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THE PRODUCTION OF $t\bar{c}$ OR $\bar{t}c$ QUARK PAIR BY e^+e^- COLLISION BASED ON THE STANDARD MODEL AND ITS EXTENSIONS

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The Production of $t\,\bar{c}$ or $\bar{t}\,c$ Quark Pair by e^+e^- Collision Based on the Standard Model and Its Extensions[1]

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Abstract

Keeping all terms of heavy flavor masses up to the lowest order and especially bearing such a massive top $m_t \geq 100GeV$ in mind, we carry out a precise calculation on the single top-quark production in e^+e^- colliders i.e. $e^+ + e^- \to t + \bar{c}(\bar{t} + c)$ in the minimal Standard Model (M.S.M.) framework at the LEP-II energy ($\sqrt{S} = 200GeV$) and the planned collider energy (e.g. $\sqrt{S} = 500GeV$). Straightforward extensions of the M.S.M. with four generations and two Higgs doublets as well are discussed.

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It is known that the minimal Stardand Model (M.S.M.) has archived a great success. However, with three generations of fundamental fermions i.e. quarks and leptons, only the sixth one, the top quark has not been discovered yet so far. Where is the top quark? The lowest bound of the top quark mass is directly obtained by experiments, i.e. the lowest bound by CDF[1] $\geq 91.GeV$. By means of a comprehensive analysis[2] with the M.S.M. and the indirect measurements, the most favourable value of the top-mass is

$$m_t = 132^{+31}_{-37}\ GeV. \tag{1}$$

If the central value is true, it is hopeful that the top quark will be discovered at Tevatron in CDF and D0 detectors in two or three years[3]. Whereas at CERN it is planned that the e^+e^- collider LEP will double its energy to $\sqrt{S} = 200GeV$, i.e. from LEP-I shifts to LEP-II. The energy is not high enough to produce a pair of top and antitop from phase space consideration, however it is available to produce such a quark pair: $t\bar{c}$ or $\bar{t}c$. Especially considering the clean environment of the e^+e^- collision, it is worth a while computing the production cross section of $t\bar{c}$ or $\bar{t}c$ at the energy of LEP-II precisely with the M.S.M. and its extension frameworks. It is because, on one hand, it will be a complement to those of $p\bar{p}$ colliders such as Tevatron for searching for the top quark, when LEP-II does seriously carry through the searching, the computed cross section is a valuable reference for the experimental study, and on the other hand, it may tell us certain hints for new physics if the cross section is measured precisely enough (as discussed below), as the nature of the process is of a high-order quantum effect i.e. through loop diagram(s) in the minimal S.M. and its certain extensions as well. Especially, any 'new' particles, when a specific extension is considered so the 'new' particles come into the calculated loop(s), thus, must influence the cross section. In fact, there

have been already a lot of efforts to make predictions based on various extensions of M.S.M., as well as quite a lot of constraints on the extensions, such as the lower bounds of the masses of the 'new' particles occurring in the extensions have been achieved. In this paper we do try two kinds of extensions: from one complex Higgs doublet and three generations of fundamental fermions in the case of the M.S.M. to two Higgs doublets and/or four generations of fermions. Moreover, since we think that to see 'new physics' in $e^+e^- \to t\bar{c}$ or $e^+e^- \to \bar{t}c$ is one of open windows better than that in $e^+e^- \to t\bar{b}$ or $e^+e^- \to \bar{t}b$ (for the later one is fixed by M.S.M. more than the former) so we incline to discuss the former one here. Considering the extensions and the e^+e^- colliders in plan for future we also try to calculate the cross section at the energy $\sqrt{S} = 500GeV$ too, although at such a high energy the top-pair production is available.

