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ANL-HEP-CP-93-95
June 1993

Monte Carlo Event Generators for Hadron-Hadron Collisions

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ABSTRACT

A brief review of Monte Carlo event generators for simulating hadron–hadron collisions is presented. Particular emphasis is placed on comparisons of the approaches used to describe physics elements and identifying their relative merits and weaknesses. This review summarizes a more detailed report[1].

1. Introduction

Today Monte Carlo event generators are essential to the design of experiments at hadronic colliders, to understand the signatures of interesting processes and how to separate these processes from potential backgrounds. Ideally such programs use QCD and electroweak theory to predict/reproduce complete events, which can be treated as equivalent to actual data for the purpose of analysis. In principle this is done by solving QCD (up to some high order), in practice this is computationally not possible. Approximations to the full PQCD theory are needed as are models for non-perturbative soft physics.

Given the use of approximations and models in Monte carlo event generation it is therefore extremely foolish to become reliant on the results of one program or approach without first exercising critical judgement as to its merits and deficiencies. The intention of this report is to provide the reader with some insight into the methodology of event generators, allowing her to make sound choices and hopefully try to ensure successful applications.

1.1. General Background

As a preliminary we quickly summarize the main components of Monte Carlo event generators. PQCD Factorization theorems[2], such as for high p_T hadron production, allow cross-sections to be decomposed into process independent, but scale dependent, structure/fragmentation functions and specific hard scattering matrix elements.

$$\sigma = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dz \sum_{ab\to cd} F_a^{h_1}(x_1,Q^2) F_b^{h_2}(x_2,Q^2) \sigma_0^{(ab\to cd)}(\hat{s},\hat{t},\hat{u}) D_h^c(z,Q^2) \qquad (1)$$

where (higher twist) corrections are suppressed by powers of the characteristic scale, $(\log Q)^m/Q^n$. The significance of Eq.(1) lies in the universality of $F_a^h(x,Q^2)$ and $D_h^c(z,Q^2)$ which facilitates a simple event generator implementation. Note that the large collinear $\log Q^2$ and soft corrections have been summed to give Q^2 dependent functions governed by Altarelli-Parisi type evolution equations[3]. For example:

$$t\frac{\partial F_a^h(x,t)}{\partial t} = \int_x^1 \frac{dz}{z} \sum_{b\to aa'} \frac{\alpha_s}{2\pi} \hat{P}_{aa'}^b(z) F_b^h\left(\frac{x}{z},t\right) \qquad (2)$$

where t is an appropriate scale variable and the $P_{bb'}^a(z)$ are the regularized splitting functions. In fact since PQCD is only applicable above a certain scale t_s it is useful to further decompose this structure function into a PQCD calculable piece and a nonperturbative piece which has to be determined from experiment.

$$F_a^h(x,t) = \int_x^1 \frac{dy}{y} \sum_b f_a^b(y,t;t_s) F_b^h\left(\frac{x}{y},t_s\right) \qquad (3)$$

It must be admitted though that the generalizations of the factorization theorems necessary for multi-particle production are not proven.

With this QCD improved parton model framework in mind we can identify the five main components in an event generator (prior to detector simulation).

1. A primary hard scatter calculated as in Eq.(1) using standard, textbook, cross-sections, (but see subsection 3.2.1).
2. (M)LLA corrections are generated via initial and final state parton showers, corresponding to the $f_a^b(y,t;t_s)$ in Eq.(3), this explicit radiation generates secondary partons.
3. Below the fixed time-like cascade cut-off t_0 a non-perturbative model for the transition, embodied in the analogue of Eq.(3) by $D_b^h(z,t_0)$, from final state partons to hadrons is employed.
4. A further model is used to treat the residual beam partons and generate a soft underlying event.
5. Finally the unstable particles produced in the previous two stages are decayed, typically according to PDG inspired tables and simple matrix elements.

Before discussing the above components in the following sections we comment on the presently available event generators.

1.2. Available Monte Carlo Programs

There are basically two types of programs available for event simulation, primarily differing in the extent to which they attempt to describe all aspects of a particle collision. At one extreme are the general purpose programs that contain a diverse menu of physics processes and aim to generate complete events. At the other extreme are specific programs that try to simulate a single class of physics process more accurately.

General purpose programs usually generate a process accurate only to first leading order and then rely upon QCD cascade approximations to simulate higher orders. All of the above five steps are followed to provide an output that can be passed directly to an experiment's detector simulation packages. The goal is to produce computer generated events that can be treated in exactly the same way as real data. The generic programs publicly available include COJETS[4], HERWIG[5], ISAJET[6] and PYTHIA[7]; together with DTUJET[10], FRITIOF[11] and HIJING[12] which only handle QCD jets and low transverse momentum events. FIELDAJET[13] is not publically available.

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The more specialized programs typically contain exact (or almost exact) matrix elements for specific processes and their end products are commonly at the parton level. Thus the majority of these programs take care only of step one above. A typical program works using the following sequence of steps:

1. Generate a point in the multi-particle phase space, for example using RAMBO[14].
2. Apply any experimental cuts to this point and reject accordingly.
3. Assign a weight to this point equal to the exact cross-section.
4. Calculate the desired observables for this event and add its weight to the appropriate bins for these distributions.

This basic procedure can then be repeated within the framework of a numerical integration package, such as VEGAS[15], to calculate the convolution over structure functions. Significant complications can arise at the one loop level when divergences have to be isolated and cancelled, usually analytically. It will immediately be realised that the events are weighted so that the output cannot be treated exactly as data. A significant advantage of using weighted events is that sampling a particular region of phase space, such as a distribution's tail, can easily be done without waste, within the above structure. At the risk of statistical inefficiency directly applying an accept/reject procedure will convert weighted events to unweighted form. The output from the specialized programs, when suitably organised, may be used as input for a general purpose event generators. Examples of such programs are EUROJET[16] and PAPAGENO[17] which both contain a diverse menu of sub-processes or NJETS[18] and VECBOS[19] which both concentrate on particular topics.

We will concentrate our discussion on the following generic programs. Note it is important to specify the version number since programs are regularly upgraded.

COJETS 6.23 An incoherent parton shower Monte Carlo for jet and $W^{\pm}/Z^0$ production. It uses a (modified) independent hadronization scheme and a parameterization of data for the underlying event.

FIELDAJET This is not a publicly available program, we comment on it as we are able.

HERWIG 5.6 A multi-process coherent, parton shower Monte Carlo; with cluster hadronization and an underlying event model based on data.

ISAJET 7.0 A multi-process, incoherent parton shower program using independent hadronization and an underlying event model based on the AGK cutting rules.

PYTHIA 5.6 A multi-process Monte Carlo, which uses JETSET 7.3 to provide coherent final state showers, string hadronization and decays. Its underlying event is based on multiple parton scattering.

DTUJET92 A non-shower Monte Carlo employing Dual Topological Unitarization to give a unified description of soft and hard QCD processes. It incorporates the programs BAMJET for string fragmentation and DECAY for resonance decays.

FRITIOF 7.2 A program using excited strings to give a unified description of soft and hard QCD scattering. It uses PYTHIA 5.6 for matrix elements, ARIADNE4.02r* for dipole showers and JETSET 7.3 for string hadronization and decays.

HIJING 1.0 A Monte Carlo providing a unified description of soft QCD and multiple scattering in an eikonal formalism. It incorporates components of ARIADNEand FR for the dipole cascades of excited strings and JETSET 7.3 their hadronization and decay.

EUROJET A multi-process matrix element Monte Carlo employing (modified) independent fragmentation and incorporating the EURODEC decay package.

PAPAGENO A multi-process, parton level, matrix element Monte Carlo.

This is summarized in the following table:

Monte Carlo Program	Hard M.E. QCD	Hard M.E. E.W.	Perturbative Cascades	Underlying Soft Event	Hadronization Model	Unstable Hadron Decays
COJETS 6.23	yes	yes	parton	param.	indep.	yes
FIELDAJET	yes	yes	parton	indep.	indep.	yes
HERWIG 5.6	yes	yes	parton	param.	cluster	yes
ISAJET 7.0	yes	yes	parton	AGK	indep.	yes
PYTHIA 5.6	yes	yes	parton	multi	string	yes
DTUJET92	yes	•	•	DTU	string	yes
FRITIOF 7.2	yes	•	dipole	multi	string	yes
HIJING 1.0	yes	•	parton	multi	string	yes
EUROJET	yes	yes	•	indep.	indep.	yes
PAPAGENO	yes	yes	•	•	•	•

Table 1: The basic program elements contained within the main Monte Carlo event generators. •'s indicate the absence of these components. See above for more comments.

