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Han–Xin HE

Fagir C. Khanna

Hiroomi Umezawa

P.O.Box 2735, Beijing 100080, The People's Republic of China

Telefax : (086)-1-2562587 Telephone : 2568348

Telex : 22040 BAOAS CN Cable : 6158

Spontaneous Breaking of Spin and Isospin Symmetries and the Collective Modes in Nuclear Matter

Hanxin He

China Institute of Atomic Energy, P.O.Box 275, Beijing
People's Republic of China ①
and Institute of Theoretical Physics, Academia Sinica, Beijing,
People's Republic of China

Fagir C. Khanna

Theoretical Physics Institute, Physics Department
University of Alberta, Edmonton, Alberta, Canada T6G 2J1
and TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3

Hiroomi Umezawa
Theoretical Physics Institute, Physics Department
University of Alberta, Edmonton, Alberta, Canada T6G 2J1

Abstract

A consistent theory of the spin and isospin excitations in an infinite nuclear system is formulated based on the Ward–Takahashi relations among various many–point Green's functions, which are derived from the requirement of the rotational invariance in spin and isospin SU(4) space. Collective modes are found to be six Nambu–Goldstone bosons, called spin, isospin, and spin–isospin waves, which appear as a result of the spontaneous breaking of the SU(4) symmetry. The W–T relations are used to determine the correlations among collective modes, and obtain the consistent expressions for static susceptibilities beyond the random–phase approximation. Then the quenching of spin–isospin excitation strength of nuclei is qualitatively discussed.

① Permanent address

1. Introduction

Investigations of nuclear spin and isospin excitations attract a great deal of attertion[1]. The interest in this subject is motivated by the universality of these collective excitations to all nuclei as well as the problem of systematically observed quenching of strength associated with some of collective excitations, i.e. the Gamow–Teller and isovector magnetic transitions. There appeared many models of describing the spin and isospin excitations trying to interpret above natue[1], and most of them were based on the random phase approximation (RPA). Probably in order to well understand the universality of nuclear spin and isospin excitations and the quenching mechanisms, one may need to further study the deeper dynamical reson for these excitations as well as the approach beyond the RPA. The recent interest in understanding the nuclear spin and isospin response to external probes also show that the collective excitations have to be studied at a level well beyond the RPA[2]. In this aspect, so far a systematical approach and a fully consistent approximation scheme is still lacking.

On the other hand, quantum field (QFT) has become a standard method to study the many–body problems in condensate matter physics. As we known from QFT, when the lowest state of a system is not an eigenstate of a continuous symmetry of the system, it is said that this symmetry is spontaneously broken. This is a common occurrence in natue. The well known examples in condensate matter physics are ferromagnetism and crystal formation. Where spin–rotational symmetry is spontaneously broken in a ferromagnet and translational symmetry is sontaneously broken in an infinite crystalline solid. Of caurse, the symmetry that is spontaneously broken is still a symmetry of the system, which is manifested through the existence of the Nambu–Goldstone boson [3] (Goldstone's theorem) being energy–gapless mode in the long wave length limit in the non–relativistic theory. In this case it is known that the symmetry properties of the system lead to a variety of exact relations among many–point Green's functions, i.e. the Ward–Takahashi (W–T) relations[4]. Some significant conclusions can be derived from these relations without any approximation. One of them is that the appearance of the N–G bosons is a result of one of the W–T relations (Goldstone's theorem[5]). In above examples, the corresponding N–G bosons are spin waves in the ferromagnet and phonons in the crystalline solid, which are well–studied collective excitation modes[6]. But none of nuclear collective excitations seem to study along this line.

Starting from this paper, we aim at the attempt to cast a unified description of the nuclear spin and isospin excitations and a fully consistent approximation scheme of the spin and isospin reponse functions or susceptibilities in the frame of modern QFT. As a starting point, in this paper we first consider a nonrelativistic infinite nuclear system in order to simpfy the treatment and make

use of the QFT method. Generalizing the study of ferromagnetism where spin rotational symmetry is spontaneously broken, we understand the spin and isospin excitations of the system under consideration as the spontaneous breakdown of the spin and isospin rotational invariance. It means that the theory or the Lagrangian of the system is spin and isospin rotationally invariant but the physically realized state does not possess such symmetry. To do this the SU(2) symmetry of the spin has to be extended to the Wigner's supermultiplet SU(4) symmetry of spin and isospin. Studying the response of the system to some spin and isospin symmetry-breaking effects and using the requirement of the rotational invariance in SU(4) space, we derive the W-T relations among many-point Green's functions. The collective modes of spin and isospin excitations thus naturally appear as a result of one of the W-T relations (Godstone's theorem). That is, these collective modes are N-G Bosons (nucleon bound state) which include spin waves, isospin-waves, and spin-isospin waves. We may simply call them σ-,τ-, and $\sigma\tau$-waves (or bosons), respectively. We hope to point out that the requirement of the rotational invariance in SU(4) space also determines many properties of σ-, τ-, and $\sigma\tau$-waves (or bosons) like nucleon-boson interactions, in particular, the correlations among collective modes.

In this paper, we also give an approach to the calculations of spin and isospin response functions and static susceptibilities. We get an expression to the static susceptibility including the corrections to RPA in terms of the nucleon self-energy. The result shows that the standard RPA susceptibility is obtained at the first approximation to nucleon self-energy, while the corrections to RPA come from the effects of the collective excitations, i.e. σ-, τ-, and $\sigma\tau$-waves, to nucleon self-energy.

This paper is orginazed as follows. In sec.2, the Ward-Takahashi relations are derived from the requirement of the rotational invariance in SU(4) space of spin and isospin by the functional integral approach. In sec. 3, the collective modes of spin and isospin excitations are studied by means of one of W-T relations, which shows why these collective modes are N-G bosons. In Sec.4, we derive several W-T relations among some many-point Green's fuctions which correspond to the interactions among nucleon and $(\sigma\tau)$- bosons. The vertices relations are given by Fourier-transforming these W-T relations into momentum space. By means of some W-T relations, we study the bound-state condition of $(\sigma\tau)$- boson and the susceptibility in Sec.5. In Sec.6, we give the expressions of the static susceptibilities in terms of nucleon self-energy, from which we obtain a consistent approximation approach beyond RPA to the calculation of susceptibilities where the corrections from $(\sigma\tau)$-bosons or spin-isospin waves are given.In Sec.7, we qualitatively discuss the quenching of spin-isospin excitation strength and the correlations among collective modes. Conclution and remark

are given in Sec.8.

2. SU(4) Symmetry and Ward-Takahashi Relations

We consider an infinite nuclear system of nonrelativistic nucleons with spin 1/2 and isospin 1/2. Their dynamics are described by the Heisenberg fields $\psi_{\lambda\mu}(x)$ and the canonical conjugates $\psi^{+}_{\lambda\mu}(x)$ where the first subscript λ denotes the spin state and the second one μ for isospin state of a nucleon. Fields $\psi_{\lambda\mu}(x)$ and $\psi^{+}_{\lambda\mu}(x)$ obey the usual equal-time anticommutotion relations

$$\{\psi_{\lambda\mu}(x),\psi^{+}_{\sigma\tau}(x')\}\delta(t-t') = \delta_{\lambda\sigma}\delta_{\mu\tau}\delta^{4}(x-x') \ . \tag{2.1}$$

The equations of motion for the Heisenberg fields can be obtained in the usual way from some appropriate Lagrangian density $L(\psi(x), \psi^{+}(x))$ for which the detail form is not needed here. $L(\psi,\psi^{+})$ has translational invariance in space-time for the infinite extended system under consideration. We assume that $L(\psi,\psi^{+})$ has separate rotational invariance in coordinate space and in SU(4) space of spin and isospin. Such decomposition is valid as long as the LS coupling is out of consideration. The 15 generators of the semi-simple Lie group SU(4) are

$$\begin{aligned} S_{\lambda} &= \int d^{4}x\psi^{+}(x)(\sigma_{\lambda}\times 1)\psi(x) \ , \\ T_{\mu} &= \int d^{4}x\psi^{+}(x)(1\times\tau_{\mu})\psi(x) \ , \\ S_{\lambda}T_{\mu} &= \int d^{4}x\psi^{+}(x)(\sigma_{\lambda}\times\tau_{\mu})\psi(x) \ , \end{aligned} \tag{2.2}$$

with $\lambda,\mu = 1,2,3$. Where nucleon fields $\psi(x)$ and $\psi^{+}(x)$ have been written in the matrix form. For the sake of convenience and simplification, we introduce the notations $Q_{\lambda\mu} = \sigma_{\lambda}\times\tau_{\mu}$ and $Q_{\lambda o} = \sigma_{\lambda}\times 1, Q_{o\mu} = 1\times\tau_{\mu}$. In this way, the 15 generators in Eq.(2.2) can be combined in the form

$$\int d^{4}x\psi^{+}(x)Q_{\lambda\mu}\psi(x) \tag{2.3}$$

with $\lambda, \mu = 0,1,2,3$.

In order to describe the collective excitations of spin and isospin for the system, let us remind the situation of the ferromagnetism. In a ferromagnet, although the Lagrangian is rotationally invariant, the ground state is not, since in it dynamics favors neighboring spins to be parallel. So that the ground state has maximum spin aligned along some, albeit arbitrary, direction. That is the ground

state is a spin–ordered state. The symmetry breaks from SU(2) to U(1), the latter being the rotation group around the chosen axis. Which axis the system chooses depends on the initial and boundary conditions. A typical procedure is to impose a weak magnetic fild h which then is gradually switched off[6]. In mathmatical treatment it corresponds to let h limit zero in the end of calculation. Now for an infinite nuclear system, the interactions among nucleons may favor the appearance of the order state of spin and isospin with the same reason as ferromagnetism. As a result, the physically realized states of the system do not possess the same symmetry as the Lagrangian, i.e. the SU(4) symmetry is spontaneously broken. To understand how the symmetry breaks, we need investigate the response of the nuclear system to some imposed external probes which then is gradually switched off. The possible external probes inducing the excitations or polarizations of spin and isospin may be kinds of electromagnetic, weak, and strong interactions (f.i. (p,n) and (n,p) reactions). Do not loss the generality, we may choose the direction of polarization of spin and isospin in the third component which is fixed by the imposed external field. The symmetry–breaking terms thus may be following forms

$$h_\sigma Q_{30}(x)\ , \qquad h_\tau Q_{03}(x)\ , \qquad h_{\sigma\tau} Q_{33}(x)\ .$$

where

$$Q_{30}(x) = \psi^+(x)\sigma_3\psi(x)\ ,$$
$$Q_{03}(x) = \psi^+(x)\tau_3\psi(x)\ ,$$
$$Q_{33}(x) = \psi^+(x)\sigma_3\tau_3\psi(x)\ ,$$

and h_σ, h_τ, $h_{\sigma\tau}$ are respectively the imposed external fields generating the spin–, isospin–, and spin–isospin polarizations.

The total Lagrangian of the system now is given by

$$L_t(x) = L(\psi(x),\psi^+(x)) + h_\sigma Q_{30}(x) + h_\tau Q_{03}(x) + h_{\sigma\tau} Q_{33}(x)\ . \tag{2.4}$$

We use functional formulation[7] of the QFT to derive the W–T relations. The generating functional of the system is defined by

$$W[\eta,\bar\eta,J] = \frac{1}{N}\int[d\psi][d\psi^+]\exp\{i\int d^4x[L(\psi,\psi^+) + h_\sigma Q_{30}(x) + h_\tau Q_{03}(x) + h_{\sigma\tau} Q_{33}(x) + \eta^+(x)\psi(x) + \psi^+(x)\eta(x) + J_{\lambda\mu}(x)Q_{\lambda\mu}(x)]\}\ , \tag{2.5}$$

with

$$Q_{\lambda\mu}(x) = \psi^+(x)Q_{\lambda\mu}\psi(x)\ . \tag{2.6}$$

Where $\eta(x)$, $\eta^+(x)$ and $J_{\lambda\mu}(x)$ are the external sources, $[d\psi][d\psi^+]$ is the measure of path integral, N is the generating functional in case $\eta = J_{\lambda\mu} = 0$. At the end of calculation, the limit $h_\sigma \to 0$, $h_\tau \to 0$, and $h_{\sigma\tau} \to 0$ should be performed. Note that η, η^+, ψ and ψ^+ anticommute with each other, while $J_{\lambda\mu}$ are the c–number sources commuting with other operators. In physics, $Q_{\lambda\mu}(x)$ describe the responses of the system to sources $J_{\lambda\mu}(x)$.