Respected with such a great mass as eq.(1) and in LEP-II, the top quark production may happen only through the single-top process. Here we consider $e^+e^- \to t\bar{c}$ or $e^+e^- \to \bar{t}c$, though there are some discussions about $t\bar{c}$ and $\bar{t}c$ production in e^+e^- colliders based on the the E_6 theory[3] and on the supersymmetry models[4]. The results of refs.[3,4] show that the production of the single t-quark inspired by the E_6 theory is observable, but depends on the parameters in it. The cross section of $e^+ + e^- \to t\bar{c}$ through Z^0 exchange based on the M.S.M. is expected being small[5], however, a complete calculation and a precise prediction of the process with the model should be performed because it is a background for certain new physics besides being a test of the M.S.M., if such experiments of the single t-quark production are carried out. Moreover the box diagrams of W-pair exchange will contribute an imaginary part when the internal lines of the bosons approaching to their mass shell(it is allowed by kinematics now), and it is interesting to know how great the imagenary part of the box diagrams is. Thus one may see that to compute the cross section of $e^+e^- \to t\bar{c}$ and $e^+e^- \to \bar{t}c$ precisely is so interesting, not only for t-quark searching but also for understanding the 'box' effects better, as well as for finding a clue to new physics.

Under the above consideration, we have completed the calculation and given a precise prediction on the production of the single t-quark associated with a charm based on the M.S.M., and its straightforward extensions to four generations with one or two Higgs doublets.

In the Feynman-t'Hooft gauge and according to the S.M., the diagrams in which an unphysical Higgs vertex $\phi^{1,2}e^+e^-$ or $\phi^{\pm}e^{\mp}\nu_e$ appears, can be omitted, since its coupling in the vertex is proportional to the electron mass which may be considered tiny in the concerned accuracy, thus there are totally twenty one diagrams contributing to the process $e^+e^- \to t + \bar{c}$ and can be divided into three categories as shown in Fig. 1. (Note: due to tiny electron mass all of the diagrams, containing one or more Higgs-electron vertices, have been ignored here.) For the first one as shown in Fig. 1.a, the spot contains ten diagrams: of them six irreducible diagrams and four reducible ones containing a reducible self-energy subdiagram of flavor-change, whereas due to the produced t and $\bar{c}$ on shell, the reducible ones do not contribute, so we shall not consider them at all. There were complete calculations on them but with massless-quark approximation[5]. For the second one as shown in Fig. 1.b, in the spot the situation is similar to the first one, not only the irreducible and reducible separation as mentioned above, but also with the same approximation to the calculations[6]. For the third category as shown in Fig. 1.c, in fact it contains only one box diagram at the lowest order. In order to pursue a systematical computation on the flavor-changing "vertices" in

the Stardand Model and without invoking the massless-quark approximation taken in refs.[5,6], all diagrams as shown in Fig.1 have been computed in ref.[7]. We shall apply the results of ref.[7] with brief illustration here only, as the calculations are very lengthy we would not repeat all details here.

The amplitude corresponding to the box diagrams (shown in Fig. 1.c) in a general form may be written as following

$$Am^{box} = c_{box} \sum_{q=d,s,b} V_{cq} \cdot V^*_{qt} \bar{v}(p_2)\gamma_\alpha\gamma_\nu\gamma_\beta L u(p_1)\bar{u}(p_3)\gamma^\alpha\gamma^\mu\gamma^\beta L v(p_4) A^{box}_{\nu\mu}, \quad (2)$$

where

$$\begin{aligned} A^{box}_{\nu\mu} = \; & b_{11}p_{1\nu}p_{1\mu} + b_{12}p_{1\nu}p_{2\mu} + b_{21}p_{2\nu}p_{1\mu} + b_{22}p_{2\nu}p_{2\mu} + b_{14}p_{1\nu}p_{4\mu} \\ & + b_{41}p_{4\nu}p_{1\mu} + b_{44}p_{4\nu}p_{4\mu} + b_{24}p_{2\nu}p_{4\mu} + b_{42}p_{4\nu}p_{2\mu} + b_0 g_{\nu\mu}, \end{aligned} \quad (3)$$

where the coefficients are integrals of the following form:

$$b_{ij} = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 f_{ij}(x_1, x_2, x_3, x_q). \quad (4)$$