We remark that many of these programs allow many more beam options than hadron–hadron collisions. Thanks to factorization this is important for tuning program parameters; however the nuclear options of DTUNUC (DTUJET's sister program), FRITIOF and HIJING are unlikely to prove useful in this respect. Finally note that fixed target experiments can also be treated by boosting to and from the Centre of Momentum (CoM) frame for event generation: necessary due to certain approximations used.

2. Available Scattering Processes

The number of processes included in the generic programs is constantly being increased so the following summary is at best a snap shot of the moment. The tables below cover the three main Monte Carlos and we comment very briefly on the available matrix elements. We statrt with processes found in the minimal standard model.

*This program differs from the standard ARIADNE by a renamed common block.

Process	Monte Carlo		
	HERWIG	ISAJET	PYTHIA
QCD			
QCD jets	yes	yes	yes
$q\bar{q}, gg \to t\bar{t}$	yes	yes	yes
$q\bar{q}, gg \to t\bar{t}\gamma\gamma$	no	no	no
$qb \to q't$	yes	no	yes
direct $\chi_c, \chi_b, J/\psi, \Psi$	no	yes	no
minimum bias	yes	yes	yes
diffractive	no	yes	yes
elastic	no	no	yes
Drell-Yan			
$q\bar{q} \to V$	yes	yes	yes
$q\bar{q} \to Vg,\ gq \to Vq$	yes	yes	yes
$gg \to VQ\bar{Q}$	no	no	yes
Prompt photon			
$qg \to q\gamma,\ q\bar{q} \to g\gamma$	yes	yes	yes
$gg \to \gamma g$	yes	no	yes
$q\bar{q} \to \gamma\gamma$	yes	yes	yes
$gg \to \gamma\gamma$	yes	no	yes
W/Z pair production			
$q\bar{q} \to VV, V\gamma$	no	yes	yes
$VV \to VV$	yes	yes	yes
$gg \to VV$	no	no	no
$qq \to VVqq$	no	no	no
Standard Model H^0			
$q\bar{q}, gg, VV \to H^0$	yes	yes	yes
$q\bar{q} \to VH^0$	no	no	yes
$q\bar{q}, gg \to t\bar{t}H^0$	no	no	yes
$H^0 \to VV, V^*V^*, f\bar{f}, \gamma\gamma, gg$	yes	yes	yes
$H^0 \to \gamma Z$	no	no	yes

Table 2: Standard Model Processes in event generators.'f' stands for fermions, 'l' for leptons,'V' for W or Z and 'Q' for heavy quarks.

Minimum Bias Events containing no (identified) hard scattering make up the bulk of the hadronic cross-section and are treated as beam fragmentation. No rigorous QCD treatment exists so the models used are fairly arbitrary, being tuned to reproduce present collider data, extrapolations to LHC/SSCenergies are suspect. See also section 6.

Diffractive Processes Again the absence of a rigorous QCD theory means that only model based parameterizations are used. No program is presently publicly available to treat hard diffraction[23,24]. See also [25].

QCD Jet Production Whilst the $O(\alpha_s^2)$ matrix elements are standard different implementations exist: HERWIG, PYTHIA and HIJING use colour decomposed expressions, the latter two neglecting interference; FRITIOF only includes diagonal, 'Rutherford' scatterings. Tree level expressions for up to six jet production exist[26] together with a partonic event generator NJETS[18]. Two one loop partonic event generators exist[27] for two jet production.

Heavy Flavour Production Here pair production is finite as $p_T \to 0$, whilst the necessary structure functions are required for flavour excitation. At LHC/SSC energies $g \to Q\bar{Q}$ in cascades is the dominant source of b-quarks. The process[28] $W + g \to Q + \bar{q}$, which appears in EUROJET, is also an important source of heavy flavours if $m_Q \gg m_q$. A partonic event generator[29] implimenting one loop corrections exists. Note large threshold corrections occur for which a specialized Monte Carlo treatment has been developed[30]. See also [31] and [32].

Drell-Yan processes When $Q^2/s \ll 1$ large loop corrections to this process exist, which can be partially incorporated using the K factor[33] $K = \exp(2\pi\alpha_s/3)$. Note the only source of transverse momentum here is initial state bremsstrahlung. Exact tree level matrix elements exist for W/Z and up to three jet production and are made available in VECBOS[19] (an interface to ISAJET exists for this program). A one loop monte carlo for $W + 1j$ is alos available[34].

Prompt Photon Production At LHC/SSC energies gluon fusion is important for both single and double photon production; often a mass quark approximation is used for the box diagram. One loop corrections and two partonic event generators[35] are available for single photon production; as is a one loop partonic event generator[36] for double photon production. See also [37]

Direct Meson Production The higher twist production of S-wave mesons is available in the TWISTER Monte Carlo[38]. Standard P-wave quarkonium production is also available however recent theoretical developments[39] are not included. Note hadronization also provides a leading twist source of such mesons.

W, Z Pair Production These processes are presently included using $q\bar{q}$ fusion rather than the full $q\bar{q} \to q\bar{q} + VV$ matrix elements of the full theory. A number of partonic monte carlos are available including VVJET[40] for pair plus jet production and GGZZ[41] for z-pair production from gltuon fussion.

Standard Model Higgs production Due to the unknown Higgs mass a large number of different production and decay channels must presently be included. We note that at intermediate mass $gg \to H$ dominates and care is need near $m_h = 2M_Z$ to allow for off resonance decay channels: $t\bar{t}$ fussion is also non-negligable, a Monte Carlo is available from the ATLAS collaboration. Above $2M_Z$ the dominant Higgs decays are to W^+W^- and Z^0Z^0 pairs so that vector boson fussion diagrams increaseingly become important, interference terms become important for $M_H > 700$ GeV/$c^2$when the resonance is nolonger narrow.

Baryon Number violation Due to the existence of the chiral anomaly, baryon (and lepton) number violation is allowed within the MSM, specifically:

$$q + q \longrightarrow (N_c N_g - 2)\bar{q} + N_g \bar{l} + n_W W^{\pm} + n_Z Z + n_H H \qquad (4)$$

It is possible to simulate this process using the HERBVI package[42], written for use with HERWIG.

A large number of high p_T process beyond the minimal standard model have been envisaged and no event generator can realistically hope to offer a comprehensive treatment. We mention some of the main variants.

Process	Monte Carlo		
	HERWIG	ISAJET	PYTHIA
Non-standard Higgs			
$Z^* \to h^0 A^0, H^0 A^0$	no	no	yes
$q\bar{q}' \to H^+$	no	no	yes
$gb \to H^- t$	no	no	yes
$\gamma^*, Z^* \to H^+ H^-$	no	no	yes
$t \to H^+ b$	no	no	yes
Supersymmetry			
$q\bar{q}, gg \to \tilde{q}\tilde{q}, \tilde{g}\tilde{g}$	no	yes	no
$qg \to \tilde{q}\tilde{g}$	no	yes	no
$q\bar{q} \to \tilde{g}\tilde{V}, \tilde{V}\tilde{V}$	no	yes	no
$qg \to \tilde{q}\tilde{V}$	no	yes	no
MSSM decay modes for $\tilde{q}, \tilde{g}, \tilde{V}$	no	yes	no
Miscellaneous processes			
$q\bar{q} \to \rho_T \to VV$	no	yes	no
$VV \to \rho_T \to VV$	no	no	yes
$q\bar{q} \to V' \to f\bar{f}$	yes	no	yes
$q\bar{q} \to V' \to VV$	no	no	yes
$VV \to V' \to f\bar{f}, VV$	no	no	no
$q\bar{q}' \to R \to q''\bar{q}'''$	no	no	yes
contact interactions	no	no	yes
leptoquarks	no	no	yes
strongly interacting V	no	no	yes
q^*(excited fermions)	no	no	yes
B number violation	yes	no	no

Table 3: Non-Standard Model Processes in event generators. Same notation as table 1. 'V' for W' or Z', 'R' for horizontal boson.

supersymmetry ISAJET and more specifically ISASUSY[43] offer the most complete treatment of SUSY, in particular the cascade decays of superparticles are carefully treated.

The Minimal SUSY model is assumed for which several parameters must be specified. A purpose built partonic event generator UA2SUSY[44] also exist.

Non-Minimal Higgs Bosons Both ISAJET and PYTHIA consider the (CP conserving) two-Higgs-doublet model, which contains three neutral Higgs, one of which is a pseudoscalars, and a pair of charged scalars. Both single and pair production processes are included.