Let us now make the variable transformations

$$\psi(x) \to e^{i\gamma\cdot Q}\psi(x)\ , \psi^+(x) \to \psi^+(x)e^{-i\gamma\cdot Q}\ , \tag{2.7}$$

and

$$\psi(x) \to e^{i(\alpha\cdot\sigma+\beta\cdot\tau)}\psi(x)\ , \psi^+(x) \to \psi^+(x)e^{-i(\alpha\cdot\sigma+\beta\cdot\tau)}\ , \tag{2.8}$$

in Eq.(2.5), respectively, where $\gamma\cdot Q = \gamma_{\lambda\mu}Q_{\lambda\mu}$. From the requirements of the rotational invariance in SU(4) space of spin and siospin, we then have

$$-i\partial W/\partial\gamma_{\lambda\mu} = 0\ , \qquad (\lambda,\mu = 0,1,2,3)\ , \tag{2.9}$$

and

$$-i\partial\partial W/\partial\alpha_i\partial\beta_j = 0\ , \qquad (i,j = 1,2,3)\ , \tag{2.10}$$

which respectively lead to

$$i\int d^4x < \eta^+(x)Q_{ik}\psi(x) - \psi^+(x)Q_{ik}\eta(x) + J_{\lambda\mu}(x)[Q_{\lambda\mu}(x)\ ,Q_{ik}(x)] >_{\eta J\lambda}$$
$$= -i\int d^4x < \psi^+(x)[h_\sigma Q_{30} + h_\tau Q_{03} + h_{\sigma\tau}Q_{33}, Q_{ik}]\psi(x) >_{\eta J\lambda} \tag{2.11}$$

and

$$i\int d^4x < \{\eta^+(x)\sigma_i\tau_j\psi(x) + \psi^+(x)\sigma_i\tau_j\eta(x) + J_{\lambda\mu}(x)\psi^+(x)[\sigma_i,\sigma_\lambda][\tau_j,\tau_\mu]\psi(x)\} >_{\eta J\lambda}$$
$$-\int d^4x\int d^4y < [\eta^+(x)\sigma_i\psi(x) - \psi^+(x)\sigma_i\eta(x) - J_{\lambda\mu}(x)\psi^+(x)[\sigma_i,Q_{\lambda\mu}]\psi(x)]$$
$$\times[\eta^+(y)\tau_j\psi(y) - \psi^+(y)\tau_j\eta(y) - J_{\lambda\mu}(x)\psi^+(y)[\tau_j,Q_{\lambda\mu}]\psi(y)] >_{\eta J\lambda}$$

$$
\begin{aligned}
&= -ih_{\sigma\tau}\int d^4x < \psi^+(x)[\sigma_3,\sigma_i][\tau_3,\tau_j]\psi(x) >_{\eta J h} \\
&+ \int d^4x \int d^4y < [\psi^+(x)[h_\sigma Q_{30} + h_{\sigma\tau} Q_{33},\sigma_i]\psi(x)(\eta^+(y)\tau_j\psi(y) \\
&- \psi^+(y)\tau_j\eta(y) + J_{\lambda\mu}(y)\psi^+(y)[Q_{\lambda\mu},\tau_j]\psi(y)) \\
&+ \psi^+(x)[h_\tau Q_{03} + h_{\sigma\tau} Q_{33},\tau_j]\psi(y)(\eta^+(x)\sigma_i\psi(x) \\
&- \psi^+(x)\sigma_i\eta(x) + J_{\lambda\mu}(x)\psi^+(x)[Q_{\lambda\mu},\sigma_i],\psi(x)) >_{\eta J h} \\
&+ \int d^4x \int d^4y < [\psi^+(x)[h_\sigma Q_{30} + h_{\sigma\tau} Q_{33},\sigma_i]\psi(x)\psi^+(y) \\
&\times [h_\tau Q_{03} + h_{\sigma\tau} Q_{33},\tau_j]\psi(y) >_{\eta J h} \quad . \qquad (2.12)
\end{aligned}
$$

Where, for convenience, we have used the compact notation

$$
\begin{aligned}
< f[\psi,\psi^+] >_{\eta J h} &= \frac{1}{N}\int[d\psi][d\psi^+] f(\psi,\psi^+) exp\{i\int d^4x[L(\psi,\psi^+) + h_\sigma Q_{30} \\
&+ h_\tau Q_{03} + h_{\sigma\tau} Q_{33} + \eta^+(x)\psi(x) + \psi^+(x)\eta(x) + J_{\lambda\mu}(x)Q_{\lambda\mu}(x)]\} \quad . \qquad (2.13)
\end{aligned}
$$

Henceforth we also use following notations

$$
< f[\psi,\psi^+] >_h = \lim_{\eta,J\to 0} < f[\psi,\psi^+] >_{\eta J h} \quad , \qquad (2.14)
$$

$$
< f[\psi,\psi^+] > = \lim_{\eta,J,h\to 0} < f[\psi,\psi^+] >_{\eta J h} \quad . \qquad (2.15)
$$

Eqs.(2.11) and (2.12) are the basic Ward–Takahashi relations resulted from the symmetry properties of the system under consideration. By taking the successive derivative of Eqs. (2.11) and (2.12) with respective to sources, we then can get a series of WT relations among the varous many–point Green's functions.

We hope to point out that the presence of the symmetry–breaking interactions (see Eq(2.4)), which distort the SU(4) invanance, leads to the appearance of the poralezation states or order states which are specified here by

$$
M_\sigma = < \psi^+(x)\sigma_3\psi(x) > \quad , \qquad (2.16)
$$

$$
M_\tau = < \psi^+(x)\tau_3\psi(x) > \quad , \qquad (2.17)
$$

$$
M_{\sigma\tau} = < \psi^+(x)\sigma_3\tau_3\psi(x) > \quad . \qquad (2.18)
$$

Where $<F>$ means the ground–state expectation value of the time–ordered

product, $<F> = <0|T[F]|0>$. Eqs. (2.16), (2.17), and (2.18) respectively imply that the rotational symmetries in spin–, isospin–, and spin–isospin–subspace are spontaneously broken. But the rotational symmetries around the third axis in these subspaces are not violated, respectively. This is easily understood from Eq.(2.11) by setting $Q_{1k} = Q_{30}$ (i.e., $\sigma_3 \times 1$), Q_{03}, and Q_{33}, respectively. It means that these third components are still good guantum numbers, and so all states may be classified as eigenstates of generators S_3, T_3, and $S_3 \times T_3$ in Eq.(2.2).

3. Collective Modes

According to Goldstone theorem, the spontaneous breaking of a continuous symmetry of a system implies that there inevitably exist a N–G boson which has no energy gap in the long wave length limit. In this section, we shall show that associating with the spontaneous breaking of SU(4) symmetry described in above section there may appear six N–G bosons which correspond to basic collective modes of spin and isospin excitations in the nonrelativistic infinite nuclear system.

To do this, let us first derive several W–T relations by means of the basic W–T relations (2.11) adn (2.12). Operating both sides of Eqs. (2.11) and (2.12) with $-i\delta / \delta J_{\lambda\mu}(x)$, respectively, and putting $J = \eta = 0$, we obtain

$$
\begin{aligned}
< [Q_{\lambda\mu}(x),Q_{1k}(x)] >_h &= -i\int d^4y < [h_\sigma Q_{30}(y) + h_\tau Q_{03}(y) \\
&+ h_{\sigma\tau} Q_{33}(y),Q_{1k}(y)]Q_{\lambda\mu}(x) >_h \quad , \qquad (3.1)
\end{aligned}
$$

and

$$
\begin{aligned}
&< [\sigma_i(x),\sigma_\lambda(x)][\tau_j(x),\tau_\mu(x)] >_h \\
&= -ih_{\sigma\tau}\int d^4y < [\sigma_3(y),\sigma_i(y)][\tau_3(y),\tau_i(y)]Q_{\lambda\mu}(x) >_h \\
&- i\int d^4y < [h_\sigma(y)Q_{30}(y) + h_{\sigma\tau} Q_{33}(y),\sigma_i(y)][Q_{\lambda\mu}(x),\tau_j(x)] >_h \\
&- i\int d^4y < [h_\tau(y)Q_{03}(y) + h_{\sigma\tau} Q_{33}(y),\tau_j(y)][Q_{\lambda\mu}(x),\sigma_i(x)] >_h \\
&+ \int d^4y d^4z < [h_\sigma(y)Q_{30}(y) + h_{\sigma\tau} Q_{33}(y),\sigma_i(y)][h_\tau(y)Q_{03}(z) \\
&+ h_{\sigma\tau} Q_{33}(z)\tau_j(z)][Q_{\lambda\mu}(x)] >_h \quad . \qquad (3.2)
\end{aligned}
$$

We now respectively consider the symmetry–breaking effects in Eq.(2.3) and write some detail relations from Eqs.(3.1) and (3.2).

(1) For the case of presence of symmetry–breaking term $h_\sigma Q_{30}(y)$. By setting $h_\tau = h_{\sigma\tau} = 0$, and putting $Q_{1k} = Q_{-0}$, $Q_{\lambda\mu} = Q_{+0}$ in Eq.(3.1), we get

$$< \sigma_3(x) >_h = 2ih_\sigma \int d^4 y < \sigma_+(y), \sigma_-(x) >_h , \tag{3.3}$$

where $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$, and the notations $Q_{10} = \sigma_1 \times 1$ given in Sec.2 have been used.

(2). For the case of presence of symmetry-breaking term $h_\tau Q_{03}(y)$. By setting $h_\sigma = h_{\sigma\tau} = 0$ and $Q_{1k} = Q_{0-}, Q_{1\mu} = Q_{0+}$ in Eq.(3.1), we get

$$< \tau_3(x) >_h = 2ih_\tau \int d_4 y < \tau_+(y), \tau_-(x) >_h , \tag{3.4}$$

where $\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$and notations $Q_{0k} = 1 \times \tau_k$ in Sec.2 have been used.

(3). For the case of presence of symmetry-breaking term $h_{\sigma\tau}Q_{33}(y)$ and $h_\sigma = h_\tau = 0$. Respectively putting $Q_{1k} = Q_{-3}, Q_{1\mu} = Q_{+0}$; $Q_{1k} = Q_{3-}$, $Q_{1\mu} = Q_{0+}$; $Q_{1k} = Q_{-0}$, $Q_{1\mu} = Q_{+3}$; $Q_{1k} = Q_{0-}$, $Q_{1\mu} = Q_{3+}$in Eq.(3.1), we have

$$< Q_{33}(x) >_h = 2ih_{\sigma\tau} \int d^4 y < \sigma_+(y), \sigma_-(x) >_h , \tag{3.5}$$

$$< Q_{33}(x) >_h = 2ih_{\sigma\tau} \int d^4 y < \tau_+(y), \tau_-(x) >_h , \tag{3.6}$$

$$< Q_{33}(x) >_h = 2ih_{\sigma\tau} \int d^4 y < Q_{+3}(y), Q_{-3}(x) >_h , \tag{3.7}$$

$$< Q_{33}(x) >_h = 2ih_{\sigma\tau} \int d^4 y < Q_{3+}(y), Q_{3-}(x) >_h . \tag{3.8}$$

By putting $\sigma_i = \sigma_+$, $\tau_j = \tau_+$ and $\sigma_\lambda = \sigma_-$, $\tau_\mu = \tau_-$ in Eq.(3.2), we have

$$\begin{aligned} < Q_{33}(x) >_h = & -4ih_{\sigma\tau} \int d^4 y < Q_{++}(y), Q_{--}(x) >_h \\ & + 2ih_{\sigma\tau} \int d^4 y < Q_{+3}(y), Q_{-3}(x) >_h \\ & + 2ih_{\sigma\tau} \int d^4 y < Q_{3+}(y), Q_{3-}(x) >_h \\ & + 4h^2_{\sigma\tau} \int d^4 y \int d^4 z < Q_{+3}(y), Q_{3+}(z), Q_{--}(x) >_h . \end{aligned} \tag{3.9}$$

If putting $\sigma_i = \sigma_+$, $\tau_j = \tau_-$ and $\sigma_\lambda = \sigma_-$, $\tau_\mu = \tau_+$ in Eq. (3.2), we get

$$\begin{aligned} < Q_{33}(x) >_h = & -4ih_{\sigma\tau} \int d^4 y < Q_{+-}(y), Q_{-+}(x) >_h \\ & + 2ih_{\sigma\tau} \int d^4 y < Q_{+3}(y), Q_{-3}(x) >_h \\ & + 2ih_{\sigma\tau} \int d^4 y < Q_{3-}(y), Q_{3+}(x) >_h \\ & + 4h^2_{\sigma\tau} \int d^4 y \int d^4 z < Q_{+3}(y), Q_{3-}(z), Q_{-+}(x) >_h . \end{aligned} \tag{3.10}$$

We now prove the existence of the collective modes of spin and isospin excitations by means of the W–T relations(3.3)–(3.10). In fact, Eq.(3.3) has the same form as that one met in the situation of ferromagnet[6]where the itinerant–electron ferromagentism is specified by $< \sigma_3(x) >$. As a result of such spontaneous polarization of the fermion's spin, there exsits a pole with energy–gapless in the transve–spin susceptibility. The pole term is known to represent a kind of N–G mode $B_\sigma(q)$ which is called spin wave. Obviously, same conclusion is suitable for the case(1) meantioned above in a nonrelativistic infinite nuclear system. Similarly, Eq.(3.4) means that the existence of the polarization state $< \tau_3(x) >$ of the considering system leads the appearance of another kind of N–G mode $B_\tau(q)$ which may be called as isospin–wave.

Now we use the W–T relatios (3.5)–(3.10) to prove the existence of N–G modes and to find all collective modes in the case of the appearance of polarization state $< Q_{33}(x) >$ in the system. The discussion is more clear in the momentum space. For this purpose, we define the propagators as

$$< \sigma_+(x), \sigma_-(y) >_h = \frac{i}{(2\pi)^4} \int d^4 y e^{-iq(x-y)} \Delta_{B(-,0)}(q) , \tag{3.11}$$

$$< \tau_+(x), \tau_-(y) >_h = \frac{i}{(2\pi)^4} \int d^4 y e^{-iq(x-y)} \Delta_{B(0,-)}(q) , \tag{3.12}$$

$$< Q_{+3}(x), Q_{-3}(y) >_h = \frac{i}{(2\pi)^4} \int d^4 y e^{-iq(x-y)} \Delta_{B(-,3)}(q) , \tag{3.13}$$

$$< Q_{3+}(x), \sigma_{3-}(y) >_h = \frac{i}{(2\pi)^4} \int d^4 y e^{-iq(x-y)} \Delta_{B(3,-)}(q) , \tag{3.14}$$

$$< Q_{++}(x), Q_{--}(y) >_h = \frac{i}{(2\pi)^4} \int d^4 y e^{-iq(x-y)} \Delta_{B(-,-)}(q) , \tag{3.15}$$

$$< Q_{+-}(x), \sigma_{-+}(y) >_h = \frac{i}{(2\pi)^4} \int d^4 y e^{-iq(x-y)} \Delta_{B(-,+)}(q) , \tag{3.16}$$

and the three point vertex functions as

$$\begin{aligned} & < Q_{--}(x), Q_{+3}(y), Q_{3+}(z) > \\ & = [\frac{i}{(2\pi)^4}]^2 \int d^4 q_1 d^4 q_2 d^4 q_3 \delta(q_1 - q_2 - q_3) e^{-iq_1 x + iq_2 y + iq_3 z} \\ & \times \Delta_{B(--)}(q_1) \Gamma^{(3)}_{\Delta}(q_1, q_2, q_3) \Delta_{B(+3)}(q_2) \Delta_{B(3,+)}(q_3) , \end{aligned} \tag{3.17}$$

and similar definition for $< Q_{-+}(x), Q_{+3}(y), Q_{3-}(z) >$. Where $q = (q_0, \vec{q})$, $\Delta_{B(i,j)}$ are

the propagators defined in momentum space, which are written in the spectrum representation as, for instanee,

$$\Delta_{B(-,3)}(q) = \frac{<0|Q_{+3}(0)|B_{(-,3)}(\bar{q})> <B_{(-,3)}(\bar{q})|Q_{-3}(0)|0>}{q_0 - [\omega_{B(-,3)}(\bar{q}^2) + 2h_{\sigma\tau}] + i\epsilon}$$
$$- \frac{<0|Q_{-3}(0)|B_{(+,3)}(\bar{q})> <B_{(+,3)}(\bar{q})|Q_{+3}(0)|0>}{q_0 + [\omega_{B(+,3)}(\bar{q}^2) - 2h_{\sigma\tau}] + i\epsilon}$$
$$+ \sum continum\ terms\ . \tag{3.18}$$

