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Spin determination of the superdeformed bands in ^{189}Hg and ^{190}Hg

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Abstract

The superdeformed bands in ^{189}Hg and ^{190}Hg are investigated. The spin value of the lowest levels observed in ^{189}Hg (fed by 366.0 keV) is assigned as 31/2 and that of ^{190}Hg (fed by 360.0 keV) is assigned to be 15, rather than the previously proposed value 14.

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Nuclear superdeformation at high-spin has been becoming one of the most attractive topics in nuclear structure physics. Now, a large number of superdeformed (SD) bands have been identified in several mass regions ($A \sim 150$, 130, and 190) [1]. It is found that there exist great regularities in these bands, which have brought to many nuclear physicists' attention [2,3,4]. It is apparent that the spin determination of the SD lavels is crucial for understanding the physics inherent in the SD bands. Unfortunately, in no case have the spins in an SD band been determined by direct experiment. A method was proposed [5] on the basis of the Harris parametrization of the dynamic moment of inertia $\mathcal{J}^{(2)}$ to determine the spins of SD bands, but it seems to possess a rather large uncertainty because $\mathcal{J}^{(2)}$ is extracted with an uncertainty of about 5% due to the error in transition energy measurement. Recently, the SD band in ^{189}Hg has been identified [6], but no spin assignment was given there.

In refs. [7, 8] an effective method has been presented to determine the spins of the SD bands directly from the transition energies. In this approach (hereafter called *ab* fitting) the observed transition energies are least squares fit to

$$E_\gamma(I) \equiv E_\gamma(I \to I-2) = a\left[\sqrt{1+bI(I+1)} - \sqrt{1+b(I-2)(I-1)}\right] \tag{1}$$

deduced from the closed two-parameter $I(I+1)$ expression [9]

$$E(I) = a\left[\sqrt{1+bI(I+1)} - 1\right], \tag{2}$$

which may be derived from the Bohr Hamiltonian for a well-deformed nucleus with small axial asymmetry. It is found that, once a correct spin value of the lowest SD level, I_0, is assigned, the calculated transition energies coincide with the observed results excellently. However, if I_0 is shifted from the correct one, even merely by ± 1, the root-mean-square (rms) deviation

$$\sigma = \left[\frac{1}{n}\sum_{i=1}^{n}\left|\frac{E_\gamma^{cal}(I_i) - E_\gamma^{exp}(I_i)}{E_\gamma^{exp}(I_i)}\right|^2\right]^{1/2} \tag{3}$$

(n being the number of fitted transtions) in the least squares fitting will increase drastically. Therefore, the I_0 value, hence all the spin values of the SD band levels, can be determined reasonably. The spins of all the SD bands in the $A \sim 190$ and 150 regions (available at that time) have been successfully assigned by this approach [8, 10]. As a supplement, the spins of the SD bands in ^{189}Hg and ^{190}Hg are assigned by the *ab* fitting in this note.

In Fig. 1 is given the plot of the rms deviation against various spin prescriptions of the SD band in ^{189}Hg. It is seen that a minimal rms deviation is reached when the spin of the lowest level observed in ^{189}Hg (fed by 366.0 keV transition), I_0, is assigned as 31/2. However, if I_0 is assumed to be 29/2 or 33/2, the rms deviation will increase immediately. Therefore, the V-shaped $\sigma - I_0$ plot favors a reasonable assignment of $I_0 = 31/2$. This assignment is consistent with the coincidence data that the known ^{189}Hg states up to $I = 25/2$ received some feeding [6].

The same assignment may be made from the analysis of the transition energies. The comparison of the calculated and observed transition energies is shown in Table I. It is clear that the calculated transition energies indeed are in good agreement with the observed values for $I_0 = 31/2$. In fact, the transition energies in the SD band of ^{189}Hg are identical to those in ^{191}Hg(band 1) [11], but with a magnification of about 104% (indicating a slight reduction of the moment of inertia in ^{189}Hg). Therefore, the same spins are expected for the corresponding transitions in the two bands. The similarity between these two bands may be attributed to a similar high-N configuration.