The vertices $Z^0 q_t \bar{q}_c$ and $\gamma q_t \bar{q}_c$ may be calculated precisely[7]. The expressions of them have the form as follows:

$$\begin{aligned} \Gamma^{Z^0(\gamma)}_\mu(p,k) = \; & c_{z(\gamma)} \sum_{q=d,s,b} V_{cq} \cdot V^*_{qt}(E_2\gamma_\mu + E_3\hat{p}\gamma_\mu\hat{p} + E_4\hat{p}\hat{k}\gamma_\mu + E_5\gamma_\mu\hat{k}\hat{p} + \\ & E_6\hat{p}k_\mu + E_7\hat{k}k_\mu + E_8\gamma_\mu\hat{p} + E_9\hat{p}\gamma_\mu + E_{10}\gamma_\mu\hat{k} + E_{11}\hat{k}\gamma_\mu)L, \end{aligned} \quad (5)$$

the coefficients E_i are integrals too, but with lower dimensions than that of b_{ij}:

$$E_i = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 f_i(x_1, x_2, x_q) \quad (6)$$

The contribution of Fig. 1.a, containing the $Z^0 q_t q_c$ vertex , to the amplitude may be expressed as

$$Am^Z = c_z \bar{v}(p_2)\gamma_\nu(T^z_3 L - Q_e)u(p_1)\bar{u}(p_3)\Gamma^{Z^0}_\mu(p_4, p_1+p_2)v(p_4)\frac{g_{\nu\mu}}{((p_1+p_2)^2 - m_z^2)}, \quad (7)$$

and the contribution of Fig. 1.b, containing the $\gamma q_t q_c$ vertex, to the amplitude:

$$Am^\gamma = c_\gamma \bar{v}(p_2)\gamma_\nu u(p_1)\bar{u}(p_3)\Gamma^\gamma_\mu(p_4, p_1+p_2)v(p_4)\frac{g^{\nu\mu}}{(p_1+p_2)^2}. \quad (8)$$

Now the whole amplitude M consists of three parts:

$$M = Am^{box} + +Am^Z + Am^\gamma. \quad (9)$$

Later on throughout the calculations, with enough accuracy we accept the approximation $m_e = 0$ and $m_q = 0, (q = u, c)$, hence

$$\begin{aligned} d\sigma(e^+ + e^- \to t + \bar{c}) &= h(S) \cdot |M|^2(|\vec{p}^*|/\sqrt{S})d\cos\theta, \\ h(S) &= \frac{1}{4}(\frac{g^2}{4\pi})^2 \frac{3.94}{128\pi} \cdot \frac{1}{S} \cdot 10^{-28}. \end{aligned} \quad (10)$$

Here $\sqrt{S}$ is the total energy of e^+e^-; θ the angle between the directions of t quark and e^+; $\vec{p}^*$ the momentum of $\bar{c}$ quark in the C.M.S. frame. For simplicity, the unit $m_W = 1$ is adopted for the calculations in this paper, but in the results we restore the explicit dependence on m_W

$$\begin{aligned} & |\vec{p}^*| = (S - m_t^2)/4\sqrt{S}, \; x_e = \sqrt{S}/m_W, \; x_\nu = m_\nu/m_W, \\ & x_b = m_b/m_W, \; x_s = m_s/m_W, \; x_p = p^*/m_W, \; x_d = m_d/m_W, \end{aligned} \quad (11)$$

and the relations

$$\begin{aligned} & p_1 \cdot p_2 = \frac{1}{2}x_e^2 m_W^2, \; p_1 \cdot p_4 = \frac{1}{2}x_e x_p(1+z)m_W^2, \\ & p_2 \cdot p_4 = \frac{1}{2}x_e x_p(1-z)m_W^2, \; z = \cos\theta \end{aligned} \quad (12)$$

will be used in the below formulae.

The square of the matrix element for the process $e^+ + e^- \to t + \bar{c}$, $|M|^2$ is easy to be obtained by a straightforward calculation.