Technicolor ISAJET provides the option of generating a technicolour ρ of arbitrary mass and width decaying into $W\,Z$ or W^+W^- pairs. The cross section is based on an elastic resonance in the WW cross section with the effective W approximation plus a W mixing term.

2.1. *Comments*

We now offer a number of comments concerning Monte Carlo generated events and their applicability/validity.

The generic programs allow higher order radiative corrections to be generated using the approximations inherent in their cascade algorithms. Typically deviations from known results occurring only in regions of phase space which are not populated, due to dynamical constraints placed on the gluon radiation. Consider Drell-Yan $W^{\pm}$ production. For $p_T \lesssim \sqrt{Q^2} \approx M_W$ the lowest order matrix element should prove adequate, except perhaps for the high precision $W^{\pm}$ mass measurement, above this an explicit jet should be included.

One loop Monte carlos are most valuable when correlations between produced particles are being measured. Otherwise using a simple k-factor with a tree level matrix element will often suffice.

It is instructive to compare[45] the Monte Carlo† estimates of the $W + n$ jet cross-sections with the exact matrix elements[19]. This is done by starting from either the $2 \to 1$ or $2 \to 2$ process and using the shower algorithm to generate additional jets. Apart from the rapidity distributions, starting with $W^{\pm} + 0$ jets leads to a poor reproduction of the higher order matrix elements, essentially due to the lack of jets above the scale M_W. Significantly better agreement with the differential distributions is obtained by starting from $W^{\pm} + 1$ jet configurations; with the possible exception of the E_T^{jet} and p_T^W distributions, which at larger values are under/over estimated respectively. This good agreement demonstrates that the prescription for gluon radiation extrapolates reliably into the non-soft, non-collinear region, in particular for initial state radiation.

In terms of the transverse energy cut, $\sqrt{p_T^2 + M_W^2} \geq E_T^{min}$, used to regularize the two-to-two cross-section the following empirical factor is need to reproduce the (lowest order) total cross-section.

$$\sigma(W + n \text{ jet}) = 1.6\left(\frac{M_W^2 + (E_T^{min})^2}{M_W^2}\right)^{2(n-1)} \times \sigma_n(W + 1 \text{ jet}) \qquad (5)$$

When $E_T^{min} < M_W$ most jets are predominantly produced with low E_T and the constant 1.6, consistent with the $O(\alpha_s)$ K-factor, is sufficient. However, when $E_T^{min} \gtrsim M_W$ the Monte Carlo progressively underestimates the exact matrix element as $n(-\text{jets})$ increases and requires

†The actual program used was HERWIG but the observations are generally applicable.

an *ad hoc* compensating factor. The discrepancies with the exact matrix element become significant when the $W^{\pm}$ and a jet do not form the hardest subsystem; this can be traced to the scale ordering enforced in the showers and the increased combinatorics for larger n. The best solution to this problem is likely to involve implementing Weak bosons, $W^{\pm}$, Z^0 and γ, in the parton showers and then include contributions beginning from the QCD two-to-two subprocesses.

In principle similar remarks also hold for photon bremsstrahlung, which becomes very important at LHC/SSC energies. However due to the need for minimum energy and isolation cuts regions of phase space where the cascade algorithms are a poor approximation are probed. A likely solution to this is to use exact matrix element matching. A further consequence is that initial state photons, only presently included in PYTHIA, are likely to be as important as final state radiation.

2.2. *Adding New Sub-Processes*

Given the modular nature of the various multi-purpose Monte Carlo programs the task of incorporating a new hard scattering sub-process is greatly eased. The first requirement is an efficient generator for the momenta of the particles involved in the scatter, distributed according to the appropriate matrix elements. This essentially reduces to the problem of finding a set of kinematic variables in terms of which the cross-section is relatively flat: some guidance can be obtained from existing sub-routines. In the case of COJETS and ISAJET, which have incoherent parton cascades, and EUROJET, which has none, it is sufficient to pass the correctly labelled partons on to the next stage of event generation. However for HERWIG and PYTHIA, which have coherent parton cascades, it is necessary to also supply the colour connections of the partons, so that correct phase space boundaries are used. For principally electroweak process where only one colour flow is involved this is straightforward: when the strong interaction plays a significant rôle then it is likely that the colour connections are no longer unique. It is now required that each colour flow is separately given a weight, as discussed in sub-subsection 3.2.1: the HERWIG and PYTHIA prescriptions differing only in the treatment of the $1/N_c$ suppressed interference terms. Finally in the case of HERWIG you may wish to include spin correlations for gluon(s) involved in a scatter, see [20]; though this can safely be ignored.

3. Parton Showers

Thanks to QCD's asymptotic freedom property[46] perturbation theory can be applied to the analysis of hard processes. For example, the ratio of hadronic to point-like muon cross-sections in e^+e^- annihilation[47] is calculable from a finite number of diagrams at each order in α_s.

$$R_{e^+e^-}(Q^2) = N_c \sum_{i=1}^{n_f} Q_i^2 \left\{ 1 + \frac{3}{4} C_F \alpha_s(Q^2) + \cdots \right\} \tag{6}$$

This is possible because for an inclusive quantity such as $R_{e^+e^-}$ the singular terms present in both real and virtual diagrams are guaranteed by unitarity to mutually cancel on quite general grounds[48]. In contrast, for semi-inclusive observables, such as jet distributions, the need to employ cut-offs as regulators means that the cancellations are only partial. This leaves behind large logarithmic remainder terms which can compensate the decrease of $\alpha_s(Q^2)$ so that an infinite number of diagrams contribute to leading order in this case.

By considering the denominator of an internal propagator two types of singularity can be identified: collinear and infra-red. In the former case large coefficients of the form $\log(Q^2/Q_0^2)$ arise, where Q_0^2 is a virtuality cut-off used to delimit the perturbative regime: $\alpha_s(Q_0^2)/\pi \lesssim 1$. Since $\alpha_s(Q^2) \sim \log^{-1}(Q^2/\Lambda^2_{\rm QCD})$ it is clear that a rearrangement of the perturbation series in $\alpha_s(Q^2)$ is required:

$$\begin{aligned} \sigma = & \sum A_n \left[\frac{\alpha_s(Q^2)}{\pi} \log_e\left(\frac{Q^2}{\mu^2}\right) \right]^n + \frac{\alpha_s(Q^2)}{\pi} \sum_n B_n \left[\frac{\alpha_s(Q^2)}{\pi} \log_e\left(\frac{Q^2}{\mu^2}\right) \right]^n \\ & + \frac{\alpha_s^2(Q^2)}{\pi^2} \sum_n C_n \left[\frac{\alpha_s(Q^2)}{\pi} \log_e\left(\frac{Q^2}{\mu^2}\right) \right]^n + \cdots \end{aligned} \tag{7}$$

The first (infinite) set of terms gives the Leading (collinear) Logarithm Approximation (LLA). Higher sets of terms are genuinely suppressed, for $\alpha_s(Q^2)/\pi < 1$, and give the Next to LLA (NLLA), *etc.* Well established techniques are now available to identify and sum using evolution equations the diagrams which give rise to these leading collinear corrections[22]. In particular in an axial gauge only 'ladder' or more generally 'rainbow' diagrams need be considered[21,49], interference or crossed rung diagrams being subleading. This ensures that the parton model language is appropriate and facilitates a description of the leading contributions in terms of a classical Markov process, the jet calculus[50], which allowed the development of Monte Carlo event generators[51,52].

The relevant evolution equations and solutions, as appropriate for Monte Carlo usage, are now outlined.