Here the pole terms containing the $1/h$-singularity have been picked up, in which $B_{(\pm,3)}(q)$ is the spin-isospin excitation states with energy $\omega_{B(\pm,3)}(q^2)$ and momentum q and the quantum number S_3 spin ± 1 and T_3 isospin being the longitudinal fluctuation. Using the definitions(3.11)-(3.17), Eqs. (3.5)-(3.10) can be respectively expressed as

$$M_h = -2h_{\sigma\tau}\Delta_{B(-,0)}(0)\ , \tag{3.19}$$
$$M_h = -2h_{\sigma\tau}\Delta_{B(0,-)}(0)\ , \tag{3.20}$$
$$M_h = -2h_{\sigma\tau}\Delta_{B(-,3)}(0)\ , \tag{3.21}$$
$$M_h = -2h_{\sigma\tau}\Delta_{B(3,-)}(0)\ , \tag{3.22}$$
$$M_h = +4h_{\sigma\tau}\Delta_{B(-,-)}(0) - 2h_{\sigma\tau}\Delta_{B(3,-)}(0) - 2h_{\sigma\tau}\Delta_{B(-,3)}(0)$$
$$- 4h^2_{\sigma\tau}\Delta_{B(-,-)}(0)\Gamma^{(3)}_{\Delta}(0,0,0)\Delta_{B(-,3)}(0)\Delta_{B(3,-)}(0)\ , \tag{3.23}$$
$$M_h = +4h_{\sigma\tau}\Delta_{B(-,+)}(0) - 2h_{\sigma\tau}\Delta_{B(-,3)}(0) - 2h_{\sigma\tau}\Delta_{B(3,-)}(0)$$
$$- 4h^2_{\sigma\tau}\Delta_{B(-,+)}(0)\Gamma^{(3)}_{\Delta}(0,0,0)\Delta_{B(-,3)}(0)\Delta_{B(3,-)}(0)\ . \tag{3.24}$$

Where $M_h = <Q_{33}(x)>_h$. Now let us show the existence of N-G bosons from Eq.(3.2) as an example. Substituting Eq.(3.18) into Eq.(3.21) and then taking the limit $h_{\sigma\tau} \to 0$, we find that

$$M_{\sigma\tau} = <Q_{33}(x)> = \lim_{h\to 0}(-2h_{\sigma\tau}\Delta_{B(-,3)}(0) \neq 0\ , \tag{3.25}$$

if only

$$\omega_{B(\pm,3)}(\bar{q}^2) = 0 \quad \text{at } q = 0.$$

In that case, we have

$$M_{\sigma\tau} = |<B_{(-,3)}(0)|Q_{(-,3)}(0)>|^2 - |<B_{(+,3)}(0)Q_{+,3}(0)>|^2\ . \tag{3.26}$$

Eq.(3.25) implies the existence of the bound states(collective states) with vanishing energies for zero momentum. This is just the statement of Goldstone's theorem. That is, these collective states are N-G bosons, which are characterized by transve-spin polarization and longitudinal isospin vibration. Without loss generality we can choose $M_{\sigma\tau} > 0$ which assumes that $<B_{(+,3)}(0)|Q_{(+,3)}(0)> = 0$. It means that $B_{(+,3)}(q)$ mode does not appear and there exists only the mode $B_{(-,3)}(q)$ which may be called as isovector-spin wave.

The discussion is similar for other expressions in Eqs.(3.19)-(3.24), from which we can prove the existence of other kinds of N-G modes, the collective modes of spin-isospin excitations. The results are described as follows. Eq.(3.19) or (3.5) shows the existence of the N-G mode $B_{(-,0)}(q)$ with S_3 spin-1 and T_3 isospin 0. That is, quantum $B_{(-,0)}(q)$ is specified by transverse-spin polarization, which may be called as isoscalar-spin wave which, of coures, is same as the spin wave appeared in the existence of polarization state $<\sigma_3(x)>$. Eq.(3.20) or (3.6) shows the existence of the N-G mode $B_{(0,-)}(q)$ with S_3 spin 0 and isospin-1. The quantum $B_{(0,-)}(q)$ is specified by transverse-isospin polarization which may be called as scalar -isospin wave which, of course, is same as the isospin wave appeared in the existence of polarization state $<\tau_3(x)>$. Eq.(3.22) or (3.8) implies the existence of the N-G mode $B_{(3,-)}$with T_3 spin-1 and S_3 spin being longitudinal vibration. Thus the quantum $B_{(3,-)}(q)$ is characterized by longitudinal isospin vibration and transverse-spin polarization, which may be called as vector -isospin wave. Eqs.(3.23) and (3.24) can be simplified in terms of Eqs.(3.21)-(3.22) and the relation $\Gamma^3_{\Delta}(0,0,0) = 0$ sicne the low-energy theorem[8] that the reactions among N-G modes vanish at the zero-energy limit. Thus, we have from Eqs.(3.23) and (3.24)

$$M_h = -4h_{\sigma\tau}\Delta_{B(-,-)}(0)\ , \tag{3.27}$$
$$M_h = -4h_{\sigma\tau}\Delta_{B(-,+)}(0)\ . \tag{3.28}$$

By means of Eqs.(3.27) and (3.28), it then is easy to prove that there exist the N-G modes $B_{(-,-)}(q)$ and $B_{(-,+)}(q)$ with the quantum numbers (S_3, T_3) being $(-1,-1)$ and $(-1,+1)$, respectively. That is, the states $|B_{(-,-)}(q)>$ and $|B_{(-,+)}(q)>$ are characterized both by transverse-spin and transverse-isospin polarizations except that their isospin polarization directions are opposite. Therefore, we may call quanta $B_{(-,-)}(q)$ and $B_{(-,+)}(q)$ as transverse spin-isospin waves.

Summarizing above discussion, we find that there exist six N-G modes $B_{(-,0)}(q)$, $B_{(0,-)}(q)$, $B_{(-,3)}(q)$, $B_{(3,-)}(q)$, $B_{(-,-)}(q)$, and $B_{(-,+)}(q)$ in the case 3 if we choose $M_{\sigma\tau} > 0$, which may be regarded as different channel of the common N-G boson, the $\sigma\tau$-boson or $\sigma\tau$-wave. Totally, there may appear six N-G modes, the collective modes of spin and isospin excitation in the system under consideration.

The results are listed in table 1. These N–G modes may be identified to well-known low energy spin, isospin and spin–isospin collective modes in nuclei. The spin wave $B_\sigma(q)$ or isoscalar–spin wave $B_{(-,0)}(q)$ are the collective modes in isoscaler M1 transition, and the isovector–spin wave $B_{(-,3)}(q)$ is that in isovector M1 transition. Isospin wave $B_\tau(q)$ or scalar–isospin wave $B_{(0,-)}(q)$ are the collective modes in the isobaric analog transition. Transverse spin–isospin waves $B_{(-,-)}(q)$ and $B_{(-,+)}(q)$, and the vector–isospin wave $B_{(3,-)}(q)$ correspond to Gamow–Teller modes which are some of the strongest excitations in charge–exchange reactions (p,n),(n,p) etc, and in inelastic scattering, (p,p′), (e,e′) etc.

4. Vertices Relations and Nucleon–($\sigma\tau$)Boson Interactions

In Sec. 3 we have found the collective modes (the N–G modes) of spin and isospin excitations in the system under consideration. To clarify the properties of these N–G bosons and their effects in the system, in this section we study the W–T relations among multi-point vertex functions in which the nucleon and N–G bosons interact with each other. These W–T relations are also important to the calculations of the propagators (two-point Green's functions) and other physical quanta such as susceptibility. Whenever making an consisitent approximation expansion, we must be careful not to violate the W–T relations, the requirements of symmetry of theory.

At first we derive the W–T relations among propagators and vertex functions, which is performed by taking the second order derivatives of the basic W–T relations (2.11) and (2.12). Operating both sides of Eqs.(2.11) and (2.12) with $\delta/\delta\eta(y)\delta/\delta\eta(x)$, and then putting $\eta = J = h = 0$, we get

$$
\begin{aligned}
&< Q_{ik}\psi(x)\psi^+(y) > - < \psi(x)\psi^+(y)Q_{ik} > \\
&\quad = \lim_{h\to 0} - i\int d^4z < [h_\sigma Q_{30}(z) + h_\tau Q_{03}(z) \\
&\quad + h_{\sigma\tau} Q_{33}(z), Q_{ik}(z)]\psi(x)\psi^+(y) >_h ,
\end{aligned}
\tag{4.1}
$$

and

$$
\begin{aligned}
&< \sigma_i\psi(x)\psi^+(y)\tau_j > - < \tau_j\psi(x)\psi^+(y)\sigma_i > \\
&- < \sigma_i\tau_j\psi(x)\psi^+(y) > - < \psi(x)\psi^+(y)\sigma_i\tau_j > \\
&= \lim_{h\to 0} \{ih_{\sigma\tau}\int d^4z < \psi(x)\psi^+(y)[Q_{30}(z),\sigma_i(z)][Q_{03}(z),\tau_j(z)] >_h \\
&+ i\int d^4z < (\tau_j\psi(x)\psi^+(y) - \psi(x)\psi^+(y)\tau_j) \\
&\times [h_\sigma Q_{30}(z) + h_\tau Q_{03}(z) + h_{\sigma\tau} Q_{33}(z), \sigma_i(z)]
\end{aligned}
$$

$$
\begin{aligned}
&+ (\sigma_i\psi(x)\psi^+(y) - \psi(x)\psi^+(y)\sigma_i) \\
&\times [h_\sigma Q_{30}(z) + h_\tau Q_{03}(z) + h_{\sigma\tau} Q_{33}(z), \tau_j(z)] >_h \\
&- \int d^4z\int d^4\zeta < \psi(x)\psi^+(y)[h_\sigma Q_{30}(z) + h_\tau Q_{03}(z) + h_{\sigma\tau} Q_{33}(z), \sigma_i(z)] \\
&\times [h_\sigma Q_{30}(\zeta) + h_\tau Q_{03}(\zeta) + h_{\sigma\tau} Q_{33}(\zeta), \tau_j(\zeta)] >_h ;
\end{aligned}
\tag{4.2}
$$

From W–T relations (4.1) and (4.2), we can write several relations among nucleon propagators and vertex functions of ($\sigma\tau$)–bosons and nucleons with different spin and isospin, which are listed as follows.

(1). For the system where $h_\sigma \neq 0$, $h_\tau = h_{\sigma\tau} = 0$, we have from Eq.(4.1)

$$
\begin{aligned}
&< \psi_{\downarrow\lambda}(x)\psi^+_{\downarrow\lambda}(y) > - < \psi_{\uparrow\lambda}(x)\psi^+_{\uparrow\lambda}(y) > \\
&\quad = \lim_{h\to 0} - 2ih_\sigma\int d^4z < \sigma_+(z)\psi_{\uparrow\lambda}(x)\psi^+_{\downarrow\lambda}(y) >_h ,
\end{aligned}
\tag{4.3}
$$

where index λ denotes isospin state.

(2) For $h_\tau \neq 0$, but $h_\sigma = h_{\sigma\tau} = 0$, we have from Eq.(4.1)

$$
\begin{aligned}
&< \psi_{\mu\downarrow}(x)\psi^+_{\mu\downarrow}(y) > - < \psi_{\mu\uparrow}(x)\psi^+_{\mu\uparrow}(y) > \\
&\quad = \lim_{h\to 0} - 2ih_\tau\int d^4z < \tau_+(z)\psi_{\mu\uparrow}(x)\psi^+_{\mu\downarrow}(y) >_h ,
\end{aligned}
\tag{4.4}
$$

where index μ denotes spin state.

(3). For $h_{\sigma\tau} \neq 0$ but $h_\sigma = h_\tau = 0$, we have from Eq.(4.1) and (4.2)

$$
\begin{aligned}
&< \psi_{\downarrow\uparrow}(x)\psi^+_{\downarrow\uparrow}(y) > - < \psi_{\uparrow\uparrow}(x)\psi^+_{\uparrow\uparrow}(y) > \\
&\quad = \lim_{h\to 0} - 2ih_{\sigma\tau}\int d^4z < Q_{+0}(z)\psi_{\uparrow\uparrow}(x)\psi^+_{\downarrow\uparrow}(y) >_h ,
\end{aligned}
\tag{4.5}
$$

$$
\begin{aligned}
&< \psi_{\uparrow\downarrow}(x)\psi^+_{\uparrow\downarrow}(y) > - < \psi_{\uparrow\uparrow}(x)\psi^+_{\uparrow\uparrow}(y) > \\
&\quad = \lim_{h\to 0} - 2ih_{\sigma\tau}\int d^4z < Q_{0+}(z)\psi_{\uparrow\uparrow}(x)\psi^+_{\uparrow\downarrow}(y) >_h ,
\end{aligned}
\tag{4.6}
$$

$$
\begin{aligned}
&< \psi_{\downarrow\lambda}(x)\psi^+_{\downarrow\lambda}(y) > - < \psi_{\uparrow\lambda}(x)\psi^+_{\uparrow\lambda}(y) > \\
&\quad = \lim_{h\to 0} - 2ih_{\sigma\tau}\int d^4z < Q_{+3}(z)\psi_{\uparrow\lambda}(x)\psi^+_{\downarrow\lambda}(y) >_h ,
\end{aligned}
\tag{4.7}
$$

$$
\begin{aligned}
&< \psi_{\mu\downarrow}(x)\psi^+_{\mu\downarrow}(y) > - < \psi_{\mu\uparrow}(x)\psi^+_{\mu\uparrow}(y) > \\
&\quad = \lim_{h\to 0} - 2ih_\tau\int d^4z < Q_{3+}(z)\psi_{\mu\uparrow}(x)\psi^+_{\mu\downarrow}(y) >_h ,
\end{aligned}
\tag{4.8}
$$

and

$$< \psi_{\uparrow\uparrow}(x)\psi^{+}_{\uparrow\uparrow}(y) > + < \psi_{\downarrow\downarrow}(x)\psi^{+}_{\downarrow\downarrow}(y) >$$
$$- < \psi_{\downarrow\uparrow}(x)\psi^{+}_{\downarrow\uparrow}(y) > - < \psi_{\uparrow\downarrow}(x)\psi^{+}_{\uparrow\downarrow}(y) >$$
$$= \lim_{\lambda\to 0}\{4ih_{\sigma\tau}\int d^4z < Q_{++}(z)\psi_{\uparrow\uparrow}(x)\psi^{+}_{\downarrow\downarrow}(y) >_{\lambda}$$
$$- 4h^2_{\sigma\tau}\int d^4z d^4\zeta < Q_{+3}(z)Q_{3+}(\zeta)\psi_{\uparrow\uparrow}(x)\psi^{+}_{\downarrow\downarrow}(y) >_{\lambda}\}, \tag{4.9}$$

$$< \psi_{\uparrow\uparrow}(x)\psi^{+}_{\uparrow\uparrow}(y) > + < \psi_{\downarrow\downarrow}(x)\psi^{+}_{\downarrow\downarrow}(y) >$$
$$- < \psi_{\downarrow\uparrow}(x)\psi^{+}_{\downarrow\uparrow}(y) > - < \psi_{\uparrow\downarrow}(x)\psi^{+}_{\uparrow\downarrow}(y) >$$
$$= \lim_{\lambda\to 0}\{4ih_{\sigma\tau}\int d^4z < Q_{+-}(z)\psi_{\uparrow\downarrow}(x)\psi^{+}_{\downarrow\uparrow}(y) >_{\lambda}$$
$$- 4h^2_{\sigma\tau}\int d^4z d^4\zeta < Q_{+3}(z)\psi_{\uparrow\downarrow}(x)\psi^{+}_{\downarrow\uparrow}(y)Q^{+}_{3-}(\zeta) >_{\lambda}\}. \tag{4.10}$$