Moreover, the kinematic moment of inertia $\mathcal{J}^{(1)}$ and dynamic moment of inertia $\mathcal{J}^{(2)}$ can be extracted from the following analytic expressions [7,8]

$$\mathcal{J}^{(1)} = \frac{\hbar^2}{ab}\left[1 - \frac{(\hbar\omega)^2}{a^2 b}\right]^{-1/2}, \tag{4}$$

$$\mathcal{J}^{(2)} = \frac{\hbar^2}{ab}\left[1 - \frac{(\hbar\omega)^2}{a^2 b}\right]^{-3/2} \tag{5}$$

and also may be estimated roughly from the observed transition energies

$$\mathcal{J}^{(1)}(I-1) \approx \frac{(2I-1)\hbar^2}{E_\gamma(I)}, \tag{6}$$

$$\mathcal{J}^{(2)}(I) \approx \frac{4\hbar^2}{\Delta E_\gamma(I)}, \tag{7}$$

where $\Delta E_\gamma(I) \equiv E_\gamma(I+2) - E_\gamma(I)$. The $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ values for ^{189}Hg extracted by these two approaches are showm in Fig. 2. It is seen that both $\mathcal{J}^{(1)}$'s and $\mathcal{J}^{(2)}$'s extracted by eqs. (6) and (7) are in good agreement with those extracted by eqs. (4) and (5) when $I_0 = 31/2$. Therefore, the analysis on the moments of inertia prefers the spin assignment of $I_0 = 31/2$. In this case, both $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ increase smoothly with increasing ω and $\mathcal{J}^{(2)}$ is always larger than $\mathcal{J}^{(1)}$, similar to those of other SD bands in this region [8].

It is interesting to note that, from eqs. (4) and (5), the ratio

$$R \equiv \sqrt{\frac{\left[\mathcal{J}^{(1)}\right]^3}{\mathcal{J}^{(2)}}} = \frac{\hbar^2}{ab} \tag{8}$$

is independent of ω (or I) within a rotational band. One may extract the R-values directly from the observed transition energies by eqs. (6) and (7). This relation has been confirmed by the extensive data on the normally deformed bands of well deformed nuclei as well as the SD bands, especially those in the $A \sim 190$ region [12]. The R vs I plots for various spin assignments in ^{189}Hg are shown in Fig. 3. Indeed an almost constant R-value is obtained for $I_0 = 31/2$, which may be regarded as an additional support to our spin determination of $I_0 = 31/2$.

Similar analysis for the SD band observed in ^{190}Hg [6] was also made. The highest transition (755.5 keV) is excluded from the ab fitting because its placement in the band is tentative [6]. The least squares fitting shows that a minimal rms deviation is achieved at $I_0 = 15$ whether or not the lowest transition (360.0 keV) is involved in the fitting. The comparison of the calculated and observed transition energies is given in Table II. It is clear that the calculated transition energies indeed are in excellent agreement with the observed values when $I_0 = 15$. The differences $|E_\gamma^{cal} - E_\gamma^{exp}|$ for $I_0 = 15$ are less than 0.4 keV (except the 402.3 keV transition), smaller than the transition energy uncertainties estimated in ref. [6]. But in case of $I_0 = 14$ or 16, the calculated transition energies will deviate appreciably from the observation. Therefore, the spin of the lowest level in this SD band may be reasonably assigned as $I_0 = 15$, rather than $I_0 = 14$ previously proposed from the Harris parametrization [13].

It is worthwhile to note that known transition in ^{190}Hg up to $14^+ \to 12^+$ in the positive-parity yrast band were also observed along with the SD cascade and the intensity distribution for the SD sequence indicates that the decay out takes place abruptly in the two lowest observed SD states [6]. Therefore, the spin assignment of $I_0 = 15$ seems more reasonable than that of $I_0 = 14$ because more angular momentum removal may be provided for the transition linking the SD band and the positive-parity yrast band.

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TABLE I. Transition energies in the SD band of ^{189}Hg. Data are taken from ref. [6].

Observed $E_\gamma(I)$ values (keV)	Calculated $E_\gamma(I)$ values (keV)					
	$I_0 = 29/2$		$I_0 = 31/2$		$I_0 = 33/2$	
	I	$E_\gamma(I)$	I	$E_\gamma(I)$	I	$E_\gamma(I)$
640.6(0.6)	61/2	639.8	63/2	641.0	65/2	642.1
604.2(0.8)	57/2	604.0	59/2	604.0	61/2	604.0
566.4(0.4)	53/2	567.0	55/2	566.3	57/2	565.6
527.8(0.6)	49/2	528.8	51/2	527.7	53/2	526.7
488.9(0.4)	45/2	489.4	47/2	488.4	49/2	487.6
448.7(0.6)	41/2	448.9	43/2	448.5	45/2	448.1
407.7(0.4)	37/2	407.4	39/2	407.9	41/2	408.3
366.0(0.4)	33/2	364.8	35/2	366.6	37/2	368.3
a ($\times 10^4$keV)		4.9831		8.0953		17.8779
b ($\times 10^{-4}$)		2.3561		1.3580		0.5776

TABLE II. Transition energies in the SD band of ^{190}Hg. The highest and the lowest transitions are excluded from the fitting. Data are taken from ref. [6].