3.1. *The Altarelli-Parisi Equations: Monte Carlo Solution*

Both the evolution of initial state structure functions[3] and final state fragmentation functions[53] are governed by a set of coupled integro-differential equations, commonly called Altarelli-Parisi equations.

$$t\frac{\partial}{\partial t} F_a^h(x,t) = \int_x^{1-\epsilon_{a'}(t)} \frac{dz}{z} \frac{\alpha_s(t,z)}{2\pi} P_{aa'}^b(z) F_b^h\left(\frac{x}{z},t\right) - F_a^h(x,t) \sum_{a\to cc'} \int_{\epsilon_c(t)}^{1-\epsilon_{c'}(t)} dz \frac{\alpha_s(t,z)}{2\pi} P_{cc'}^a(z) \tag{8}$$

$$t\frac{\partial}{\partial t} D_h^a(x,t) = \int_x^{1-\epsilon_{b'}(t)} \frac{dz}{z} \frac{\alpha_s(t,z)}{2\pi} P_{bb'}^a(z) D_h^b\left(\frac{x}{z},t\right) - D_h^a(x,t) \sum_{a\to cc'} \int_{\epsilon_c(t)}^{1-\epsilon_{c'}(t)} dz \frac{\alpha_s(t,z)}{2\pi} P_{cc'}^a(z) \tag{9}$$

with the (unregularized) splitting functions are given by:

$$\begin{aligned} P_{qq}^q(z) &= C_R[z^2 + (1-z)^2] & C_R &= 1/2 \\ P_{qg}^q(z) &= C_F \frac{1+z^2}{(1-z)} = P_{gq}^q(1-z) & C_F &= \frac{N_c^2-1}{2N_c} \\ P_{gg}^g(z) &= C_A 2\left[\frac{(1-z)}{z} + \frac{(1-z)}{z} + z(1-z)\right] & C_A &= N_c \end{aligned} \tag{10}$$

To lowest order the initial and final state splitting kernels have exactly the same form, but begin to differ at $O(\alpha_s^2)$. This separation of the radiation into intial and final state is actually gauge dependent and strictly only their sum is observable. Indeed by a suitable (axial) gauge choice it is possible to arrange for one parton in a hard scattering not to radiate at all: this possibility is pursued in COJETS[4]. The choice and exact definition of the evolution variable, t, and splitting fraction, z, is implementation specific, see below.

Final State Showers Introducing a Sudakov form factor[54] defined as:

$$\log_e \Delta^a(t,t_0^a) = -\int_{t_0^a}^t \frac{dt'}{t'} \sum_{a\to cc'} \int_{\epsilon_c(t')}^{1-\epsilon_{c'}(t')} dz \frac{\alpha_s(t,z)}{2\pi} P_{cc'}^a(z) \tag{11}$$

Eq.(9) can be re-written as an integral equation, with series solution:

$$D_h^a(x,t) = \Delta^a(t,t_0^a)D_h^a(x,t_0^a) + \int_{t_0^a}^{t}\frac{dt_1}{t_1}\int_x^{1-\epsilon_{a'}(t_1)}\frac{dz_1}{z_1}\Delta^a(t,t_1)\frac{\alpha_s(t_1,z)}{2\pi}P_{aa'}^b(z_1)\Delta^b(t_1,t_0^b)D_h^b\left(\frac{x}{z_1},t_0^b\right)$$
$$+\int_{t_0^a}^{t}\frac{dt_1}{t_1}\int_x^{1-\epsilon_{a'}(t_1)}\frac{dz_1}{z_1}\int_{t_0^b}^{t_1}\frac{dt_2}{t_2}\int_{x/z_1}^{1-\epsilon_{b'}(t_2)}\frac{dz_2}{z_2}$$
$$\times\Delta^a(t,t_1)\frac{\alpha_s(t_1,z_1)}{2\pi}P_{aa'}^b(z_1)\Delta^b(t_1,t_2)\frac{\alpha_s(t_2,z_2)}{2\pi}P_{bb'}^c(z_2)\Delta^c(t_2,t_0^c)D_h^c\left(\frac{x}{z_1z_2},t_0^c\right)$$
$$+\cdots \tag{12}$$

Here each term corresponds to a possible parton cascade in an event generator, and is implemented as a series of iterated steps.

Initial State Showers Employing exactly the same method as in the final state case the following series solution to Eq.(8) is obtained:

$$F_a^h(x,t) = \Delta^a(t,t_s^a)F_a^h(x,t_s^a) + \int_{t_s^a}^{t}\frac{dt_1}{t_1}\int_x^{1-\epsilon_{b'}^a(t_1)}\frac{dz_1}{z_1}\Delta^a(t,t_1)\frac{\alpha_s(t_1,z_1)}{2\pi}P_{bb'}^a(z_1)\Delta^b(t_1,t_s^b)F_b^h\left(\frac{x}{z_1},t_s^b\right)$$
$$+\int_{t_s^a}^{t}\frac{dt_1}{t_1}\int_x^{1-\epsilon_{b'}^a(t_1)}\frac{dz_1}{z_1}\int_{t_s^b}^{t_1}\frac{dt_2}{t_2}\int_{x/z_1}^{1-\epsilon_{c'}^b(t_2)}\frac{dz_2}{z_2}$$
$$\times\Delta^a(t,t_1)\frac{\alpha_s(t_1,z_1)}{2\pi}P_{bb'}^a(z_1)\Delta^b(t_1,t_2)\frac{\alpha_s(t_2,z_2)}{2\pi}P_{cc'}^b(z_2)\Delta^c(t_2,t_s^c)D_c^h\left(\frac{x}{z_1z_2},t_s^c\right)$$
$$+\cdots \tag{13}$$

This form is appropriate for forward evolution. An equivalent solution, suitable for backward evolution[55], can be obtained from Eq.(13) by inserting one, judiciously written as a ratio of structure functions, and manipulating:

$$1 = \Pi^a(t,t_s;x) + \int_{t_s^a}^{t}\frac{dt_1}{t_1}\int_x^{1-\epsilon_{a'}^a(t_1)}\frac{dz_1}{z_1}\Pi^a(t,t_1;x)\frac{\alpha_s(t_1,z_1)}{2\pi}P_{aa'}^b(z_1)\frac{F_b^h(x/z_1,t_s^b)}{F_a^h(x,t_1)}\Pi^b\left(t_1,t_s;\frac{x}{z_1}\right)$$
$$+\int_{t_s^a}^{t}\frac{dt_1}{t_1}\int_x^{1-\epsilon_{a'}^a(t_1)}\frac{dz_1}{z_1}\int_{t_s^b}^{t_1}\frac{dt_2}{t_2}\int_{x/z_1}^{1-\epsilon_{b'}(t_2)}\frac{dz_2}{z_2}\Pi^a(t,t_1;x)\frac{\alpha_s(t_1,z_1)}{2\pi}P_{aa'}^b(z_1)\frac{F_b^h(x/z_1,t_1)}{F_a^h(x,t_1)}$$
$$\times\Pi^b\left(t_1,t_2;\frac{x}{z_1}\right)\frac{\alpha_s(t_2,z_2)}{2\pi}P_{bb'}^c(z_2)\frac{F_c^h(x/(z_1z_2),t_s^c)}{F_a^h(x/z_1,t_2)}\Pi^c\left(t_2,t_s^c;\frac{x}{z_1z_2}\right)$$
$$+\cdots \tag{14}$$

Where the usual time-like, Sudakov form factor, Eq.(11), has been replaced by:

$$\Pi^a(x,t,t_s^a) = \Delta^a(t,t_s^a)\frac{F_a^h(x,t_s^a)}{F_a^h(x,t)} \tag{15}$$

which represents the probability of parton a with momentum fraction x at scale t having come from the scale t_s^a, without resolvable radiation.

Photon and $W^{\pm}/Z^0$ radiation can also be included in the cascades[64] by introducing the appropriate splitting functions, for example:

$$\frac{\alpha_{EM}}{2\pi}P_{q\gamma}^q(z) = \frac{\alpha_{EM}}{2\pi}Q_q^2\frac{1+z^2}{1-z} \tag{16}$$

Note a fixed $\alpha_{EM}(0)$ is used, $P_{\gamma f}^{\gamma}(z)$ is deemed negligible and a separate cut-off on photon radiation $Q_0 \gtrsim m_e$ is allowed. For heavy gauge bosons a longitudinal polarisation must be included in the effective W approximation[65] which is accurate only for very high energies and fails to treat recoils adequately.

The physical interpretation and numerical implementation of these solutions to the evolution equations can be found elsewhere and is not discussed here. We now continue by giving a theory outline motivating the angular ordering modification to the above cascade prescription.