The above formulae are also suitable for the four-generation case, provided an extension on the CKM matrix is involved. Furthermore, in general it is impossible

to calculate out the integrals occurring in the calculations analytically, therefore we need to evaluate the integrals numerically. In fact as the integrals are principle ones it is not trivial to correctly evaluate the principle-value integrals numerically, especially here they have singularities in a high-dimensional region. In ref.[8] we solved this problem by suggesting a general method. Hence, by using the method, all integrations may be carried out without difficulties.

In this paper, the following parameters are input into the numerical calculation:

$$m_W = 81.GeV, \qquad m_Z = 92.GeV, \qquad m_s = 0.5GeV,$$
$$m_b = 5.GeV, \quad \frac{G_F}{\sqrt{2}} = \frac{e^2}{8m_W^2 \sin^2\theta_W}, \quad \sin^2\theta_W = 0.23 \tag{13}$$

The numerical results of $e^+ + e^- \to t + \bar{c}$ with a varied top mass are presented in Tab.1, where "Data 1" are those based on the minimal S.M., while "Data 2", those based on the extended one with four-generation fermions but still with one Higgs doublet. We should note that for the "Data 2" case, in the present calculations the fourth "bottom" quark mass $m_{b'}$ and the related K.M. matrix elements, as one of the prior and favorable choices, are taken as the following:

$$V_{cs} \cdot V_{st}^* = -0.5, \quad V_{cb} \cdot V_{bt}^* = 0.05,$$
$$V_{cb'} \cdot V_{b't}^* = 0.45, \quad m_{b'} = 160.GeV. \tag{14}$$

Tab.1

$E(e+e-)$	200. GeV	200. GeV	500. GeV	500. GeV
M_t GeV	$\sigma(10^{-5}pb)$	$\sigma(10^{-4}pb)$	$\sigma(10^{-5}pb)$	$\sigma(10^{-4}pb)$
	Data 1	Data 2	Data 1	Data 2
115.	1.36	1.99	$1.04 \cdot 10^{-3}$	1.57
140.	.74	1.96	.869	1.97
165.	.71	1.43	1.09	2.45
190.	.041	.15	2.63	3.85

The results (Tab.1) show that the cross section $\sigma(e^+e^- \to t\bar{c})$ for the M.S.M is much smaller than that in the four generation extension. The key point is easy

to understand: In the M.S.M., we roughly have

$$|M|^2 \propto |\sum_{q=d,s,b} V_{cq} V_{tq}^* f(x_q)|^2 \simeq |V_{cs} V_{ts}^* (x_b^2 - x_s^2) f'(x_b)|^2, \tag{15}$$

where a very small factor $(x_b^2 - x_s^2) = \frac{m_b^2 - m_s^2}{m_W^2}$ is contributed by the GIM-like cancellation [9] (the unitarity of the CKM matrix), and another factor $V_{cs}V_{ts}^*$ containing an off-diagonal CKM matrix element V_{ts}^* is also very small as known by indirect measurements via the unitarity of the CKM matrix, i.e. numerically these two factors:

$$(x_b^2 - x_s^2) = \frac{m_b^2 - m_s^2}{m_W^2} \simeq 0.004.$$
$$0.058 \times 0.9752 > V_{cs}V_{ts}^* > 0.029 \times 0.9734, \tag{16}$$

The second constraint above is in fact from the unitarity of three generations. If the 4th generation exists, probably it will be changed greatly. The two factors act together so as to make the final results so small.

In the case for the extension of the four generations, as heavy quarks will decouple when their masses become very large, i.e

$$f(x_q)_{x_q \to \infty} \to 0, \tag{17}$$

hence the 4th generation term of $f(x_b')$ will not give a big contribution to the cross section, in other words, the result is not sensitive to the mass of 4th generation quark b' when $m_{b'} \gg m_b$, and we obtain

$$|M|^2 \propto |\sum_{q=s,b} V_{cq} V_{tq}^* f(x_q)|^2. \tag{18}$$

However, unlike in the M.S.M. case $V_{cs}V_{ts}^* \simeq -V_{cb}V_{tb}^*$, now in the four-generation case in general, $V_{cs}V_{ts} \not\simeq V_{cb}V_{tb}$, that breaks the GIM cancellation for the three-generation case, of cause the final result still depends on the choice of the extented

CKM matrix elements when the 4th generation is involved. In Tab.1 it is that the most favourable choice on the matrix elements for the problem is adopted, so it is almost the maximum value of the cross section for the extension.

