge group $\mathcal{G}$, its representation takes the folllowing form

$$\Omega_{g'} = exp - i\alpha^a Q^a \tag{37}$$

with α^a are real constants, which is unitary for the hermicity of $\hat{Q}^a$. Note that the assumption that gauge symmetry is not broken or equivalently eqs.(34) and (35) is important for the representation to be unitary. Otherwise the zero-norm

states will appear in the Hilbert space of the theory and the charge operators $\hat{Q}_i$ are not physical operators, and hence it is impossible for them to generate unitary reresentation of gauge group $\mathcal{G}$ in the physical Hilbert space. So under global gauge transformations, the transformation behaviour for a quantum state $|\hat{A}>$ and quantum holonomy operator $\hat{f}(\hat{A},\alpha)$ take the following forms respectively

$$\begin{aligned} |\hat{A}> &\longrightarrow |\hat{A}>^{g'} = \Omega_{g'}^{-1}|\hat{A}> \\ \hat{f}(\hat{A},\alpha) &\longrightarrow [f(\hat{A},\alpha)]^{g'} = \Omega_{g'}^{-1}\hat{f}(\hat{A},\alpha)\Omega_{g'} \end{aligned} \tag{38}$$

Hence we have

$$\begin{aligned} [Z(\alpha)]^{g'} &= (<\hat{A}|\hat{f}(\hat{A},\alpha)|\hat{A}>)^{g'} \\ &= (<\hat{A}|)^{g'}[\hat{f}(\hat{A},\alpha)]^{g'}(|A>)^{g'} \\ &= <\hat{A}|\Omega_{g'}\Omega_{g'}^{-1}\hat{f}(\hat{A},\alpha)\Omega_{g'}\Omega_{g'}^{-1}|\hat{A}> \\ &= Z(\alpha) \end{aligned} \tag{39}$$

that is, $Z(\alpha)$ is invariant under global gauge transformation. Note that $Z(\alpha)$ is matrix-valued. By means of Wigner-Eckart theorem, we obtain that

$$Z(\alpha)_{ij} = C(\alpha)\delta_{ij} \tag{40}$$

with that

$$C(\alpha) = <\hat{A}^{(\lambda)}|\hat{f}(\hat{A},\alpha)|\hat{A}^{(\rho)}> \delta_{\lambda\rho} \tag{41}$$

where λ,ρ stand for irreducible representation and $|A^{(\lambda)}>$ is a state in the Hilbert space of three-dimensional general covariant field theory and belongs to λ irreducible representation of gauge group G for the fact that the representation provided by the Hilbert space is in general reducible.

As for the reasons that the vanishing or not of $C(\alpha)$ depends on the closing or not of the path α, it can be explained as following. If there exist no Wilson loops or holonomy operators in the universe, the Hilbert space of the theory only contains

one vaccum state $|0>$. In the case that they are present, they will play the role of matter fields and after quantization they will generate the Hilbert space through their action on the vaccum state. Let's see Witten's quantization idea. The three dimensional manifold can be chopped into pieces along the time evolution direction. Since we assume that the links are located in a finite region of three dimensional space-time manifold, at an arbitrary time t, the state of the system evoluting from the vaccum at time t=-∞, which is represented by a transverse surface Σ with no punctures on it, now is represented by a punctured surface Σ created by the intersection with it of the links or arbitrary paths on which Wilson line or holonomy operators are defined. The obserbles—holonomy operators acting as creating operators generates the present state. In the polarization that A is chosen coordinate, the state functional at time t takes the following form[6]

$$\begin{aligned} |\Psi[A,t]> &= <A,t|\Psi> = \int dA' <A,t|A',t_0=-\infty><A',t_0=-\infty|\Psi> \\ &= \int dA' K(A,t;A',t_0=-\infty)\Psi(A',t_0=-\infty) \\ &= \int \mathcal{D}A exp(i\int_{t_0=-\infty}^{t} S[A]dt + \int_\Sigma B[A])\Psi(A,t_0=-\infty)\Pi_n Pexp\int_{P_n}^{Q_n} A^{(n)} \end{aligned} \tag{42}$$