3.2. *Coherence Effects*

The treatment of soft gluons[56], which because of their long wavelengths probe an events global (colour) structure, necessaryly involves quantum interference effects. However it has been shown that classical Markov process, modified by a simple angular ordering[57] constraint, is capable of including soft coherence. In the eikonal approximation, the cross-section for $\{p_i\}$ hard partons and one soft gluon, $g_\mu = \omega(1-\hat{\underline{k}})$, $\omega \ll E_i$, is given by[58]:

$$d\sigma^{(N+g)} = -\frac{\alpha_s}{2\pi}\sum_{i\neq j}\hat{T}^i.\hat{T}^j\omega^2\left[\frac{p_i.p_j}{p_i.kp_j.k} - \frac{1}{2}\frac{p_i^2}{(p_i.k)^2} - \frac{1}{2}\frac{p_j^2}{(p_j.k)^2}\right]\frac{d\omega}{\omega}\frac{d\Omega}{2\pi}d\sigma^{(N)} \tag{17}$$

Introducing angular variables ($\omega^2\times$) the term in square brackets is usefully rewritten:

$$\frac{1}{2}\left[\frac{1}{\zeta_{ik}}\left\{1-\frac{1}{\gamma_i^2\zeta_{ik}}+\frac{(\zeta_{ij}-\zeta_{ik})}{\zeta_{jk}}\right\}+(i\leftrightarrow j)\right] \quad \text{where,} \quad \zeta_{ij}\equiv\frac{p^i.p^j}{E_iE_j} = 1-v_iv_j\cos\theta_{ij}$$
$$=\frac{1}{2}\left[\frac{1}{\zeta_{ik}}\left\{\frac{v_i-\cos\theta_{ik}}{1-v_j\cos\theta_{ik}}+\frac{(\cos\theta_{ik}-v_j\cos\theta_{ij})}{1-v_j[\cos\theta_{ij}\cos\theta_{ik}-\sin\theta_{ij}\sin\theta_{ik}\cos\phi_i]}\right\}+(i\leftrightarrow j)\right] \tag{18}$$

The separation is such that the first term is only collinear singular for k parallel to p_i: it naturally represents emission off parton i. The dependence on the azimuthal angle ϕ_i is illustrated below, for massless partons.

Figure 1: The radiation function $2\zeta_{ik}\times$ Eq.(18) as a function of ϕ_i. Left $\pi/2 = \theta_{ij} > \theta_{ik}$: dashed line $\theta_{ik} = \pi/8$; solid line $\theta_{ik} = \pi/4$; and dotted line $\theta_{ik} = 3\pi/8$. Right $\pi/4 = \theta_{ij} < \theta_{ik} = \pi/2$. (Note: $v_i = 1 = v_j$).

Upon azimuthal averaging it is quickly seen that no net radiation occurs for $\theta_{ij} < \theta_{ik}$. In fact for successive soft gluon radiation ($\omega_1 \gg \omega_2$) it can be shown that the dominant region of phase space is equivalent to a sequence of branchings with decreasing opening angles: this is the basis for angular ordering. Including parton masses introduces a 'dead cone' around

the heavy flavour, so that, approximating slightly, radiation is restricted to the region:

$$R_{ij}^{(i)}: \quad \frac{m_i^2}{E_i^2} < \zeta_{ik} < \zeta_{ij} \quad \Longleftrightarrow \quad v_j \cos\theta_{ij} < \cos\theta_{ik} < v_i \tag{19}$$

This screened cone is illustrated below.

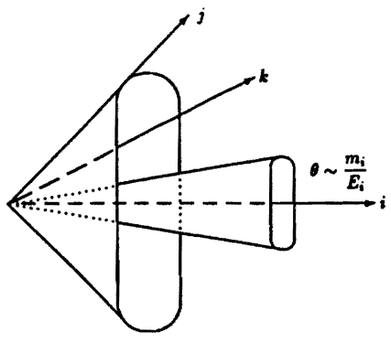


Figure 2: The screened cone $R_{ij}^{(i)}$ for emission of the soft gluon k associated with parton i, whose interference partner is j.

Note that apart from the kinematically depleted central region for heavy quarks no other differences arise *w.r.t.* the radiation expected off a light quark.

Since after azimuthal averaging the distribution Eq.(18) is equal to the $z \to 1$ limit of $P_{ig}^{i}(z)$ it is sufficient to impose the restriction Eq.(19) on the standard final state branching algorithms above. The treatment of initial state radiation is more complex, due to the different kinematics. However the result is a similar ordering in the angles φ, between the time-like parton at a branching and the incoming hadron, which are required to decrease away from the hard scattering[59]. This is adequate for $x \to 1$, however when $x \to 0$ in the structure function extra complications arise[60] and only a rather inefficient Monte Carlo algorithm is currently available[30]. Evidence of the need for colour coherence in hadronic collisions has been seen by CDF in three jet events[61].

Finally it can be seen from Eq.(18) and fig. 1 that a non-trivial azimuthal distribution is implied. A further influence on the orientation of branching planes comes from the gluon spin (direction ϕ_0) as follows:

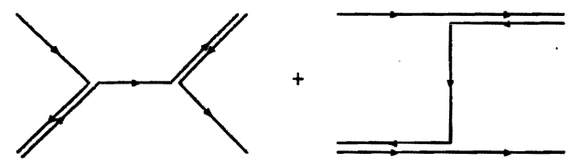
$$\mathcal{P}(z,\phi) = \begin{cases} P_{gg}^{g}(z) + 2z(1-z)\cos 2(\phi-\phi_0) & \propto 1 \quad +1/9\cos 2(\phi-\phi_0) \quad g \longrightarrow gg \\ P_{q\bar{q}}^{g}(z) - 2z(1-z)\cos 2(\phi-\phi_0) & \propto 1 \quad -\cos 2(\phi-\phi_0) \quad \longrightarrow q\bar{q} \end{cases} \tag{20}$$

The numerical values are for $z = 1/2$ which maximizes the correlations. The proper inclusion of these correlations which couple all azimuthal angles in an event requires a non-Markov algorithm: a review can be found in [20].

3.2.1. Initial Colour Flows

In QCD it is typical that more than one colour flow is involved in the amplitude for a hard subprocess. This leads to an ambiguity in initially deciding which partons are colour connected. Whilst this is not an issue for an incoherent algorithm, such as COJETS or ISAJET, a prescription is required if inter-jet coherence is to be modelled as in HERWIG and PYTHIA.

Consider quark-gluon scattering. Three diagrams contribute to the amplitude: s-channel quark exchange, with colour factor $T^a T^b$; u-channel quark exchange, with colour factor $T^b T^a$; and t-channel gluon exchange, with colour factor $if_{abc}T^c = T^a T^b - T^b T^a$. The two colour flows for this amplitude are illustrated below.



$T^a T^b(\mathcal{M}_s + \mathcal{M}_t)$ $\qquad$ $T^b T^a(\mathcal{M}_u - \mathcal{M}_t)$

Figure 3: The two colour flows contributing to the amplitude for quark gluon scattering

A natural prescription for giving weights to these two options is to use, $|\mathcal{M}_s + \mathcal{M}_t|^2$ and $|\mathcal{M}_u - \mathcal{M}_t|^2$. However this neglects the $1/N_c$ suppressed interference between the two colour topologies. Two attitudes are now possible: in PYTHIA the default is to neglect this correction[62] (perhaps even positing a string inspired superselection rule to eliminate it[76], this could not be established within PQCD); or, as in HERWIG, to distribute it between the two weights[63]. This has been found to be unique if: i) the pole structure and crossing symmetry are preserved; ii) each weight is separately positive definite; and iii) the sum over colour configurations yields the exact differential cross-section.

4. Comparison of the Cascade Algorithms

In the following table we detail the main features of the commonly used cascade algorithms before commenting on their merits.

The first major difference in the time-like parton cascades is the choice of evolution variable, in particular HERWIG's. (Note all the z definitions are equivalent for $E \to \infty$). This choice has the advantage of automatically incorporating angular ordering. In PYTHIA an unattractive veto has to be imposed on the angles which may over restrict the phase space requiring a larger value of α_s to compensate. Both COJETS and ISAJET remain incoherent algorithms. A purported disadvantage of the HERWIG scheme is that the actual parton momenta remain implicit until the completed cascade can be reconstructed; however this is unlikely to inconvenience the casual user.

	PYTHIA	Interference	HERWIG
T^aT^b	$\frac{C_F}{N_c}\frac{-u}{s}\frac{(u^2+s^2)}{t^2}$	$+\frac{1}{N_c^2}\frac{u^2}{t^2}$	$\frac{u^2}{2}\frac{(u^2+s^2)}{-ust^2}+\frac{1}{2N_c^2}\frac{u}{s}$
T^bT^a	$\frac{C_F}{N_c}\frac{-s}{u}\frac{(u^2+s^2)}{t^2}$	$+\frac{1}{N_c^2}\frac{s^2}{t^2}$	$\frac{s^2}{2}\frac{(u^2+s^2)}{-ust^2}+\frac{1}{2N_c^2}\frac{s}{u}$
Sum	$\frac{C_F}{N_c}\frac{(u^2+s^2)^2}{-ust^2}$	$+\frac{1}{N_c^2}\frac{(u^2+s^2)}{t^2}$	$\frac{(u^2+s^2)}{-2ust^2}\left[u^2+s^2-\frac{t^2}{N_c^2}\right]$

Table 4: The colour flow weights used in PYTHIA and HERWIG, given as: $\pi\alpha_s^2/s^2)^{-1}.d\sigma(qg\to qg)/dt$.