In the calculation, we do find that the box diagram contributes an imaginary part to the amplitude substantially as expected before the precise calculation when the center mass system energy reaches so high such as $\sqrt{S} = 200GeV$ and higher.

Now let us consider the extensions with two Higgs doublets. The remarkable difference from the M.S.M, now is one couple of physical charged Higgs' will be involved in the model and its coupling to the third (the fourth also) quarks-top and bottom is comparatively strong (proportional to m_t) that in Fig. 1.a and Fig. 1.b more diagrams, involving at least one charged Higgs exchange in the spot, will be added and the cross section of the single-top production is expected to undergo a substantial change. After a precise calculation with care we do find the cross section increasing greatly due to the charged Higgs contributions when choosing the most favourable but tolerable parameters for the extension.

As done above in the M.S.M. case, we ignore all diagrams in which as long as a $H\bar{e}e$ or $H^+\bar{e}\nu$ or $H^-\bar{\nu}e$ vertex is involved, as their contributions to the amplitude are proportional to the electron mass. As a matter of fact, the diagrams, inside the spot in Fig. 1.a and Fig. 1.b with a charged Higgs coupling to the bottom b or b' (if there is the four th generation) and a very heavy top t, especially when b' exists and $m_{b'} = 160GeV$ (even greater) for example, contribute to the amplitude so greatly that it may dominate over the rest diagrams and make the process being visible in LEP-II (Note that if there is no b' the cross-section is not so great). We plot the numerical results versus the parameters $\tan\beta$[10] and top-mass in Fig. 2. and Fig. 3 respectively, based on the extension with two possible models (Model I and Model II, see ref.[10]) and four generations as well. One may see from the Figs.2,3. that only for the one with two Higgs doublets of the considered extensions, the single-t production may be visible at LEP-II when the parameter $\tan\beta$ falls into the region $\tan\beta \leq 0.1$ for Model-I and $\tan\beta \geq 10.$ for Model-II.

In conclusion, we have found that if m_t is in the region as shown in eq.(1) (not very big), the single-t quark production with a charm quark in e^+e^- colliders, $\sigma(e^+e^- \to t\bar{c})$, is very small according to the minimal Stardand Model prediction, that it cannot be seen in LEP II even in those colliders ($\sqrt{S} = 200GeV, 500GeV$ and higher) which are now under consideration. Even for the extension only generations being extented, the cross section may reach to such an order $3.4 \times 10^{38}\ cm^2$, much larger than that of the M.S.M., but still beyond the capacity of LEP-II facilities. Thus if the single-t quark production with a charm quark $e^+e^- \to t\bar{c}$ is found at LEP II, there must be something beyond the minimal Stardand Model and the simplest four-generation extension as well, while one of possibilities is the Higgs sector extended, i.e. two electroweak complex doublets instead of one.

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Figure Captions

1. The Feynman diagrams for $e^+e^- \to t\bar{c}$.

2. $\sigma(e^-e^+ \to t\bar{c})$ as a function of $\tan\beta = \frac{v_2}{v_1}$ with $\sqrt{S} = 200GeV$, $m_t = 140GeV$ and $m_{b'} = 160GeV$ (the thin line represents that with $m_H = 50GeV$ in Model I; the thick one with $m_H = 100GeV$ in Model I; the dashed one with $m_H = 50GeV$ in Model II; the dotted one with $m_H = 100GeV$ in Model II.)

3. The dependence of $\sigma(e^-e^+ \to t\bar{c})$ on the top mass m_t. The solid curve represents that with $\tan\beta = 0.02, m_{b'} = 160GeV$ for Model I. The dashed-dotted one with $\tan\beta = 100, m_{b'} = 160GeV$ for Model II. The dotted one with $\tan\beta = 100, m_{b'} = 200GeV$.