where $B[A]$ denotes the boundary terms associated with the transverse surface Σ, n the different components of links or different paths and P_n, Q_n the puncture points created by the intersection of the nth component of links or paths with Σ. More exactly the state should be generated by the action on vaccum state of the holonomy operators defined on the projection on the transverse surface Σ of the links or paths located in three dimensional space time region of less time t[7]

$$\Psi[A,t] = \Pi_n Pexpi\int_{P_n}^{Q^n} \Sigma_{i=1,2}A_i(x_1,x_2)dx^i\Psi[A,t=-\infty] \tag{43}$$

where x_i, i=1,2 chracterize the punctures position in Σ. On the other hand, the state represented by punctured surface can be obtained through the vaccum state at $t=+\infty$ for the links or paths on which quantum holonomy operators are defined

locate in a finite region. that is, it is generated by the action on the vaccum state at $t = +\infty$ of holonomy operators defined on the other part of the links or path cutted by the transverse surface. The states generated in this way also constitute the representation of gauge group $\mathcal{G}$. The quantum $Z(\alpha)$ or $C(\alpha)$ is exactly the expectation value of holonomy operator $f(A, \alpha)$ between of these two kind of states due to the cutting of transverse surface. From above statement, it can be easily seen that if the path is not closed, the irreducible representation provided by two kinds of state space cannot be equivalent, hence according to eq.(41), $C(\alpha)$ must vanish. Conversely, if the path is a contour, the irreducible representation should be equivalent for that these two kinds of Hilbert space are created from the holonomy operators inheriting the same representation from gauge group $\mathcal{G}$. So we arrive at the conclusion eq.(25).

Let's see the case that quantum holonomy operator is defined on a many-component link L. From the defination of a link, it is well known that the components of a link don't intersect but can link with each other, hence the expectation value of quantum holonomy operator

$$Z(L) =< Pexp\int_L A >=< \Pi_{i=1}^n Pexp\int_{\alpha_i} A > \tag{44}$$

can be evaluated without any path ambigurity for the path ordering in the defination of holonomy operator already shows the sequence of transport of the connection on the path provided by the components of a link L. Morever the quantum holonomy operator defined on different components of L can be in different representation of gauge group G. Under global gauge transformation, they are covariantly transformed according to the representation to which the connection A belongs:

$$Pexp\int_{\alpha_i} A^{(\rho),(i)} \longrightarrow \Omega_{g'}^{(\rho),(i)^{-1}} Pexp\int_{\alpha_i} A^{(\rho),(i)} \Omega_{g'}^{(\rho),(i)} \tag{45}$$

where ρ specifies the representation and i the ith component. Repeating above performation, we can reach the following result

$$Z(L) = C(\alpha_1, \alpha_2, ...\alpha_n) 1_{\rho_1} \otimes 1_{\rho_2} \otimes ... \otimes 1_{\rho_n} \tag{46}$$

or

$$Z(L)_{A_1 A_2 ... A_n, B_1 B_2 ... B_n} = C(\alpha_1, \alpha_2, ...\alpha_n) \delta_{A_1 B_1} \delta_{A_2 B_2} ... \delta_{A_n B_n} \tag{47}$$

This can also be straightforwdly predicted from the result eqs.(24) and eqs.(40), which are suit for a single knot. Note that the coefficient scalar function $C(\alpha_1, \alpha_2, ...\alpha_n)$ doesn't not simply depend on each isolated component α_i (i=1,2,...n), but also has relation with the linking among the components. That is, in general

$$C(\alpha_1, \alpha_2, ...\alpha_n) \neq C(\alpha_1) C(\alpha_2) ... C(\alpha_n) \tag{48}$$

unless every components of L don't link with each other. If one of the components is open (of course, in this case, the geometric object is not a link), $C(\alpha_1, \alpha_2, ...\alpha_n)$ must vanish due to the local gauge transformation property of holomomy operator defined on this one open component[1]. This many-component result eqs.(46) and (47) furtherly implies that the connection between quantum holonomy and Hopf algebra is worth investigating.[8]

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