A related issue concerns the vertex kinematics which depend in detail on the choice of t and z. Using ISAJET as example we have at the branching, $a\to b(z)+c(1-z)$:

$$0\le p_T^2 = z(1-z)t_a-(1-z)t_b-zt_c \tag{21}$$

This places upper limits on the daughters' maximum virtualities, to be used in the cascades, of zt_a and $(1-z)t_a$ respectively. However even if the subsequent t_b and t_c satisfy these constraints it is not sufficient to guarantee Eq.(21) holds. Elaborate schemes have therefore had to be devised in COJETS, ISAJET and PYTHIA to cope with this problem. Fortunately HERWIG's choice of variables avoids the difficulty posed by such a joint constraint.

Excepting COJETS all cascades use the branching p_T^2 as α_s argument. This enables hard infra-red $\log(1-z)$ terms, associated with higher order corrections, to be resummed and in the $g\to gg$ case certain $\log z$ terms also. In fact in HERWIG the treatment of the semi-inclusive $z\to 1$ region is sufficiently accurate (NLLA) that Λ_{QCD} can be measured using using fits to data. The Monte Carlo Λ being related to the $\overline{MS}$ value via:

$$\Lambda_{MC} = \exp\left(\frac{67-3\pi^2-10N_f/3}{2(33-2N_f)}\right)\Lambda_{\overline{MS}} \tag{22}$$

In the two coherent schemes only HERWIG treats the effects of heavy flavours, the so called 'dead-cone'. Concentrating on b-quarks which hadronize (thus breaking the coherence with the jet) before decaying into a fixed average number of particles, an energy independent depletion in b-jet multiplicities *w.r.t.* equal energy light quark jets should be expected and is seen[66]. Both HERWIG and, as an option, PYTHIA include soft azimuthal correlations using the dipole formula Eq.(18) in their cascades. Likewise both HERWIG and, as an option, PYTHIAinclude azimuthal spin correlations. However PYTHIA only treats intra-jet angles using a nearest neighbours approximation whilst HERWIG correlates all angles, including inter-jet effects, using a non-Markov approach.

A similar set of observations also hold for the space-like parton cascades. Here only HERWIG includes the effects of colour coherence, which motivates its choice of evolution

Feature	Monte Carlo			
	COJETS	HERWIG	ISAJET	PYTHIA
Time-like Cascades: $i\longrightarrow jk$				
Evolution var. t	Q^2	$E_i\sqrt{\xi_{jk}}$	Q^2	Q^2
Splitting frac. z	P^+_{i+1}/P^+_i	E_j/E_i	P^+_j/P^+_i	E_j/E_i
α_s scale	Q^2	$z(1-z)Q^2$	Q^2	$z(1-z)Q^2$
Ang. ordering	no	automatic	no	veto on: $\frac{z_1(1-z_1)}{Q_1^2}>\frac{1-z_2}{z_2Q_2^2}$
Dead Cone	no	yes	no	no
Soft Correlations	no	dipole approx.	no	o dipole approx.
Spin Correlations	no	fully included	no	o nearest neighbour
photon rad.	1γ from M.E.	yes	yes, plus $W^\pm$, Z^0	yes
Space-like Cascades: $j(+k)\longleftarrow i$				
Direction	forward	backward	backward	backward
Evolution var. t	Q^2	$E_i\sqrt{\xi_k}$	Q^2	Q^2
Splitting frac.	P^+_j/P^+_i	E_j/E_i	P^+_j/P^+_i	$\hat{s}_j/\hat{s}_i$
α_s scale	Q^2	z	Q^2	$(1-z)Q^2$
Ang. ordering	no	automatic	no	no
Soft correlations	no	dipole approx.	no	no
Spin correlations	no	fully included	no	o nearest neighbour
photon rad.	no	no	no	$q\to q+\gamma$ only

Table 5: Main features of the commonly used branching algorithms. A o indication non-default option.

variable. The presently used algorithm is precise in the $z\to 1$ limit but is an approximation in the small z region. Experimentally the need for coherence has recently been demonstrated by comparing the observed distribution of a soft third jet in $p\bar{p}$ collisions with HERWIG predictions and either ISAJET or an incoherent version of HERWIG. The need for coherence in final state radiation is well established by LEP, though independent fragmentation can be shown to give a moderately useful approximation.

All the models include photon (and Z^0, $W^\pm$) radiation to some extent. Unfortunately due to the need for minimum energy and isolation cuts, regions of phase space where the cascade approximations are likely to be inefficient and inaccurate are probed. Note also that photons in initial and final state radiation should be equally important. In e^+e^- data it was found necessary to include matrix element matching procedures, which would also benefit gluon radiation.

Main Parton Shower Summary It is clear that HERWIG has the most sophisticated parton showers, based on a careful analysis of PQCD. From the outset soft gluon coherence has been made intrinsic to both the initial and final state cascades, indeed it is hard wired. Special efforts have also been made to fully include the quantum mechanical effects of gluon

spins. It should provide accurate quantitative predictions at higher energies.

Initially PYTHIA did not contain showers which were added to better reproduce energy dependence in jet events. Latter, as the need arose, angular ordering was imposed on the cascades. though not as yet in the intial state. A relatively large number of options, to test various approximations, are made available to the user.

COJETS represents an early approach and has remained steadfast in its use of both forward evolution for initial states and incoherent independent jet fragmentation. Special effort has focussed on modifying the hadronization model to mimic soft coherence. It is not clear whether reparameterization, of model elements, to reproduce data at higher energies can be sustained.

ISAJET provides a rather basic incoherent shower algorithms for independently fragmenting jets. As such energy extrapolations ought to be regarded as qualitative whilst soft particle/jet distributions should be treated with caution. The inclusion of γ, Z^0, $W^{\pm}$ radiation in final states is noteworthy.

Other Approaches to Showers Lastly two other approaches to shower evolution are available in existing Monte Carlos:

ARIADNE[67] which is used by FRITIOF. Here the basic units are not partons but the colour dipoles lying between them. The dipoles subdivide according to specific rules, equivalent to the more usual Altarelli-Parisi approach[68], with k_T^2 ordering automatically ensuring angular ordering. The dipoles may be viewed as describing the PQCD evolution of a string. Treating extended hadrons requires some modifications[69] and is particularly troublesome for Drell-Yan.

NLLjet describes e^+e^- and ep collisions[70,71] in a Next to LLA, which is suitable for extracting Λ_{QCD}. It uses a generalization of the jet calculus[72] which includes one-to-three parton splitting functions.

5. Hadronization Models

The confinement transition from the quark and gluon degrees of freedom appropriate in perturbation theory to the hadrons observed by real world experiments is not understood. In this strongly interacting transition regime we presently rely on models, which to varying degrees reflect possible scenarios for the QCD dynamics. Three main schools of thought dominate model building: cluster, independent and string hadronization.

Before beginning our discussion it is appropriate to reiterate that the fragmentation of a parton ($q/\bar{q}$ or g), produced in a hard sub-process, into a hadron is generally described as proceeding in two stages. First is a parton cascade, formulated according to PQCD, which evolves the primary parton from the hard scattering scale, Q^2, into a secondary parton at a (fixed) cut-off scale, Q_0^2. It is during this calculable stage that many of the event's main features are determined: energy dependences, event activity, jet profiles, multiplicities, *etc.* In a separate second stage, carried out from the fixed, low virtuality scale, Q_0^2, a model is employed to convert the secondary partons into hadrons. The second stage is essentially energy independent and assumed to be local in nature. This separation of the fragmentation

function was embodied in the analogue of Eq.(3) and is shown more schematically below:

$$D_a^h(x, Q^2) = (\text{PQCD evolution: } Q^2 \to Q_0^2) \otimes (\text{model: parton} \to \text{hadron})\Big|_{Q_0^2} \qquad (23)$$

One immediate consequence of this picture is that EUROJET (and PAPAGENO) are rather handicapped by their lack of parton cascades. In particular the collision energy dependence, of for example multiplicity or out of event plane activity, will be incorrect and program parameters will require retuning for different energies. Accurate quantitative extrapolations to higher energies are therefore not possible. Similar considerations imply that when higher order matrix element calculations are employed in conjunction with, say, the JETSET hadronization model, for example as used at LEP in α_s determinations, then the default parameters (tuned assuming cascades) must be reset.