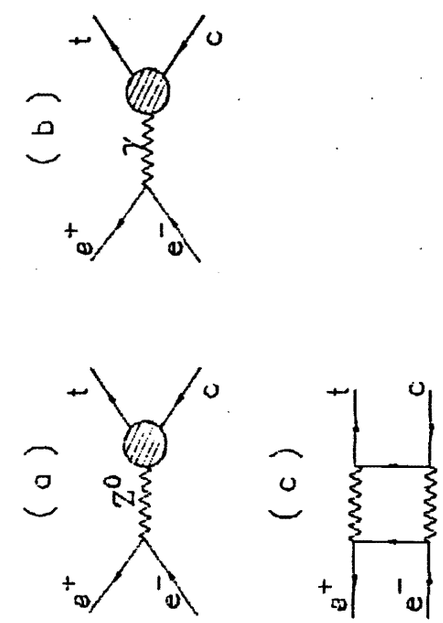

Fig. 1

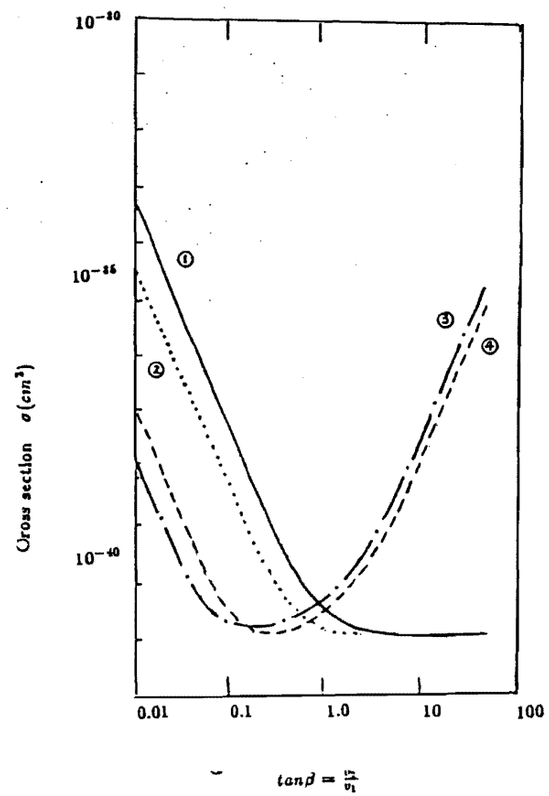
10^{-30}
10^{-35}
10^{-40}
Cross section $\sigma(cm^2)$
①
②
③
④
0.01
0.1
1.0
10
100
$tan\beta = \frac{v_2}{v_1}$

Fig.2

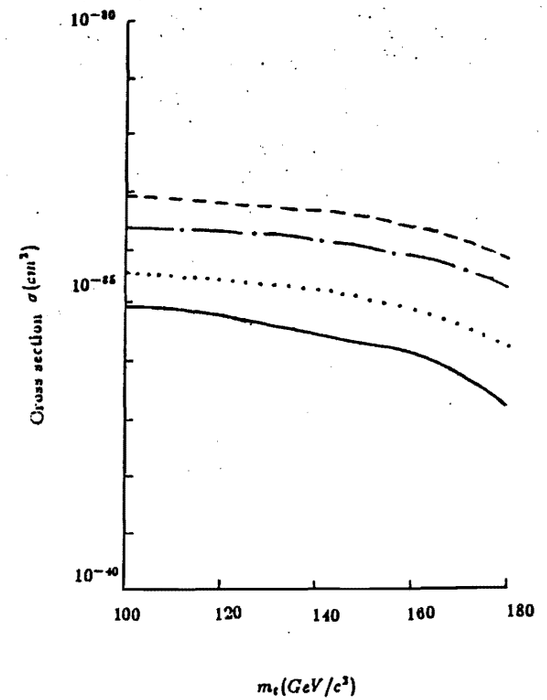
10^{-30}
10^{-35}
10^{-40}
Cross section $\sigma(cm^2)$
100
120
140
160
180
$m_t(GeV/c^2)$

Fig.3