And Eqs. (4.9) (4.10) satisfy

$$< \psi_{\uparrow\uparrow}(x)\psi^{+}_{\uparrow\uparrow}(y) > = < \psi_{\downarrow\downarrow}(x)\psi^{+}_{\downarrow\downarrow}(y) > ,$$
$$< \psi_{\downarrow\uparrow}(x)\psi^{+}_{\downarrow\uparrow}(y) > = < \psi_{\uparrow\downarrow}(x)\psi^{+}_{\uparrow\downarrow}(y) > . \tag{4.11}$$

Next, let us derive the W−T relations among many−point vertex functions which is relative to third order derivatives. Operating $\delta / \delta\eta(y)\, \delta / \delta\eta^{+}(x)\delta / \delta J_{\mu\lambda}(z)$ both sides of Eqs.(2.11) and (2.12), and then putting $\eta = J = h = 0$, we obtain following relations:

(1)When $h_{\sigma} \neq 0$ but $h_{\tau} = h_{\sigma\tau} = 0$, we have

$$< (\sigma_{+}\psi(x)\psi^{+}(y) - \psi(x)\psi^{+}(y)\sigma_{+})\sigma_{-}(z) > - < \psi(x)\psi^{+}(y)\sigma_{3}(z) >$$
$$= \lim_{\lambda\to 0} - 2ih_{\sigma}\int d^4\zeta < \sigma_{+}(\zeta)\psi(x)\psi^{+}(y)\sigma_{-}(z) >_{\lambda} . \tag{4.12}$$

(2)When $h_{\tau} \neq 0$ but $h_{\sigma} = h_{\sigma\tau} = 0$, we have

$$< (\tau_{+}\psi(x)\psi^{+}(y) - \psi(x)\psi^{+}(y)\tau_{+})\tau_{-}(z) > - < \psi(x)\psi^{+}(y)\tau_{3}(z) >$$
$$= \lim_{\lambda\to 0} - 2ih_{\tau}\int d^4\zeta < \tau_{+}(\zeta)\psi(x)\psi^{+}(y)\tau_{-}(z) >_{\lambda} . \tag{4.13}$$

(3)When $h_{\sigma\tau} \neq 0$ but $h_{\sigma} = h_{\tau} = 0$, we have

$$< (Q_{+3}\psi(x)\psi^{+}(y) - \psi(x)\psi^{+}(y)Q_{+3})Q_{-0}(z) > - < \psi(x)\psi^{+}(y)Q_{33}(z) >$$
$$= \lim_{\lambda\to 0} - 2ih_{\sigma\tau}\int d^4\zeta < Q_{+0}(\zeta)\psi(x)\psi^{+}(y)Q_{-0}(z) >_{\lambda} , \tag{4.14}$$

$$< (Q_{3+}\psi(x)\psi^{+}(y) - \psi(x)\psi^{+}(y)Q_{3+})Q_{0-}(z) > - < \psi(x)\psi^{+}(y)Q_{33}(z) >$$
$$= \lim_{\lambda\to 0} - 2ih_{\sigma\tau}\int d^4\zeta < Q_{0+}(\zeta)\psi(x)\psi^{+}(y)Q_{0-}(z) >_{\lambda} , \tag{4.15}$$

$$< (Q_{+0}\psi(x)\psi^{+}(y) - \psi(x)\psi^{+}(y)Q_{+0})Q_{-3}(z) > - < \psi(x)\psi^{+}(y)Q_{33}(z) >$$
$$= \lim_{\lambda\to 0} - 2ih_{\sigma\tau}\int d^4\zeta < Q_{+3}(\zeta)\psi(x)\psi^{+}(y)Q_{-3}(z) >_{\lambda} , \tag{4.16}$$

$$< (Q_{0+}\psi(x)\psi^{+}(y) - \psi(x)\psi^{+}(y)Q_{0+})Q_{3-}(z) > - < \psi(x)\psi^{+}(y)Q_{33}(z) >$$
$$= \lim_{\lambda\to 0} - 2ih_{\sigma\tau}\int d^4\zeta < Q_{3+}(\zeta)\psi(x)\psi^{+}(y)Q_{3-}(z) >_{\lambda} , \tag{4.17}$$

and

$$< (Q_{++}\psi(x)\psi^{+}(y) + \psi(x)\psi^{+}(y)Q_{++})Q_{--}(z) >$$
$$- < (Q_{+0}\psi(x)\psi^{+}(y)Q_{0+} + Q_{0+}\psi(x)\psi^{+}(y)Q_{+0})Q_{--}(z) >$$
$$- < \psi(x)\psi^{+}(y)Q_{33}(z) >$$
$$= \lim_{\lambda\to 0}\{ - 4ih_{\sigma\tau}\int d^4\zeta < Q_{++}(\zeta)\psi(x)\psi^{+}(y)Q_{--}(z) >_{\lambda}$$
$$- 2ih_{\sigma\tau}\int d^4\zeta < Q_{+3}(\zeta)(Q_{0+}\psi(x)\psi^{+}(y) - \psi(x)\psi^{+}(y)Q_{0+})Q_{--}(z) >_{\lambda}$$
$$- 2ih_{\sigma\tau}\int d^4\zeta < Q_{3+}(\zeta)(Q_{+0}\psi(x)\psi^{+}(y) - \psi(x)\psi^{+}(y)Q_{+0})Q_{--}(z) >_{\lambda}$$
$$+ 4h^2\int d^4\xi d^4\zeta < Q_{+3}(\zeta)Q_{3+}(\xi)\psi(x)\psi^{+}(y)Q_{--}(z) >_{\lambda} . \tag{4.18}$$

Using the same procedure as that in the second derivatives, from the W−T relations (4.12)−4.18) we can derive a series of relations among many−point vertex functions for $(\sigma\tau)$−bosons and nucleons with different spin and isospin. Here we only write somes as examples to explain the physical meaning implied from these W−T relations. We have from Eq. (4.16)

$$< \psi_{\downarrow\lambda}(x)\psi^{+}_{\uparrow\lambda}(y)Q_{-3}(z) > - < \psi_{\uparrow\lambda}(x)\psi^{+}_{\uparrow\lambda}(y)Q_{33}(z) >$$
$$= \lim_{\lambda\to 0} - 2ih_{\sigma\tau}\int d^4\zeta < Q_{+3}(\zeta)\psi_{\uparrow\lambda}(x)\psi^{+}_{\uparrow\lambda}(y)Q_{-3}(z) >_{\lambda} , \tag{4.19}$$

and from Eq.(4.18)

$$< \psi_{\downarrow\downarrow}(x)\psi^{+}_{\uparrow\uparrow}(y)Q_{--} > - < \psi_{\uparrow\uparrow}(x)\psi^{+}_{\uparrow\uparrow}(y)Q_{33}(z) >$$

$$= \lim_{\lambda \to 0} \{ -4ih_{\sigma\tau} \int d^4\zeta < Q_{++}(\zeta)\psi_{\uparrow\uparrow}(x)\psi^+_{\uparrow\uparrow}(y)Q_{--}(z) >_\lambda$$

$$- 2ih_{\sigma\tau} \int d^4\zeta < Q_{+3}(\zeta)\psi_{\uparrow\downarrow}(x)\psi^+_{\uparrow\uparrow}(y)Q_{--}(z) >_\lambda$$

$$- 2ih_{\sigma\tau} \int d^4\zeta < Q_{3+}(\zeta)\psi_{\downarrow\uparrow}(x)\psi^+_{\uparrow\uparrow}(y)Q_{--}(z) >_\lambda$$

$$+ 4h^2_{\sigma\tau} \int d^4\zeta d^4\xi Q_{+3}(\zeta), Q_{3+}(\xi)\psi_{\uparrow\uparrow}(x)\psi^+_{\uparrow\uparrow}(y)Q_{--}(z) >_\lambda \quad . \qquad (4.20)$$

The first and second terms of the l.h.s. of Eqs. (4.19)–(4.20) express the processes of emission or absorption of the $(\sigma\tau)$–bosons and the longitudinal spin–isospin fluctuation, respectively. The integral parts in Eqs.(4.19)-(4.20) are the vertex functions for the nucleon–$(\sigma\tau)$ bosons scattering processes. Thus, Eqs.(4.19)–(4.20) and hence the W–T relations (4.14)–(4.18) show following processes: If there appears the longitudinal spin–isospin polarization in a nuclear system, this longitudinal fluctuation will be absorbed due to nucleons, and so form $(\sigma\tau)$–bosons with different channels. The $(\sigma\tau)$–bosons may be emited also due to nucleons, and then appear the processes of nucleon–$(\sigma\tau)$boson scattering and that of absorption of the $(\sigma\tau)$–bosons due to other nucleons. The later will again produce the longitudinal spin–isospin fluctuation. This is the qualitative picture for the appearance and the propagation of spin–isospin waves (or bosons), the collective excitations of spin–isospin, in the nuclear matter. The W–T relations (4.12) and (4.13) show the similar processes if only instead of spin–isospin fluctuation by spin and isospin fluctuations, respectivly.

The physical meaning of above W–T relations will be more clear when the discussion is in the momentum space. For this purpose and also for the convenience of later discussion, we now transform the W–T relations(4.3)–(4.20) to momentum space. To do this, we introduce the momentum representation of the nucleon propagators as

$$< \psi_{\mu\lambda}(x)\psi^+_{\mu\lambda}(y) > = \frac{i}{(2\pi)^4} \int d^4p e^{-ip(x-y)} S_{\mu\lambda}(p) \ , \qquad (4.21)$$

and the momentum representation of three–point vertex functions as

$$< Q_{+3}(z)\psi_{\uparrow\lambda}(x)\psi^+_{\downarrow\lambda}(y) >$$

$$= [\frac{i}{(2\pi)^4}]^2 \int d^4p d^4q e^{-i(p+q)x + ipy + iqz} [S_{\uparrow\lambda}(p+q)S_{\downarrow\lambda}(p)$$

$$+ S_{\uparrow\lambda}(p+q)\Gamma_{(-,3)}(p+q;p,q)S_{\downarrow\lambda}(p)\Delta_{B(-,3)}(q)] \ , \qquad (4.22)$$

$$< Q_{++}(z)\psi_{\uparrow\uparrow}(x)\psi^+_{\downarrow\downarrow}(y) >$$

$$= [\frac{i}{(2\pi)^4}]^2 \int d^4p d^4q e^{-i(p+q)x + ipy + iqz} [S_{\uparrow\uparrow}(p+q)S_{\downarrow\downarrow}(p)$$

$$+ S_{\uparrow\uparrow}(p+q)\Gamma_{(-,-)}(p+q;p,q)S_{\downarrow\downarrow}(p)\Delta_{B(-,-)}(q)] \ . \qquad (4.23)$$

Where spin (isospin) index $\mu(\lambda) = \uparrow, \downarrow$. $\Gamma^{(3)}_{(-,3)}$, and $\Gamma^{(3)}_{(-,-)}$ are the three point vertex functions in momentum space in which the subscripts (–,3), (–,–), etc label $(\sigma\tau)$–boson states of verter functions. The four–point Green's functions are expessed as

$$< Q_{+3}(z)Q_{3+}(\zeta)\psi_{\uparrow\uparrow}(x)\psi^+_{\downarrow\downarrow}(y) >$$

$$= < \psi_{\uparrow\uparrow}(x)\psi^+_{\uparrow\uparrow}(\zeta) > < \psi_{\uparrow\downarrow}(\zeta)\psi^+_{\uparrow\downarrow}(y) > < \psi_{\downarrow\downarrow}(z)\psi^+_{\downarrow\downarrow}(y) >$$

$$+ < \psi_{\uparrow\uparrow}(x)\psi^+_{\uparrow\uparrow}(z) > < \psi_{\downarrow\uparrow}(z)\psi^+_{\downarrow\uparrow}(\zeta) > < \psi_{\downarrow\downarrow}(\zeta)^+_{\downarrow\downarrow}(y) >$$

$$+ [\frac{i}{(2\pi)^4}]^3 \int d^4p_1 d^4p_2 d^4q_1 d^4q_2 \delta(p_1 - q_1 - p_2 - q_2)$$

$$\times e^{-ip_1x + iq_1\zeta + ip_2y + iq_2z} S_{\uparrow\uparrow}(p_1)\Delta_{B(3,-)}(q_1)$$

$$\times [\Gamma^{(4)}_{(-,3)\uparrow\uparrow,\downarrow\downarrow(3,-)}(p_1,q_1;p_2,q_2)$$

$$+ \Gamma^{(3)}_{(-,3)}(p_1;q_2,p_2-q_2)S_{\downarrow\uparrow}(p_1-q_2)$$

$$\times \Gamma^{(3)}_{(3,-)}(p_1-q_2;p_2,q_1)$$

$$+ \Gamma^{(3)}_{(3,-)}(p_1;q_1;p_1-q_1)S_{\uparrow\downarrow}(p_1-q_1)$$

$$\times \Gamma^{(3)}_{(-,3)}(p_1-q_1;p_2,q_2)]S_{\downarrow\downarrow}(p_2)\Delta_{B(-,3)}(q_2) \ . \qquad (4.24)$$