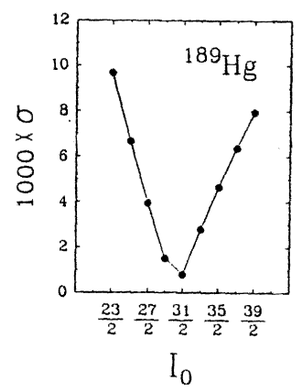
Observed	Calculated $E_\gamma(I)$ values (keV)					
$E_\gamma(I)$ values	$I_0 = 14$		$I_0 = 15$		$I_0 = 16$	
(keV)	I	$E_\gamma(I)$	I	$E_\gamma(I)$	I	$E_\gamma(I)$
(755.5)(1.5)						
728.0(0.6)	36	726.8	37	728.3	38	729.6
696.2(0.8)	34	696.1	35	696.4	36	696.8
663.7(0.4)	32	663.9	33	663.5	34	663.1
629.9(0.6)	30	630.4	31	629.5	32	628.6
594.4(0.4)	28	595.5	29	594.3	30	593.2
558.1(0.6)	26	559.2	27	558.1	28	557.1
521.1(0.4)	24	521.6	25	520.8	26	520.1
482.7(0.4)	22	482.6	23	482.5	24	482.5
443.1(0.4)	20	442.3	21	443.2	22	444.1
402.3(0.4)	18	400.8	19	403.0	20	405.0
360.0(0.4)						
a ($\times 10^4$keV)		4.1677		5.4296		7.4340
b ($\times 10^{-4}$)		2.8658		2.0761		1.4346

Figure Captions

Fig. 1 The rms deviations for various spin assignments for the SD band in ^{189}Hg. I_0 is the spin value prescribed to the lowest level observed [i.e. $E_\gamma(I_0+2 \to I_0) = 366.0$ keV].

Fig. 2 The kinematic and dynamic moments of inertia for the SD band in ^{189}Hg. The $\mathcal{J}^{(1)}$ values extracted by eq. (6) are denoted by circles (for $I_0 = 31/2$), diamonds ($I_0 = 29/2$), and triangles ($I_0 = 33/2$), while the $\mathcal{J}^{(2)}$ extracted by eq. (7) (independent of the spin assignment) are denoted by circles. The values extracted by eqs. (4) and (5) are represented by solid lines. The errors of $\mathcal{J}^{(1)}$ extracted by eq. (6) are too small ($\leq 0.2\hbar^2\mathrm{MeV}^{-1}$) to be drawn in the figure. The rotational frequency is given by $\hbar\omega(I) = (1/2)[\hbar\omega(I+1) + \hbar\omega(I-1)] = (1/4)[E_\gamma(I+2 \to I) + E_\gamma(I \to I-2)]$.

Fig. 3 The ratios $R \equiv \sqrt{[\mathcal{J}^{(1)}]^3/\mathcal{J}^{(2)}}$ for various spin assignments for the SD band in ^{189}Hg.



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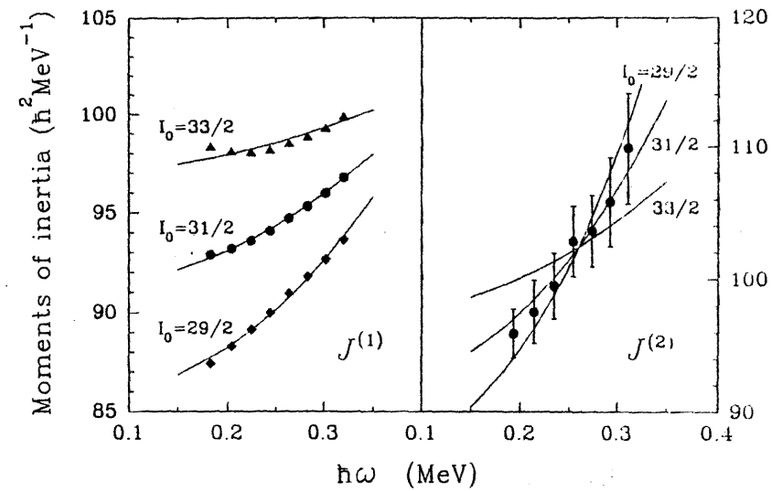

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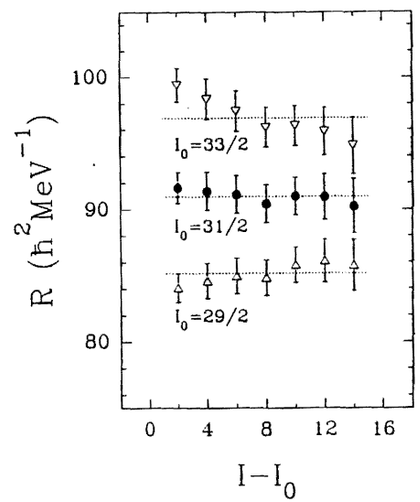

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