It is an implicit assumption of Monte Carlo event generator authors that it is PQCD which is responsible for an event's characteristics and that the hadronization (model) causes little disruption of the event properties arising in the first stage of fragmentation. Thus in each type of model it is the quantum numbers of a small number of neighbouring partons which are responsible for the properties of a produced hadron. This gives rise to an approximate local conservation of flavour, momentum, *etc.* A more precise formulation of this concept, loosely called Local Parton Hadron Duality (LPHD), has been provided by the St. Petersburg group[73]. They showed that the x-spectrum of π's, K's, p's *etc.*, measured in e^+e^- jets differ from the parton distributions predicted by resummed PQCD , with $Q_0 \approx m_h$, only by jet energy independent constants. However questions have been raised about the validity of this reduction of hadronization to a normalization constant, both in its 'hard line' interpretation above, and in the extent to which this is realised in the models at present energies[74].

Finally, and perhaps most importantly, it should be emphasized that at the present time each hadronization model *only provides a parameterization of present data based upon at best an educated guess and at worst a well defined prejudice.* It should go without saying therefore that any serious analysis which may be sensitive to hadronization effects must be based on at least two models. (Altering the free parameters of one model is unlikely to be representative of potential effects).

5.1. *Comparison of Main Hadronization Model Features*

The details of cluster, independent and string hadronization are widely available and we confine ourselves here to a comparison of their relative merits. In particular we will try to ascertain to what extent the models explain or merely describe the hadronization process. We will concentrate on identified particle distributions.

In all cases the schemes are based on a small number of, recursively applied, branchings, where at each iterative step probabilistic rules are applied to select flavours, spins and momenta. The motivation for these rules vary from the QCD inspired complex dynamics of strings and minimalism of clusters to the simple expediency of independent fragmentation. The main features are summarized in the following table.

The first serious difference between the three schemes concerns the extent to which they are covariant. This is not an issue for strings, nor for clusters, however both momentum and to a lesser extent quantum number conservation are problematic for independent hadronization. Whilst, essentially, *ad hoc* remedies for these ailments exist they are not always

Feature	Hadronization Model		
	Cluster	Independent	String
Principle	very simple	simple	complex
Lorentz invariant	yes	no	yes
Flavour, charge, *etc.*, conservation	automatic	on *ad hoc* basis	automatic
Mass dependence via	hadrons	quarks	quarks
Strangeness suppression	predicted	free param.	free params.
Baryon suppression	predicted	free param.	free params.
J^P ratios	predicted	free params.	free params.
Limited P_T	natural	built in	built in
Fragmentation function	N/A	free	restricted by L-R symmetry
Cut-off (Q_0) dependence	significant	very strong	modest
Stability	infrared sensitive	collinear sensitive	stable
Limitations	'string-like' decay for massive clusters	requires large cut-off Q_0	none

Table 6: Comparison of the three main hadronization schemes' major features.

implemented: also, physical observables are known to be sensitive to the details of the solution adopted[75]. It may be noted that as currently implemented strong isospin conservation can prove troublesome for all models.

The second major area of difference are in the rules for flavour, spin and momentum selection. Only cluster hadronization provides a succinct basic algorithm, using just the phase space available to the produced particles and isotropic decays. Since clusters are typically light, limited transverse momentum is automatic; hadrons with non-zero strangeness and baryon number are suppressed because they are heavier; and the spin ratios of iso-flavour hadrons follow partly from the $2S+1$ factor and partly from the larger masses of higher spin states. This absence of free parameters may be traced in part to the emphasis placed on observable hadron properties and further enhances the models predictive power, for examples the production rates of tensor mesons.

The contrast with both independent and string hadronization is marked. In principle string hadronization is at an advantage because it is based upon a semi-quantitative picture. The problem is that it is formulated in terms of indirectly measurable quantities. For example, a flavour independent, Gaussian p_T distribution is predicted, but its width is deemed free[‡]; whilst the Gaussian form is simply an assumption in independent hadronization. The situation is even worse for the flavour selection rules. Again, for strings the tunnelling mechanism predicts a Gaussian suppression of heavier flavours; however given the uncertain quark and, especially, diquark masses this can only be taken as a guiding framework, resulting in a

[‡]Actually a width, $\sqrt{\kappa/\pi}$, is predicted but proves to be too small

reliance on a truly byzantine parameterization[§]. To emphasize this point, 13 inputs are needed in the basic LUND string model[8] to describe the 13 (assuming $u \to d$ isospin[¶]) $L=0$ light (u,d,s) hadrons. In practice this aspect of string hadronization is little different from, the somewhat simpler, independent hadronization scheme which has no underlying mechanism. No dynamical explanation has been found with which to select hadron spins and these are just chosen according to relative weights in both models. In addition longitudinal fragmentation functions are also required, though the consistency of the string's causal and Lorentz structure leads to a unique family of L-R symmetric functions[77].

As an aside doubts have been raised by the authors[76] as to the validity of the tunnelling mechanism for describing the transverse momentum distribution of string fragments. Though an attractive, alternative description in terms of (nonperturbative) gluon emission has not been forthcoming. A second worry concerns pions whose small spatial size, especially when compared to the neglected string width, is not mandated by the Goldstone theorem.

The issue of stability, *w.r.t* collinear and soft partons, arises for cluster and independent hadronization. Specifically the spectrum of clusters is sensitive to the emission of soft gluons. The simplest illustration is provided by a colour singlet, Z^0 say, decay into two quarks: this gives a cluster of mass, M^2, at rest. Now add a central, very soft gluon: this gives two clusters of mass $2Mm$ and momenta $\pm M_{\mathbb{R}}$ in the parent rest frame. The infra-red instability may be regarded either as a serious problem or perhaps as a warning that it is important to treat the perturbation theory correctly. Indeed it is well known that many observables, such as $\langle n \rangle$, are infra-red sensitive; it should also be noted that the first configuration discussed above is highly Sudakov suppressed, PQCD does not like isolated colour charges. In independent fragmentation a similar problem arises when one final state parton is replaced by two parallel partons of equal net energy which gives a different multiplicity: $\propto \log(E/\langle m_h \rangle)$, for fragmentation functions behaving as $1/z$ as $z \to 0$. The collinear instability arise essentially because the two partons are oblivious to each others presence.

Finally it must be admitted that cluster hadronization is not as simple in practice as in principle. Difficulties occur for both light and very heavy clusters which have necessitated the introduction of a limited number of free parameters. Light clusters require a one-body decay option, which proves influential in the distribution of fast particles. In particular this feature can be exploited to gain some control over the stiffness of $b-$flavoured hadron spectra. (*c.f.* the cluster-like decay of light strings.) Heavy clusters render implausible the isotropic, two-body decay mechanism and a cluster fission option is needed, whose most notable feature is the appearance of a preferred axis, aligned in what would be the string direction. Given the rarity of heavy clusters, the flavour properties of their fission are minor; though the threshold for splitting does influence multiplicities. By comparison arbitrarily massive strings can be handled.

Main Hadronization Model Summary Independent hadronization is basically an empirical scheme with no basis in QCD theory (indeed it is colour blind!); a fact reflected in the extreme arbitrariness of its parameters. It is adequate for describing data's global features,

[§]Even given precise (di-)quark masses uncertainties may still remain from the use of an idealised infinite length string in the tunnelling calculations.

[¶]The number of hadrons increases to 27 if u and d are distinguished

though less so for soft/inter-jet distributions, and in conjunction with a perturbative cascade to extrapolate to higher energies can be expected to yield reliable results.

String hadronization is a fully covariant scheme based on the complex dynamics of colour flux tubes, as anticipated from QCD. This provides a general framework for assigning flavours, momenta, *etc.*, however uncertainties, particularly in quark masses, plague its implementation. Consequently a very large number of parameters are needed to describe identified particle properties, which it does quite successfully. It is the most popularly used scheme.

Cluster hadronization is in principle a remarkably simple, compact and predictive scheme, founded on the properties of colour singlet clusters proved in PQCD[78]. By weighting potential decay channels by the phase space available, it relies only on measured hadron properties thus successfully removing many ambiguities. Unfortunately the scheme is sensitive to the tails of the cluster mass spectrum so that practicalities have somewhat corrupted its initial purity. It remains however rather successful.