$\Gamma^{(4)}_{(-,3)\uparrow\uparrow,\downarrow\downarrow(3,-)}$ is the momentum representation of the corresponding vertex function for the nucleon–$(\sigma\tau)$ boson scattering process. The five–point Green's fuctions in momentum spance can be similarly defined. Here Green's functions were decomposed into the proper vertices. By means of the expressions (4.21)–(4.24), we rewrite the W–T relation Eqs.(4.3)–(4.10) in momentum space as follows:

(1)When $M_\sigma = < \sigma_3(x) > \neq 0$, we have

$$S^{-1}_{\downarrow\downarrow}(p) - S^{-1}_{\uparrow\downarrow}(p) = M_\sigma \Gamma^{(3)}_{(-,0)}(p,o,;p) \ . \qquad (4.25)$$

(2) When $M_\tau = \langle \tau_3(x) \rangle \neq 0$, we have

$$S^{-1}_{\downarrow\downarrow}(p) - S^{-1}_{\uparrow\uparrow}(p) = M_\tau \Gamma^{(3)}_{(0,-)}(p,o;;p) \ . \tag{4.26}$$

(3) When $M_{\sigma\tau} = \langle Q_{33}(x) \rangle \neq 0$, we have

$$S^{-1}_{\downarrow\uparrow}(p) - S^{-1}_{\uparrow\uparrow}(p) = M_{\sigma\tau} \Gamma^{(3)}_{(-,3)}(p,o;;p) \ , \tag{4.27}$$

$$S^{-1}_{\uparrow\downarrow}(p) - S^{-1}_{\uparrow\uparrow}(p) = M_{\sigma\tau} \Gamma^{(3)}_{(0,-)}(p;p,o) \ , \tag{4.28}$$

$$S^{-1}_{\downarrow\downarrow}(p) - S^{-1}_{\uparrow\downarrow}(p) = M_{\sigma\tau} \Gamma^{(3)}_{(-,3)}(p;p,o) \ , \tag{4.29}$$

$$S^{-1}_{\downarrow\downarrow}(p) - S^{-1}_{\downarrow\uparrow}(p) = M_{\sigma\tau} \Gamma^{(3)}_{(0,-)}(p;p,o) \ . \tag{4.30}$$

and

$$\begin{aligned} & S_{\uparrow\uparrow}(p) + S_{\downarrow\downarrow}(p) - S_{\downarrow\uparrow}(p) - S_{\uparrow\downarrow}(p) \\ & \quad = M_{\sigma\tau} S_{\uparrow\uparrow}(p) \Gamma^{(3)}_{(-,-)}(p;p,o) S_{\downarrow\downarrow}(p) \\ & \quad + M^2_{\sigma\tau} S_{\downarrow\uparrow}(p) \{ \Gamma^{(4)}_{(-,3)\uparrow\uparrow,\downarrow\downarrow(0,-)}(p,o;p,o) \\ & \quad + \Gamma^{(3)}_{(-,3)}(p,0;p) S_{\downarrow\uparrow}(p) \Gamma^{(3)}_{(0,-)}(p,0;p) \\ & \quad + \Gamma^{(3)}_{(0,-)}(p;0,p) S_{\uparrow\downarrow}(p) \Gamma^{(3)}_{(-,3)}(p;p,o) \} S_{\downarrow\downarrow}(p) \ . \end{aligned} \tag{4.31}$$

and Eq.(4.31) satisfies

$$S_{\uparrow\uparrow}(p) = S_{\downarrow\downarrow}(p), \qquad S_{\downarrow\uparrow}(p) = S_{\uparrow\downarrow}(p) \ . \tag{4.32}$$

Similarly, we can rewrite the W−T relations (4.12)−(4.20) in the momentum space. For instance, from Eq(4.19), we have

$$\begin{aligned} & S_{\downarrow\downarrow}(p) S_{\uparrow\downarrow}(p) \Delta^{-1}_{B(-,3)}(q) + \Gamma^{(3)}_{(-,3)}(p,q;p+q) - \Delta^{-1}_{B(-,3)}(q) \\ & \quad - \Delta_L(q) \Delta^{-1}_{B(-,3)}(q) \Gamma^{(3)}_{(l)}(p,q;p+q) \\ & = - M_{\sigma\tau} \Gamma^{(4)}_{(-,3)\uparrow\downarrow,\uparrow\downarrow(-,3)}(p,o;p+q,o) \ . \end{aligned} \tag{4.33}$$

Eq.(4.20) leads to

$$\begin{aligned} & S_{\downarrow\downarrow}(p) S^{-1}_{\uparrow\uparrow}(p) \Delta^{-1}_{B(-,-)}(q) + \Gamma^{(3)}_{(-,-)}(p,q;p+q) - \Delta^{-1}_{B(-,-)}(q) \\ & \quad - \Delta_l(q) \Delta^{-1}_{B(-,-)}(q) \Gamma^{(3)}_{(l)}(p,q;p+q) \\ & = - M_{\sigma\tau} \{ \Gamma^{(4)}_{(-,-)\uparrow\uparrow,\uparrow\uparrow(-,-)}(p,o;p+q,o) \\ & + S^{-1}_{\uparrow\uparrow}(p) S_{\uparrow\downarrow}(p) \Gamma^{(4)}_{(-,3)\uparrow\downarrow,\uparrow\uparrow(-,-)}(p,q;p+q,o) \\ & + S^{-1}_{\uparrow\uparrow}(p) S_{\downarrow\uparrow}(p) \Gamma^{(4)}_{(0,-)\downarrow\uparrow,\uparrow\uparrow(-,-)}(p,q;p+q,o) \} \\ & - M^2_{\sigma\tau} \Gamma^{(5)} p,q,o;o,p+q) \ . \end{aligned} \tag{4.34}$$

Where $\Gamma^{(4)}_{(-,-)\uparrow\uparrow,\uparrow\uparrow(-,-)}$, etc are the vertex functions for the scattering processes of nucleon−$(\sigma\tau)$ boson with different states, $\Gamma^{(5)}$ is five−point vertex function, Δ_l is the propagator of the longitudinal spin−isospin fluctuation, and $\Gamma^{(3)}_l$ is the vertex function for the process of emission or absorption of the longitudinal spin−isospin fluctuation due to nucleons.

We may use Feynman diagrams to describe these W−T relations. The nucleon propagators of Eq.(4.21) and the $(\sigma\tau)$−boson propagators of Eqs.(3.11)−(3.16) are given in Figs.1(a) and 1(b), respectively. The W−T relation (4.19) or (4.33) is illustrated in Fig.2, while Eq.(4.20) or (4.34) is shown in Fig.3.

From above discussion we can see that the W−T relations among many−point Green's functions given in this section describe the constraint relations among nucleon−$(\sigma\tau)$ boson interaction processes. In the limit of zero energy−momentum transfer, these W−T relations are much simplified since the energy−gapless excitation mode of $(\sigma\tau)$−bosons, i.e. $\Delta^{-1}_B(q)$ and $\Delta_l(q)$ vanish at $q=0$. For example, the first, third and fourth terms in the left sides of Eqs.(4.33) and (4.34) disappear in the limit $q \to 0$. These simplified W−T relations will be useful to the successive approximation calculations of the Green's functions.

5. Bound–State Condition and Susceptibility

We have already shown in section 3 that the cellective modes of spin and isospin excitations in the infinite nuclear system are N–G modes which are nucleon bound states. Let us now derive the bound–state condition which is well known to be determined by the pole singularities of the propagators. Remember that $Q_{lk}(x)=\psi^{+}(x)Q_{lk}\psi(x)$ are the Heisenberg operators for the bound states where $(lk)=(\pm,0),(0,\pm),(\pm,3),(3,\pm),(\pm,\pm)$. Using the relations

$$
\begin{aligned}
&< Q_{+0}(x)Q_{-0}(y)> = -\lim_{\epsilon\to 0} < Q_{+0}(x)\psi_{\uparrow\mu}(x)\psi^{+}_{\downarrow\mu}(x+\epsilon)>,\\
&< Q_{0+}(x)Q_{0-}(y)> = -\lim_{\epsilon\to 0} < Q_{0+}(x)\psi_{\lambda\uparrow}(x)\psi^{+}_{\lambda\downarrow}(x+\epsilon)>,\\
&< Q_{+3}(x)Q_{-3}(y)> = -\lim_{\epsilon\to 0}\sum_{\mu}\eta_{\mu} < Q_{+3}(x)\psi_{\uparrow\mu}(x)\psi^{+}_{\downarrow\mu}(x+\epsilon)>,\\
&< Q_{3+}(x)Q_{3-}(y)> = -\lim_{\epsilon\to 0}\sum_{\lambda}\eta_{\lambda} < Q_{3+}(x)\psi_{\lambda\uparrow}(x)\psi^{+}_{\lambda\downarrow}(x+\epsilon)>,\\
&< Q_{++}(x)Q_{--}(y)> = -\lim_{\epsilon\to 0} < Q_{++}(x)\psi_{\uparrow\uparrow}(x)\psi^{+}_{\downarrow\downarrow}(x+\epsilon)>,\\
&< Q_{+-}(x)Q_{-+}(y)> = -\lim_{\epsilon\to 0} < Q_{+-}(x)\psi_{\uparrow\downarrow}(x)\psi^{+}_{\downarrow\uparrow}(x+\epsilon)> .
\end{aligned}
\tag{5.1}
$$

and Eqs. (3.11)–(3.16) and (4.22)–(4.23), we obtain

$$
\begin{aligned}
\triangle_{B(-,0)}(q) = D_{\sigma}(q)[1 - \frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\mu}(p+\frac{q}{2})\\
\times \Gamma^{(3)}_{(-,0)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\downarrow\mu}(p-\frac{q}{2})]^{-1} ,
\end{aligned}
\tag{5.2}
$$

$$
\begin{aligned}
\triangle_{B(0,-)}(q) = D_{\tau}(q)[1 - \frac{-i}{(2\pi)^4}\int d^4p S_{\lambda\uparrow}(p+\frac{q}{2})\\
\times \Gamma^{(3)}_{(0,-)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\lambda\downarrow}(p-\frac{q}{2})]^{-1} ,
\end{aligned}
\tag{5.3}
$$

$$
\begin{aligned}
\triangle_{B(-,3)}(q) = D_{\sigma\tau(3)}(q)[1 - \frac{-i}{(2\pi)^4}\sum_{\mu}\eta_{\mu}\int d^4p S_{\uparrow\mu}(p+\frac{q}{2})\\
\times \Gamma^{(3)}_{(-,3)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\downarrow\mu}(p-\frac{q}{2})]^{-1} ,
\end{aligned}
\tag{5.4}
$$

$$
\begin{aligned}
\triangle_{B(3,-)}(q) = D_{\sigma(3)\tau}(q)[1 - \frac{-i}{(2\pi)^4}\sum_{\lambda}\eta_{\lambda}\int d^4p S_{\lambda\uparrow}(p+\frac{q}{2})\\
\times \Gamma^{(3)}_{(3,-)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\lambda\downarrow}(p-\frac{q}{2})]^{-1} ,
\end{aligned}
\tag{5.5}
$$

$$
\begin{aligned}
\triangle_{B(-,-)}(q) = D_{\sigma\tau}(q)[1 - \frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\uparrow}(p+\frac{q}{2})\\
\times \Gamma^{(3)}_{(-,-)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\downarrow\downarrow}(p-\frac{q}{2})]^{-1} ,
\end{aligned}
\tag{5.6}
$$

$$
\begin{aligned}
\triangle_{B(-,+)}(q) = D_{\sigma\tau(+)}(q)[1 - \frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\downarrow}(p-\frac{q}{2})\\
\times \Gamma^{(3)}_{(-,+)}(p-\frac{q}{2},q;p+\frac{q}{2})\times S_{\downarrow\uparrow}(p+\frac{q}{2})]^{-1} .
\end{aligned}
\tag{5.7}
$$

Where

$$D_{\sigma}(q) = \frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\mu}(p+\frac{q}{2})S_{\downarrow\mu}(p-\frac{q}{2}) , \tag{5.8}$$

$$D_{\tau}(q) = \frac{-i}{(2\pi)^4}\int d^4p S_{\lambda\uparrow}(p+\frac{q}{2})S_{\lambda\downarrow}(p-\frac{q}{2}) , \tag{5.9}$$

$$D_{\sigma\tau(3)}(q) = \frac{-i}{(2\pi)^4}\sum_{\mu}\eta_{\mu}\int d^4p S_{\uparrow\mu}(p+\frac{q}{2})S_{\downarrow\mu}(p-\frac{q}{2}) , \tag{5.10}$$

$$D_{\sigma(3)\tau}(q) = \frac{-i}{(2\pi)^4}\sum_{\lambda}\eta_{\lambda}\int d^4p S_{\lambda\uparrow}(p+\frac{q}{2})S_{\lambda\downarrow}(p-\frac{q}{2}) ; \tag{5.11}$$

$$D_{\sigma\tau}(q) = \frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\uparrow}(p+\frac{q}{2})S_{\downarrow\downarrow}(p-\frac{q}{2}) , \tag{5.12}$$

$$D_{\sigma\tau(+)}(q) = \frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\downarrow}(p-\frac{q}{2})S_{\downarrow\uparrow}(p+\frac{q}{2}) , \tag{5.13}$$

and

$$
\begin{aligned}
\eta_{\lambda(\mu)} = 1 \quad for \quad \lambda(\mu)=\uparrow,\\
-1 \quad for \quad \lambda(\mu)=\downarrow .
\end{aligned}
\tag{5.14}
$$

In Eqs (5.2)–(5.13), the integration path for dp_0 is taken along the half circle in the upper plane for complex p_0.

The condition for the bound-state energy of $(\sigma\tau)$-bosons is given by the pole singularities of the propagators (5.2)–(5.7), i.e. the dispersion equations

$$\Delta^{-1}_{B(l,k)}(q) = 0 \ , \tag{5.15}$$

with $(l,k) = (-,0)$, $(0,-)$, $(-,3)$, $(3,-)$, $(-,-)$ and $(+,-)$. It leads to following equations:

$$\frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\mu}(p+\frac{q}{2})\Gamma^{(3)}_{(-,0)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\downarrow\mu}(p-\frac{q}{2}) = 1 \ , \tag{5.16}$$

$$\frac{-i}{(2\pi)^4}\int d^4p S_{\downarrow\uparrow}(p+\frac{q}{2})\Gamma^{(3)}_{(0,-)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\downarrow\downarrow}(p-\frac{q}{2}) = 1 \ , \tag{5.17}$$

$$\frac{-i}{(2\pi)^4}\sum_{\mu}\eta_{\mu}\int d^4p S_{\uparrow\mu}(p+\frac{q}{2})\Gamma^{(3)}_{(-,3)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\downarrow\mu}(p-\frac{q}{2}) = 1 \ , \tag{5.18}$$

$$\frac{-i}{(2\pi)^4}\sum_{l}\eta_{l}\int d^4p S_{\downarrow\uparrow}(p+\frac{q}{2})\Gamma^{(3)}_{(3,-)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\downarrow\downarrow}(p-\frac{q}{2}) = 1 \ , \tag{5.19}$$

$$\frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\uparrow}(p+\frac{q}{2})\Gamma^{(3)}_{(-,-)}(p+\frac{q}{2};p-\frac{q}{2},q)\times S_{\downarrow\downarrow}(p-\frac{q}{2}) = 1 \ , \tag{5.20}$$

$$\frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\downarrow}(p-\frac{q}{2})\Gamma^{(3)}_{(-,+)}(p-\frac{q}{2},q;p+\frac{q}{2})\times S_{\downarrow\uparrow}(p+\frac{q}{2}) = 1 \ . \tag{5.21}$$

They determine the spectra $\omega_{B(l,k)}$ of the correcsponding spin and isospin excitations of the system. At $q=0$, the bound-state energy vanishes, i.e., $\omega_{B(l,k)}(0)=0$, as we have shown in section 3. In this case, Eqs(5.16)–(5.21) become so called gap equations which determine the polarization of the system in question.

In fact, the polarizations M_{σ}, M_{τ}, and $M_{\sigma\tau}$ can be calculuted from Eqs.(2.16), (2.17), and (2.18) by the similar procedure as done in Eq.(5.1). We have

$$M_{\sigma} = \frac{-i}{(2\pi)^4}\int d^4p[S_{\uparrow\mu}(p) - S_{\downarrow\mu}(p)] \ , \tag{5.22}$$

$$M_{\tau} = \frac{-i}{(2\pi)^4}\int d^4p S_{\downarrow\uparrow}(p) - S_{\downarrow\downarrow}(p) \ , \tag{5.23}$$

$$M_{\sigma\tau} = \frac{-i}{(2\pi)^4}\int d^4p[S_{\uparrow\uparrow}(p) - S_{\uparrow\downarrow}(p) - S_{\downarrow\uparrow}(p) - S_{\downarrow\downarrow}(p)] \ , \tag{5.24}$$

Combining the W–T relations (4.25)–(4.31) with Eqs.(5.22)–(5.24), we obtain a set of gap equations which are same as Eqs.(5.16)–(5.21) at $q=0$. This shows the energy-gapless of spin and isospin excitations, implied just as by the Gddstone theorem.

The response of the system to external field h ($h = h_{\sigma}$, h_{τ}, or $h_{\sigma\tau}$) can be described by formula (5.2)–(5.7) if all quanta $S_{\lambda\mu}$ and $\Gamma^{(3)}$ are calculated in the existence of external field h. To explicitly label this point, we write the response functions or susceptibilities as

$$\Delta_{\sigma}(q,h) = D_{\sigma}(q,h)[1 - \frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\mu}(p+\frac{q}{2})\Gamma^{(3)}_{(-,0)}(p+\frac{q}{2};p-\frac{q}{2},q)$$
$$\times S_{\downarrow\mu}(p-\frac{q}{2})]^{-1} \ , \tag{5.25}$$

$$\Delta_{\sigma\tau}(q,h) = D_{\sigma\tau}(q,h)[1 - \frac{-i}{(2\pi)^4}\int d^4p S_{\uparrow\uparrow}(p+\frac{q}{2})\Gamma^{(3)}_{(-,-)}(p+\frac{q}{2};p-\frac{q}{2},q)$$
$$\times S_{\downarrow\downarrow}(p-\frac{q}{2})]^{-1} \ , \tag{5.26}$$

and the similar expressions for $\Delta_{\tau}(q,h)$, $\Delta_{\sigma\tau(3)}(q,h)$ $\Delta_{\sigma(3)\tau}(q,h)$ and $\Delta_{\sigma\tau(+)}(q,h)$, corresponding to Eqs.(5.2)–(5.14). Where use was made of the notations $\sigma = B(-,0)$, $\tau = B(0,-)$, $\sigma(3)\tau = B(3,-)$, $\sigma\tau(3) = B(-,3)$, $\sigma\tau = B(-,-)$ and $\sigma\tau(+) = B(-,+)$ in order to specify the response functions. To solve exactly these equations are very difficulty, since we need $\Gamma^{(4)}$ in order to get $\Gamma^{(3)}$, while we need $\Gamma^{(5)}$ to get $\Gamma^{(4)}$, and so on. To simplify the discussion, here we consider the situation of zero energy-momentum transfer limit. In this case, the W–T relations among vertex functions, i.e., Eqs. (4.25)–(4.34), may be used to write the formulas for the static response functions or susceptibilities $\Delta_{\sigma}(h)$,...$\Delta_{\sigma\tau}(h)$. Even so, it is still hard to solve exactly. Therefore, making certain approximation is needed, which will be examined in next section.

6.The Corrections of Spin–Isospin Waves to Susceptibilities

So far our discussions are based on the requirements of the basic SU(4) rotational invariance of the theory, and so the obtained results including the expressions for the susceptibilities are exact for the system under consideration. To

solve the susceptibility expressions in static limit is still very difficult task. In this section, we examine a consistent approximation apptoach to the calculation of the static susceptibilities in the infinite nuclear system of nonrelativistic nucleons. To begin with let us consider a simplified model of the infinite nuclear matter in which only cenltral force between nucleons is considered. The Lagrangian for this model is given by

$$L(x) = \sum_{\alpha} \psi_{\alpha}^{+}(x)[i\partial_{t} - \epsilon(\nabla)]\psi_{\alpha}(x) - \frac{1}{2}\sum_{\alpha\beta}\int d^{4}y \psi_{\alpha}^{+}(x)\psi_{\beta}^{+}(y) \times V_{c}(x-y)\psi_{\beta}(y)\psi_{\alpha}(x) + h\psi_{\alpha}^{+}(x)\sigma_{3}\tau_{3}\psi_{\alpha} , \tag{6.1}$$

Where the subscript $\alpha(\beta)$ of the Heisenberg fields $\psi_{\alpha}(x)$ labels the spin and isospin states of a nucleon, i.e., $\alpha(\beta)=\lambda\mu$. We now only consider the symmetry-breaking term $h\psi^{\dagger}(x)\sigma_{3}\tau_{3}\psi(x)$. The discusion for the case of existence of term $h_{\sigma}\sigma_{3}(x)$ or $h_{\tau}\tau_{3}(x)$ is similar and more simpler. The equations of motion corresponding to Eq.(6.1) is

$$i\partial_{t} - \epsilon(\nabla)\ \psi_{\alpha}(x) = J_{\alpha}[\psi;x] - h\sigma_{3}\tau_{3}\psi_{\alpha}(x) , \tag{6,2}$$

with

$$J_{\alpha}[\psi;x] \equiv \sum_{\beta}\int d^{4}y \psi_{\beta}^{+}(y)V_{c}(x-y)\psi_{\beta}(y)\psi_{\alpha}(x) , \tag{6.3}$$

For the convenience of discussion, we set $V_{c}(x-y) = g_{c}\delta(x-y)$.

To calculate nucleon propagator $S_{\lambda\mu}(p)$ in the expressions of susceptibilities, we use the Dyson equations[9] which are given by

$$S_{\uparrow\uparrow}(p) = S_{+,+}(p) + S_{+,+}(p)\Sigma_{\uparrow\uparrow}(p)S_{\uparrow\uparrow}(p), \tag{6.4a}$$

$$S_{\uparrow\downarrow}(p) = S_{+,-}(p) + S_{+,-}(p)\Sigma_{\uparrow\downarrow}(p)S_{\uparrow\downarrow}(p), \tag{6.4b}$$

$$S_{\downarrow\uparrow}(p) = S_{-,+}(p) + S_{-,+}(p)\Sigma_{\downarrow\uparrow}(p)S_{\downarrow\uparrow}(p), \tag{6.4c}$$

$$S_{\downarrow\downarrow}(p) = S_{-,-}(p) + S_{-,-}(p)\Sigma_{\downarrow\downarrow}(p)S_{\downarrow\downarrow}(p), \tag{6.4d}$$

where $S_{\pm,\pm}$ are the propagators of bare nucleon with different spin and isospin states:

$$S_{ab}(p) = \frac{\theta(\tilde{\epsilon}(\vec{p}))}{p_{0} - \tilde{\epsilon}(\vec{p}) + i\epsilon} + \frac{\theta(-\tilde{\epsilon}(\vec{p}))}{p_{0} - \tilde{\epsilon}(\vec{p}) - i\epsilon} , \tag{6.5}$$

with

$$\tilde{\epsilon}(p) = \epsilon(p) - 2(a \times b)h . \tag{6.6}$$

Where a(b) = +,−. The self-energy parts $\Sigma_{\lambda\mu}(p)$ are defined by

$$< J_{\lambda\mu}[\psi;x]\psi_{\lambda\mu}^{+}(y) > = \frac{i}{(2\pi)^{4}}\int d^{4}p e^{-ip(x-y)}\Sigma_{\lambda\mu}(p)S_{\lambda\mu}(p) . \tag{6.7}$$

Now let us concentrate our attention on the calculation of the static susceptibilities. Using the W−T relations (4.25)−(4.31), and the following relations

$$\frac{1}{2h}(S_{\uparrow\mu}^{-1}(p) - S_{\downarrow\mu}^{-1}(p)) = -\frac{1}{2h}(\Sigma_{\uparrow\mu}(p) - \Sigma_{\downarrow\mu}(p)) , \tag{6.8a}$$

$$\frac{1}{2h}(S_{\lambda\uparrow}^{-1}(p) - S_{\lambda\downarrow}^{-1}(p)) = -\frac{1}{2h}(\Sigma_{\lambda\uparrow}(p) - \Sigma_{\lambda\downarrow}(p)) , \tag{6.8b}$$

which are derived by inverting Eqs.(6.4a)−(6.4d), we may write the expressions for the static susceptibilities in the form

$$\Delta_{\alpha}(h) = \frac{D_{\alpha}^{(0)}(h)}{1 + \kappa_{\alpha}(h)} , \tag{6.9}$$

Where index α represents σ, τ, $\sigma\tau(3)$, $\sigma(3)\tau$, $\sigma\tau$, $\sigma\tau(+)$. $D_{\alpha}^{(0)}(h)$ are given by Eqs.(5.8)−(5.13) at q = 0 case, while the nucleon propagators may take the lowest order form or what you need. $\kappa_{\alpha}(h)$ are given by

$$\begin{aligned}\kappa_{\beta}(h) = &- D_{\beta}^{(0)}(h)D_{\beta}^{-1}(h)\Delta_{\beta}^{-1}(h)\frac{i}{(2\pi)^{4}}\int d^{4}p \\ &\times \{S_{\uparrow\uparrow}(p)\frac{1}{2h}(\Sigma_{\uparrow\uparrow}(p) - \Sigma_{\downarrow\uparrow}(p))S_{\downarrow\uparrow}(p) \\ &+ S_{\uparrow\downarrow}(p)\frac{1}{2h}(\Sigma_{\downarrow\downarrow}(p) - \Sigma_{\uparrow\downarrow}(p))S_{\downarrow\downarrow}(p)\} \\ &- (D_{\beta}(h) - D_{\beta}^{(0)}(h))D_{\beta}^{-1}(h) ,\end{aligned} \tag{6.10}$$

for $\alpha = \beta = \sigma, \tau, \sigma\tau(3)$, and $\sigma(3)\tau$; and

$$\begin{aligned}\kappa_{\sigma\tau}(h) = &- D_{\sigma\tau}^{(0)}(h)D_{\sigma\tau}^{-1}(h)\{\Delta_{\beta}^{-1}(h)\frac{i}{(2\pi)^{4}}\int d^{4}p \\ &\times [S_{\uparrow\uparrow}(p)\frac{1}{4h}(\Sigma_{\uparrow\uparrow}(p) - \Sigma_{\downarrow\uparrow}(p))S_{\downarrow\uparrow}(p)\end{aligned}$$

$$+ S_{\uparrow\downarrow}(p)\frac{1}{4h}(\Sigma_{\downarrow\downarrow}(p) - \Sigma_{\uparrow\downarrow}(p))S_{\downarrow\downarrow}(p)]$$

$$+ 4h\Delta_{\sigma\tau}(h)\Delta^{-1}_{\sigma\tau(3)}(h)\Delta^{-1}_{\sigma(3)\tau}(h)\frac{i}{(2\pi)^4}\int d^4p \times [S_{\uparrow\uparrow}(p)$$

$$\times \frac{1}{2h}(\Sigma_{\uparrow\uparrow}(p) - \Sigma_{\downarrow\uparrow}(p))S_{\downarrow\uparrow}(p)\frac{1}{2h}(\Sigma_{\downarrow\uparrow}(p) - \Sigma_{\downarrow\downarrow}(p))S_{\downarrow\downarrow}(p)$$

$$+ 4h\Delta_{\sigma\tau}(h)\Delta^{-1}_{\sigma\tau(3)}(h)\Delta^{-1}_{\sigma(3)\tau}(h)\frac{i}{(2\pi)^4}\int d^4p\ S_{\uparrow\uparrow}(p)$$

$$\times \frac{1}{2h}(\Sigma_{\uparrow\uparrow}(p) - \Sigma_{\downarrow\uparrow}(p))S_{\uparrow\downarrow}(p)\frac{1}{2h}(\Sigma_{\uparrow\downarrow}(p) - \Sigma_{\downarrow\downarrow}(p))S_{\downarrow\downarrow}(p)$$

$$+ 4h\Delta_{\sigma\tau}(h)\frac{i}{(2\pi)^4}\int d^4p S_{\uparrow\uparrow}(p)\Gamma^{(4)}(p,o;p,O)S_{\downarrow\downarrow}(p)\}$$

$$- (D_{\sigma\tau}(h) - D^{(0)}_{\sigma\tau}(h))D^{-1}_{\sigma\tau}(h)\ , \qquad (6.11)$$

for $\alpha = \sigma\tau$, and a similar form for $\alpha = \sigma\tau(+)$.