Finally it should be noted that the physics environment provided by hadronic collisions, unlike e^+e^-, is not clean so that good model discriminants have not yet been found for this arena. Further, the little understood, potential for interplay between the physics of hadronization and that of the soft underlying event still allows theorists scope for evolving their models.

5.2. *Alternative Schemes*

A number of alternatives to those provided by the 'big three' hadronization schemes exist in the literature. Typically these work within the existing frameworks whilst offering innovations designed to overcome specific defects. Three schemes which are, partially, implemented in event generators are mentioned below

The Montevay Scheme In answer to the lack of Lorentz invariance in independent fragmentation the fully covariant Montevay scheme has bee proposed[80]. It is a hybrid model which uses an initial string network to select a frame in which four momentum and flavour conservation can be naturally implemented. It is also collinear safe. Unfortunately its complexity for multiparton final states has limited its (partial) use to a non-default PYTHIA option.

Hybrid String-Cluster Model The e^+e^- event generator CALTECH–II provides an attempt to combine the desirable properties of both strings and clusters in its hadronization model[79]. First strings form, their development being characterized by their break up into clusters possessing a continuous mass spectrum: this avoids infra-red sensitive gluon splittings. Second the clusters decay into hadrons (and sub-clusters), basically according to the phase space available to the final state; thereby avoiding the need for many parameters.

The UCLA Flavour Model This is a variant of the LUND string model in which measured hadron properties are used in flavour selection[81]. The left-right symmetric fragmentation function, integrated over z and p_T^2, is reinterpreted as a hadron mass distribution, which together with trivial Clebsh-Gordon and spin counting factors gives a two parameter model. This is applied iteratively to several hadrons at once thus including shared heavy flavour suppression.

6. Underlying Event

The final issue to be resolved when simulating a full hadronic collision is the treatment of the beam remnants, that is the particles which are left in the incoming hadrons after extracting the parton shower/hard scattering initiators. Essentially by definition, the relevant processes only involve relatively small momentum transfers and necessarily probe non-perturbative physics. Therefore once again programs rely on models rather than first principles calculations. Three generic approaches are used: a parameterization of existing data; a multiple scattering, mini-jet model; and dual topological unitarization models.

It may be remarked at this point that whilst the basic physics is assumed to be rather similar to the soft hadronic collisions which dominate minimum bias data significant differences occur when a high Q^2, hard scattering is present. In particular the associated particle/energy flow in high Q^2 events, for example on the wings of high p_T jets, is significantly larger than in an equivalent $\sqrt{S}$, minimum bias event. This 'pedestal effect' has been seen for jets[82,83], W/Z production[84] and Drell-Yan pairs[85]. Typically an enhancement of between 1.5 and 4 is required. Both experimentally[82] and theoretically[86] a two component picture is favoured in which QCD brehmstrahlung provides a small component that grows with Q^2 and soft physics a contribution that appears to saturate for sufficiently large Q^2.

Finally it must again be repeated given the lack of an agreed QCD method of calculation and consequent reliance on models that caution must be exercised whenever an event generator's results are thought to be sensitive to the underlying event structure. In particular extrapolations to higher $\sqrt{s}$ should be based on more than one model‖.

6.1. *Comparison of Models*

Both HERWIG and COJETS employ *ad hoc* models based on parameterizations of (then existing) data. HERWIG adapts the GENCL Monte Carlo[88] to utilize its own cluster hadronization scheme, COJETS works directly in of hadrons. Both assume a flat central rapidity plateau, limited transverse momenta and special prescriptions for the leading hadrons. HERWIG assumes a negative binomial multiplicity distribution and COJETS a KNO scaling form. No supporting theory is attempted and so energy extrapolations are particularly open to question. It is interesting to note that in HERWIG's case no enhancement of the underlying event was found necessary to reproduce the plateau seen in hard scatterings, soft gluon radiation proving adequate[89]. A characteristic asymmetry in the average multiplicity in a jet's two shoulders is anticipated.

A significantly more ambitious approach[90], based on perturbative estimates for multiple semi-hard parton scatterings, is available in PYTHIA and HIJING. The mini-jet cross-section as a function of a p_T^{min} cut-off, normalized to the fitted inelastic, non-diffractive cross-section, is used as scattering probability. Multiple scatters above p_T^{min} are generated from a Poisson type distribution and simplified string drawings used. As an option a double Gaussian spatial distribution for the partons within a hadron can be folded into the probabilities above. If a hard scattering occurs then a large hadronic overlap is likely and double scatterings can be anticipated: this naturally gives an enhanced underlying event. A particular feature to

‖A related divergence in the predictions for the γp total cross-section as measured at HERA[87] may serve as a cautionary warning.

be expected in such a multiple scattering model are correlations in azimuth, and to a lesser extent rapidity, in minijet observables[91].

Another approach embodying a smooth transition from soft to hard scattering events is the FRITIOF excited string model. In a soft collision, net colour neutral, multiple gluon exchange is assumed to yield two high mass excited strings which radiate according to the colour dipole model ARIADNE, before hadronizing. To include possible hard scatterings a p_T is selected using the minijet cross-section and then accepted or rejected (and a soft event generated) according to whether the first branching in the subsequent cascades has a smaller or larger k_t. This provides a rather elegant model rooted in the phenomenology of non-perturbative strings.

The ISAJET description of soft collisions is based on the AKG model[92] for a unified description of total and elastic cross-sections. The basic unit is a cut Pommeron, which gives rise to chain of hadrons uniformly distributed in rapidity and with a Poissonian multiplicity distribution. The number of such Pommerons, k, and the mean hadron p_T is adjusted separately for soft and hard scattering events so as to reproduce data. After separately assigning leading baryons the Pommerons are assigned rescaled longitudinal momentum fractions from a uniform distribution. Finally each Pommeron is fragmented in its own CoM frame, using an independent fragmentation function made energy dependent to reproduce the observed rise in dN/dy with s. Since no attempt is made to include low p_T jets which contribute significantly to LHC/SSC cross-sections the ISAJET model is only partially complete.

A significantly more elaborate two component scheme is employed by DTUJET to simultaneously describe mini-jet type hard scatterings and soft scatterings. Employing cut diagrams based on soft and hard Pommerons, including a triple Pommeron vertex and Pommeron loop, allows low and high single/double diffraction events to be simulated. The eikonal approximation is used to enforce unitarity and provide a unified treatment. Since free parameters are fixed using a mini-jet prediction of the hard scattering cross-section extrapolations to high energy are particularly unstable *w.r.t.* changes in the (gluon) structure function used. See also [93].

Acknowledgements

The preparation of this document would not have been possible without the gracious help of the following people, who provided the write-ups of the individual programs: Ian Hinchliffe, Roberto Odorico, Hong Pi, Frank Page, Johannes Ranft, Mike Seymour, Törbjorn Sjöstrand and Xin-Nian Wang.

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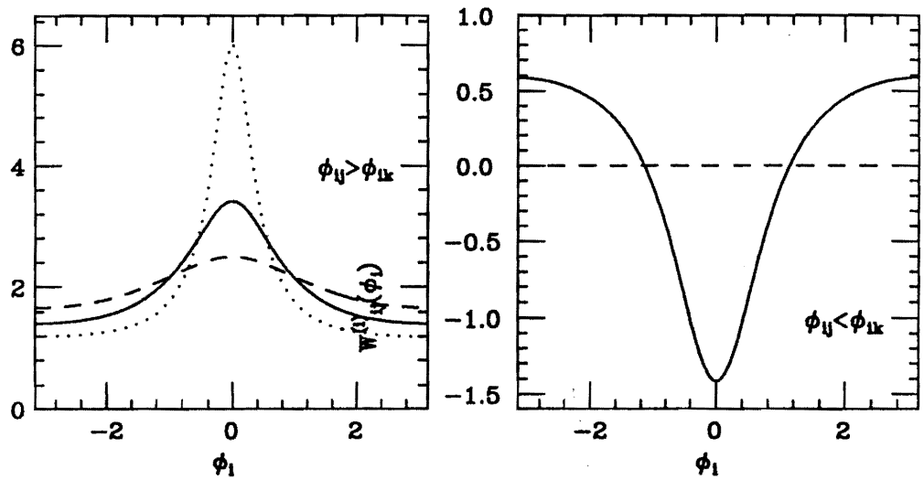
6
4
2
0
−2
0
2
ϕ_1
$\phi_{1j}>\phi_{1k}$
$W^{(1)}_{1j}(\phi_1)$
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2
0
2
ϕ_1
$\phi_{1j}<\phi_{1k}$