Thus the calculation of the static susceptibilities is now reduced to the problem of calculating the nucleon self-energy in the presence of some disturbing external field. The formula (6.9) is still a formal but consistent expression for the static susceptibilities, since it involves $\Delta_\alpha(h)$ self in κ_z and the as yet undertermined propagators $\Delta_\alpha(k)$ in $\Sigma_{\lambda\mu}(p)$ as shown later. To solve this problem, we make the approximation expansions for the nucleon self-energy as

$$\Sigma_{\lambda\mu}(p) = \Sigma^{(1)}_{\lambda\mu}(p) + \Sigma^{(2)}_{\lambda\mu}(p) + \dots\ . \qquad (6.12)$$

Where the indexes (1),(2) represent the first and second order approximation, respectively. Substituting Eq.(6.12) into Eqs.(6.4a)–(6.4b), we get the following suceessive approximation expressions for the nucleon propagators:

$$S^{(1)}_{\uparrow\uparrow}(p) = S_{+,+}(p) + S_{+,+}(p)\Sigma^{(1)}_{\uparrow\uparrow}(h)S^{(1)}_{\uparrow\uparrow}(p)\ , \qquad (6.13)$$

$$S^{(2)}_{\uparrow\uparrow}(p) = S^{(1)}_{\uparrow\uparrow}(p) + S^{(1)}_{\uparrow\uparrow}(p)\Sigma^{(2)}_{\uparrow\uparrow}(p,h)S^{(2)}_{\uparrow\uparrow}(p)\ , \qquad (6.14)$$

.....

Where "....." denote the expressions for $S^{(1)}_{\uparrow\downarrow}$, $S^{(1)}_{\downarrow\uparrow}$, $S^{(1)}_{\downarrow\downarrow}$, $S^{(2)}_{\uparrow\downarrow}$, $S^{(2)}_{\downarrow\uparrow}$, $S^{(2)}_{\downarrow\downarrow}$, and the more high order approximation of $S_{\lambda\mu}$.

The nucleon self-energy in n-th order approximation can be calculated directly from Eq.(6.7).

The first order nucleon self-energy parts are given by

$$\Sigma^{(1)}_{\uparrow\uparrow}(h) = -g_c\frac{i}{(2\pi)^4}\int d^4k(S_{\downarrow\uparrow}(k) + S_{\uparrow\downarrow}(k) + S_{\downarrow\downarrow}(k))\ , \qquad (6.15a)$$

$$\Sigma^{(1)}_{\downarrow\uparrow}(h) = -g_c\frac{i}{(2\pi)^4}\int d^4k(S_{\uparrow\uparrow}(k) + S_{\uparrow\downarrow}(k) + S_{\downarrow\downarrow}(k))\ , \qquad (6.15b)$$

$$\Sigma^{(1)}_{\uparrow\downarrow}(h) = -g_c\frac{i}{(2\pi)^4}\int d^4k(S_{\downarrow\downarrow}(k) + S_{\uparrow\uparrow}(k) + S_{\downarrow\uparrow}(k))\ , \qquad (6.15c)$$

$$\Sigma^{(1)}_{\downarrow\downarrow}(h) = -g_c\frac{i}{(2\pi)^4}\int d^4k(S_{\uparrow\downarrow}(k) + S_{\uparrow\uparrow}(k) + S_{\downarrow\uparrow}(k))\ . \qquad (6.15d)$$

We obtain the second order nucleon self-energy parts as follows:

$$\Sigma^{(2)}_{\uparrow\uparrow}(p) = g_c^2\frac{i}{(2\pi)^4}\int d^4k\{S_{\downarrow\uparrow}(p-k)(\Delta_\sigma(k) - D_\sigma(k))$$
$$+ S_{\downarrow\uparrow}(p-k)(\Delta_{\sigma\tau(3)}(k) - D_{\sigma\tau(3)}(k))$$
$$+ S_{\uparrow\downarrow}(p-k)(\Delta_\tau(k) - D_\tau(k))$$
$$+ S_{\uparrow\downarrow}(p-k)(\Delta_{\sigma(3)\tau}(k) - D_{\sigma(3)\tau}(k))$$
$$+ S_{\downarrow\downarrow}(p-k)(\Delta_{\sigma\tau}(k) - D_{\sigma\tau}(k))\}$$
$$+ g_c^2\frac{i}{(2\pi)^4}\int d^4k(S_{\uparrow\uparrow}(p-k)(\Delta_{\downarrow\uparrow}(k) + \Delta_{\uparrow\downarrow}(k) + \Delta_{\downarrow\downarrow}(k))\ , \qquad (6.16)$$

$$\Sigma^{(2)}_{\downarrow\uparrow}(p) = g_c^2\frac{i}{(2\pi)^4}\int d^4k(S_{\uparrow\uparrow}(p+k)(\Delta_\sigma(k) - D_\sigma(k))$$
$$+ S_{\uparrow\uparrow}(p+k)(\Delta_{\sigma\tau(3)}(k) - D_{\sigma\tau(3)}(k))$$
$$+ S_{\uparrow\downarrow}(p+k)(\Delta_{\sigma\tau(+)}(k) - D_{\sigma\tau(+)}(k))$$
$$+ S_{\downarrow\downarrow}(p+k)(\Delta_\tau(k) - D_\tau(k))$$
$$+ S_{\downarrow\downarrow}(p+k)(\Delta_{\sigma(3)\tau}(k) - D_{\sigma(3)\tau}(k))\}$$
$$+ g_c^2\frac{i}{(2\pi)^4}\int d^4k[S_{\downarrow\uparrow}(p+k)(\Delta_{\uparrow\downarrow}(k) + \Delta_{\uparrow\downarrow}(k)) + S_{\downarrow\uparrow}(p-k)\Delta_{\downarrow\uparrow}(k)]\ , \qquad (6.17)$$

$$\Sigma^{(2)}_{\uparrow\downarrow}(p) = g_c^2\frac{i}{(2\pi)^4}\int d^4k\{S_{\uparrow\uparrow}(p+k)(\Delta_\tau(k) - D_\tau(k))$$
$$+ S_{\uparrow\uparrow}(p+k)(\Delta_{\sigma(3)\tau}(k) - D_{\sigma(3)\tau}(k))$$
$$+ S_{\downarrow\uparrow}(p-k)(\Delta_{\sigma\tau(+)}(k) - D_{\sigma\tau(+)}(k))$$

$$+ S_{\downarrow\downarrow}(p-k)(\Delta_{\sigma}(k) - D_{\sigma}(k))$$
$$+ S_{\downarrow\downarrow}(p-k)(\Delta_{\sigma\tau(3)}(k) - D_{\sigma\tau(3)}(k))\}$$
$$+ g_c^2 \frac{i}{(2\pi)^4} \int d^4k [S_{\uparrow\downarrow}(p+k)\Delta_{\uparrow\uparrow}(k) + S_{\uparrow\downarrow}(p-k)(\Delta_{\downarrow\uparrow}(k) + \Delta_{\downarrow\downarrow}(k))] \ , \tag{6.18}$$

$$\Sigma^{(2)}_{\downarrow\downarrow}(p) = g_c^2 \frac{i}{(2\pi)^4} \int d^4k \{S_{\uparrow\uparrow}(p+k)(\Delta_{\sigma\tau}(k) - D_{\sigma\tau}(k))$$
$$+ S_{\downarrow\uparrow}(p+k)(\Delta_{\tau}(k) - D_{\tau}(k))$$
$$+ S_{\downarrow\uparrow}(p+k)(\Delta_{\sigma(3)\tau}(k) - D_{\sigma(3)\tau}(k))$$
$$+ S_{\uparrow\downarrow}(p+k)(\Delta_{\sigma}(k) - D_{\sigma}(k))$$
$$+ S_{\uparrow\downarrow}(p+k)(\Delta_{\sigma\tau(3)}(k) - D_{\sigma\tau(3)}(k))\}$$
$$+ g_c^2 \frac{i}{(2\pi)^4} \int d^4k S_{\downarrow\downarrow}(p+k)(\Delta_{\uparrow\uparrow}(k) + \Delta_{\downarrow\uparrow}(k) + \Delta_{\uparrow\downarrow}(k)) \ . \tag{6.19}$$

Where $D_x(k)$, $x = \sigma, \tau, \sigma\tau(3), \sigma(3)\tau, \sigma\tau, \sigma\tau(+)$, are given by Eqs.(5.8)–(5.13). $\Delta_x(k)$ are $(\sigma\tau)$–boson propagators in momentum space, which are defined by Eqs.(3.11)–(3.16). $\Delta_{\uparrow\uparrow}(k)$, $\Delta_{\downarrow\uparrow}(k)$, $\Delta_{\uparrow\downarrow}(k)$ and $\Delta_{\downarrow\downarrow}(k)$ are defined as

$$< 0|T[\psi_{\lambda\mu}(x)\psi^{+}_{\lambda\mu}(x)\psi_{\lambda\mu}(y)\psi^{+}_{\lambda\mu}(y)]|0 >$$
$$= \frac{i}{(2\pi)^4} \int d^4k \Delta_{\lambda\mu}(k) e^{-ik(x-y)} \ , \tag{6.20}$$

with $\lambda(\mu) = \uparrow, \downarrow$, which are, in fact, the nucleon density correlation functions. Graphically the first order nucleon self–energy $\Sigma^{(1)}_{\uparrow\uparrow}$ and the second order nucleon self–energy $\Sigma^{(2)}_{\uparrow\uparrow}$ are respectively shown in Figs(4) and (5).

Using the expansion expressions for $\Sigma_{\lambda\mu}$ and $S_{\lambda\mu}$, we may write κ_x as

$$\kappa_x = \kappa^{(1)}_x + \kappa^{(2)}_x + \dots \ , \tag{6.21}$$

Where $\kappa^{(1)}_x$ corresponds to that in which the nucleon propagator and self–energy take the first order approximation given respectively by Eqs.(6.13) and (6.15), while $\kappa^{(2)}_x$ is that in which $S_{\lambda\mu}$ and $\Sigma_{\lambda\mu}$ are taken up to the second order approximation given by Eqs.(6.14) and (6.16)–(6.19). Substituting Eqs.(6.15a)–(6.15d) into Eq.(6.10) and (6.11), we have

$$\kappa^{(1)}_{\sigma}(h) = M^{-1}_{\sigma\tau}(h) \frac{i}{(2\pi)^4} \int d^4p \{S^{(1)}_{\uparrow\uparrow}(p)$$
$$\times (-g_c \frac{i}{(2\pi)^4} \int d^4k (S_{\downarrow\uparrow}(k) - S_{\uparrow\uparrow}(k)) S^{(1)}_{\downarrow\uparrow}(p)$$
$$+ S^{(1)}_{\uparrow\downarrow}(p)(-g_c \frac{i}{(2\pi)^4} \int d^4k (S_{\uparrow\downarrow}(k) - S_{\downarrow\downarrow}(k)) S^{(1)}_{\downarrow\downarrow}(p)\}$$
$$= g_c (D^{(0)}_{\sigma\tau(3)}(h) + D^{(0)}_{\sigma}(h))/2 \equiv g_c D^{(0)}(h) \ . \tag{6.22}$$

Where we have used Eqs.(3.19)–(3.22) and (3.27)–(3.28) and (5.24), and ignored the terms $\sim 0(g_c^2)$ which may be attributed to $\kappa^{(2)}_{\sigma}$. Thus, Eq.(6.9) to the first order approximation leads to

$$\Delta_{\sigma}(h) = \frac{D^{(0)}_{\sigma}(h)}{1 + g_c D^{(0)}(h)}$$
$$= \frac{D^{(0)}(h)}{1 + g_c D^{(0)}(h)} \left(\frac{D^{(0)}_{\sigma}(h)}{D^{(0)}(h)}\right) \ . \tag{6.23}$$

which is the standard RPA results[1]. when $g_c = 0$, Eq.(6.23) gives

$$\Delta^{(0)}_{\sigma}(0) = D^{(0)}_{\sigma}(0) \ , \tag{6.24}$$

which of course is the results for free nucleon gas[10].

The static susceptibilities to the second order approximation have the form

$$\Delta_{\sigma}(0) = \frac{D^{(0)}_{\sigma}(h)}{1 + g_c D^{(0)}(h) + \kappa^{(2)}_{\sigma}} \ . \tag{6.25}$$

Hence $\kappa^{(2)}_x$ represents the correction to the RPA, and it can be obtained from Eqs.(6.10)–(6.11) in which the nucleon propagators and self–energies take $S^{(1)}_{\lambda\mu}$ and $\Sigma^{(1)}_{\lambda\mu}$ in those terms $\sim 0(g_c^2)$. The obtained expressions for $\kappa^{(2)}_{\sigma}$ and $\kappa^{(2)}_{\sigma\tau}$ are still rather formal since they involve undetermined propgators $\Delta_x(k)$ and $\Delta_{\lambda\mu}(k)$ as seen from Eqs.(6.16)–(6.19) and (6.10)–(6.11). Here we do not like to perform the detail approximation calculation for $\kappa^{(2)}_{\sigma}$, which will be examined elsewhere. What we are now interested in is to analyse the mechanism given by $\kappa^{(2)}_x$ From the expressions of $\Sigma^{(2)}_{\lambda\mu}$, namely Eqs.(6.16)–(6.19) as well as Fig.(7), we see that $\Sigma^{(2)}_{\lambda\mu}$ come from the contributions of the spin–isospin waves (or bosons) to the

$S^{(2)}_{\lambda\mu}$ and $\Sigma^{(2)}_{\lambda\mu}$ as well as

nucleon self-energies in infinite nuclear medium. Hence, $k_\alpha^{(2)}$ represent the corrections of spin-isospin waves to the susceptibilities, which are in fact equivalent to the contributions from 2p2h excitations.

7. Quenching of Spin-Isospin Transitions, and the Correlations among Collective Modes

In this section, we first brifely discuss how the approach given in this paper is used to describe the quenching of spin-isospin transitions including Gamow-Teller, isoscaler and isvector magnetic, and Fermi-type transitions. And then we discuss the correlations among collective modes.

(1). Quenching of Gamow-Teller transition.

Empirically one finds that the GT strength in heavy nuclei carries only about 60 per cent of the sum rule result[(I)(II)]. To interpret the quenching of GT strength, we may use the results of last section. Recalling the definition for the GT excitation strength

$$\begin{aligned} B_\pm(GT) &= \langle i|\bar{\sigma}\tau_\mp|j\rangle \langle j|\bar{\sigma}\tau_\pm|i\rangle \\ &= \langle i|\sigma_\mp\tau_\mp|j\rangle \langle j|\sigma_\pm\tau_\pm|i\rangle \\ &+ \langle i|\sigma_\pm\tau_\mp|j\rangle \langle j|\sigma_\mp\tau_\pm|i\rangle \\ &+ \langle i|\sigma_3\tau_\mp|j\rangle \langle j|\sigma_3\tau_\pm|i\rangle \ , \end{aligned} \tag{7.1}$$

we may write the corresponding expression for $B_\pm(GT)$ in our approach as

$$B_\pm(GT) = \Delta_{\sigma\tau}(q) + \Delta_{\sigma\tau(\pm)}(q) + \Delta_{\sigma(3)\tau(\pm)}(q) \ . \tag{7.2}$$

In the limit $q \to 0$, by using the formula (6.25) we obtain, for instance

$$\begin{aligned} B_+(GT) &= \frac{D_{\sigma\tau}}{1 + g_c D^{(0)} + \kappa_{\sigma\tau}^{(2)}} \\ &+ \frac{D_{\sigma\tau(+)}}{1 + g_c D^{(0)} + \kappa_{\sigma\tau(+)}^{(2)}} + \frac{D_{\sigma(3)\tau}}{1 + g_c D^{(0)} + \kappa_{\sigma(3)\tau}^{(2)}} \ . \end{aligned} \tag{7.3}$$

The RPA result is

$$B_+(GT) = (D_{\sigma\tau} + D_{\sigma\tau(+)} + D_{\sigma(3)\tau})\frac{1}{1 + g_c D^{(0)}} \ , \tag{7.4}$$

which gives the familiar quenching factor. Of course, this only is a qualitative explanation. Quantitative interpretation needs to calculate $\kappa^{(2)}$ and to include the contribution from Δ-isobar degree of freedom.

(2) Quenching of magnetic transitions[(I)].

According to the definition of the magnetic spin operator

$$g^{(S)}\bar{\sigma} + g^{(V)}\bar{\sigma}\tau_3 \ ,$$

we may write the relative isoscaler, and isovector magnetic spin excitation strength parts B(M1;S) and B(M1;V) as follows:

$$B(M1;S) = \Delta_\sigma(q) + \Delta_l(q) \ , \tag{7.5}$$

$$B(M1;V) = \Delta_{\sigma\tau(3)}(q) + \Delta_d(q) \ . \tag{7.6}$$

Where Δ_l and Δ_d respectively denote the longitudinal spin and spin-isospin fluctuations, as shown in section 4. In the limit $q \to 0$, we have from Eq.(6.25)

$$B(M1;S) = \frac{D_\sigma}{1 + g_c D^{(0)} + \kappa_\sigma^{(2)}} \ , \tag{7.7}$$

and

$$B(M1;V) = \frac{D_{\sigma\tau(3)}}{1 + g_c D^{(0)} + \kappa_{\sigma(3)}^{(2)}} \ , \tag{7.8}$$

which give the qualitative interpretation for the quenching of magnetic spin transitions.

(3) Quenching of Fermi Transition

Similar to above discussion, we may write the Ferim excitation strength in $q \to 0$ limit as

$$B(F) = \frac{D_\tau}{1 + g_c D^{(0)} + \kappa_\tau^{(2)}} \ , \tag{7.9}$$

Maybe since empirically the GT transitions dominate by far over the F-transitions, so far no one seems to discuss the quenching of F-transition. Here we hope to point out that the existence of the quenching of F-transition, like the

quenching of other spin–isospin transitions, is a natural and necessarian result in our approach.

(4) The correlations among collective modes.

We have found in section 3 that the appearance of the collective modes of spin and isospin excitations, i.e., spin–isospin waves, is a result of the sponontaneous breaking of SU(4)– rotational invariance, which is described by the W–T relations (3.5)–(3.10). We notice that Eqs.(3.9) and (3.10) show the coupling relations among different modes, which in fact describe the correlations among $(\sigma_\pm \tau_\pm)$, $\sigma(3)\tau$ and $\sigma\tau(3)$ modes. Eqs (3.7) and (3.8) show that $\sigma(3)\tau$ and $\sigma\tau(3)$ modes may be excited individually. Thus, Eqs.(3.9) and (3.10) show that $(\sigma_\pm \tau_\pm)$–modes are accompanied by $\sigma(3)\tau$ and $\sigma\tau(3)$ modes. Physically this means that the GT excitations are always accompanied by the magnetic spin excitations. From the calculation for the static susceptibilities or the GT excitation strength we find that the correlation between GT and M1 does not appear in the normal RPA result as shown by Eqs(6.23) and (7.4), and should appear in the result beyond RPA as seen by Eqs.(6.25) and (7.3). This is easy to understand in physics. In fact, the M1 operator $\vec{\sigma}\tau_1$ is just an isospin rotation of the GT operator $\vec{\sigma}\tau_\pm^{(12)}$. This rotation are now performed by propagating the spin–isospin waves appeared in the treatment beyond RPA. It is very interesting to find the correlation nature in experiment.

8. Conclusions

We investigated the spin and isospin excitations in an infinite nonrelativistic nuclear system by using the quantum field theory method. The Ward–Takahashi relations among various many–point Green's functions were derived from the requirement of the rotational invariance in the spin and isospin SU(4) space in the presence of some symmetry breaking force. We found that there exist six N–G bosons:$(\sigma\tau)$–bosons or $(\sigma\tau)$–waves, i.e., the spin, isospin, and spin–isospin waves in six channels, which appear to be a result of one of the W–T relations, namely, the result of the spontaneous breaking of the SU(4) symmetry. These collective modes may be identified to well–known low–energy nuclear spin and isospin excitations.

The W–T relations also determine many useful properties of $(\sigma\tau)$–bosons, e.g., nucleon–$(\sigma\tau)$ boson interactions in the zero–momentum limit, etc. Some of the W–T relations in momentum space were used to obtain a consistent expression for the static susceptibility in terms of nucleon self–energy. Then we gave a qualitative interpretation for the quenching of spin–isospin excitation strength of nuclei. In particular, the W–T relations determine the correlations among collective modes.

We hope to emphasize that the above rather formal results are exact and model independent for an infinite nonrelativistic nuclear system, since our discussions were based on the requirements of symmetry of the system with spontaneously broken symmetry.

Finally we would like to mention the effect of some explicit symmetry breaking forces included in the effective interaction, i.e., the spin and isospin dependent terms. The situation seems to be similar to that of the spontaneous breakdown of chiral symmetry in which a N–G boson acquires mass due to the explicit breaking of chiral symmetry explaining the pion with a finite mass. In analogy it should be stressed that explicit breaking of SU(4) symmetry may lead to a finite energy of those $(\sigma\tau)$–bosons, which will be discussed in the further work.

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Table Caption

Table 1. Collective modes of spin and isospin excitation.

Figure Captions

Fig.1. (a)Nucleon propagators.
(b)Boson propagators.

Fig.2 Diagram of Eq.(4.19).

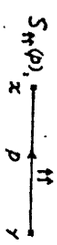
Fig.3. Diagram of Eq.(4.20).

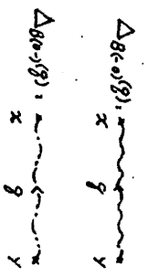
Fig.4 The first order nucleon self-energy part $\Sigma^{(1)}_{\uparrow\uparrow}$.

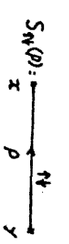
Fig.5. The second order nucleon self-energy part $\Sigma^{(2)}_{\uparrow\uparrow}$: corrections of self-energy parts from spin-isospin waves.

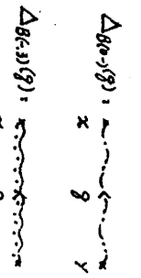
Table 1.

Order State	Symmetry Breaking	N-G Modes	Physical Identifications
$\langle \sigma_j(x) \rangle$	$SU(2)_\sigma$ to $U(1)_\sigma$	$B_\sigma(\vec{q})$, Spin wave	Isoscalar M1 excitation state
$\langle \tau_j(x) \rangle$	$SU(2)_\tau$ to $U(1)_\tau$	$B_\tau(\vec{q})$, Isospin wave	Isobaric analog transition state
$\langle Q_{33}(x) \rangle$	$SU(4)$ to		
	$U(1)_\sigma \otimes 1$	$B_{(-,0)}(\vec{q})$, (B_σ), Isoscalar-spin wave	Isocalar M1 excitation state
	$+ \; 1 \otimes U(1)_\tau$	$B_{(0,-)}(\vec{q})$, (B_τ), Scalar-isospin wave	Isobaric analog trasition state
	$+ \; U(1)_\sigma \otimes U(1)_\tau$	$B_{(-,3)}(\vec{q})$, $(B_{\sigma\tau_3})$, Isovector-spin wave	Isovector M1 excitation state
		$B_{(3,-)}(\vec{q})$, $(B_{\sigma_3\tau})$, Vector-isospin wave	Gamow-Teller state
		$B_{(-,-)}(\vec{q})$, $(B_{\sigma\tau})$, (Transverse) Spin-isospin wave	
		$B_{(-,+)}(\vec{q})$, $(B_{\sigma\tau_+})$,	









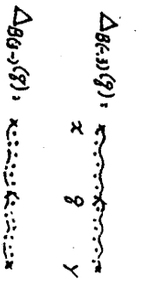


Fig. 1.(a)

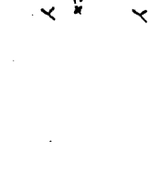


Fig. 1.(b)

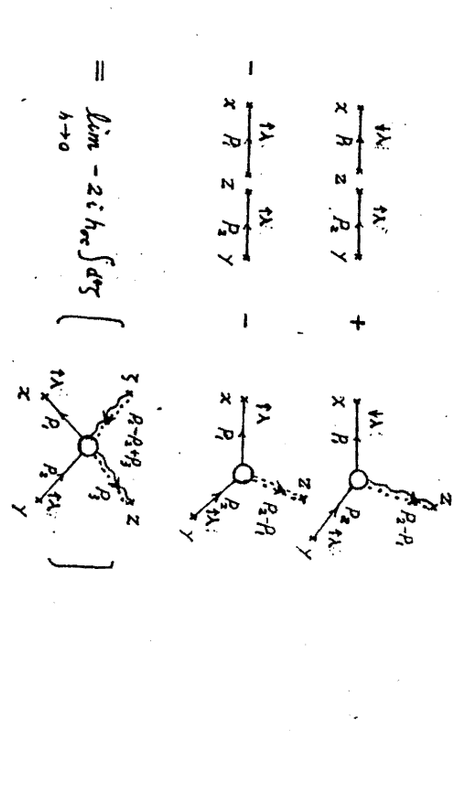


Fig. 2.

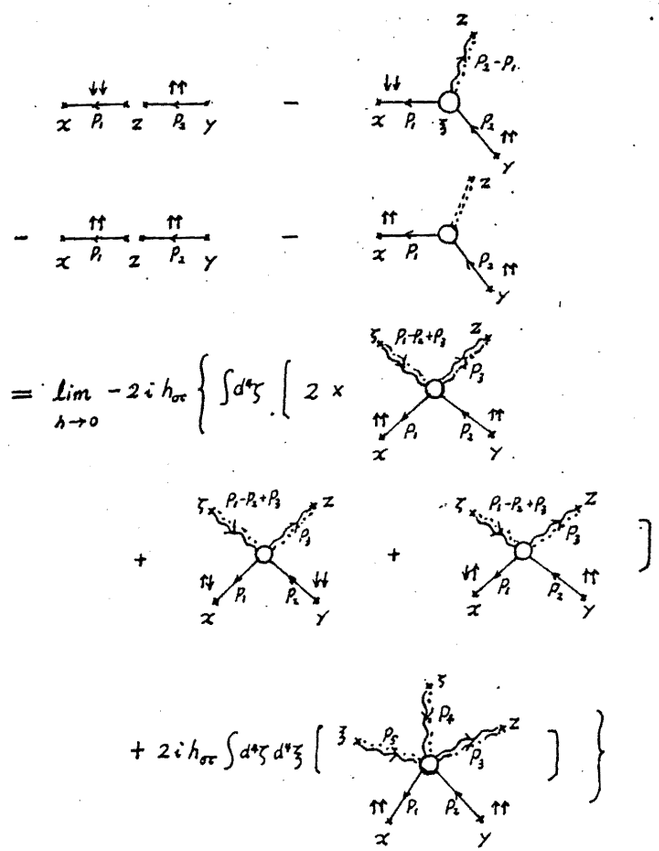


Fig. 3.

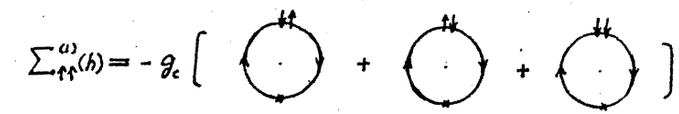


Fig. 4.

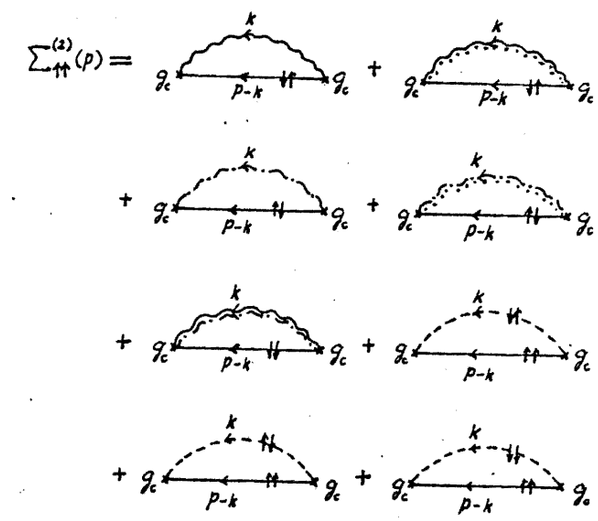


Fig. 5.