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HEAVY FERMION MASS PREDICTIONS AND RENORMALIZATION GROUP EQUATIONS

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ABSTRACT

It has recently become popular to predict the low energy fermion mass spectrum based on a high scale, $M_X \approx 10^{16}$ GeV/c^2, ansatz for the Yukawa couplings. The evolution to low energy being achieved using renormalization group equations, with supersymmetry extant above a breaking scale $M_S \sim 10^3$ GeV/c^2. In this paper it is shown how the heavy fermion and Higgs masses are determined by fixed points and thus will obtain in many specific models. This gives $m_t \approx 184$ GeV/c^2, $M_h \approx 122$ GeV/c^2 and imposing a hierarchy of scalar vacuum expectation values $m_b \approx 4.1$, $m_\tau \approx 1.78$ GeV/c^2. The dependence on $\alpha_s(M_{Z^0})$ and M_S is elucidated.

It is presently en vogue to predict/parameterize fermion masses and mixings using a relatively simple unification scale, M_X, ansatz for the Yukawa coupling matrices: U, D and E. After (supersymmetric) RGE evolution of the scale dependent matrices and gauge couplings to low energy, diagonalization gives the spectrum[1]:

$$M_U^2 = S_U(UU^\dagger)S_U^\dagger; \qquad M_D^2 = S_D(DD^\dagger)S_D^\dagger; \qquad V_{CKM} = S_U S_D^\dagger \tag{1}$$

where properly masses are given implicitly by the eigen-values as $m_f = g_f(m_f)v/\sqrt{2}$.

Here I report on an analysis adopting a different perspective in which 'quasi-fixed points' of the non-linear RGEs themselves determine results, independently of boundary conditions. To exploit this feature, at least initially, we need large Yukawa couplings: for g_b and g_τ this requires a small associated VEV, v_2. Taking $V_{tb} = 1$, we treat the dominant third generation masses in isolation, an approximation sanctioned by nature: [1] gives full details and further references. Initially though it is instructive to consider simpler cases: first the top and gauge coupling evolution ($g_b^2 = 0 = g_\tau^2$).

$$16\pi^2 \frac{dg_t^2}{dt} = 2g_t^2[C_0 g_t^2 - \sum_{i=1}^{3} C_i g_i^2]; \qquad 16\pi^2 \frac{dg_i^2}{dt} = 2b_i g_i^4 \qquad \text{where, } t = \log(E) \tag{2}$$

This may be solved, by considering $1/g_t^2$, and using an integrating factor:

$$g_t^2(t) = \frac{g_t^2(t_X)}{e^{X(t)}}\left[1 - \frac{2C_0}{16\pi^2}\int_{t_X}^{t} ds \frac{g_t^2(t_X)}{e^{X(s)}}\right]^{-1} \quad \text{where,} \quad e^{X(s)} = \prod_{i=1}^{3}\left(\frac{g_i^2(s)}{g_i^2(t_X)}\right)^{C_i/b_i} \tag{3}$$

Neglecting weak corrections, $C_{1,2} = 0$, this can be exactly integrated:

$$\frac{g_t^2(t)}{g_t^2(t_X)} = \left(\frac{g_3^2(t)}{g_3^2(t_X)}\right)^{\frac{C_3}{|b_3|}}\left\{1 - \frac{C_0}{(C_3 - |b_3|)}\frac{g_t^2(t_X)}{g_3^2(t_X)}\left[1 - \left(\frac{g_3^2(t)}{g_3^2(t_X)}\right)^{\frac{C_3}{|b_3|}-1}\right]\right\}^{-1} \xrightarrow{g_3^2(t) \gg g_3^2(t_X)} \frac{(C_3 - |b_3|)}{C_0}\frac{g_3^2(t)}{g_t^2(t_X)} \tag{4}$$

[1] Observe that any matrices in the set $R_1\{U, D\}R_2^\dagger$, with $R_{1,2}$ unitary, give equivalent results.

Initially $g_t^2(t_X) > 1$ in Eq.(2) so the C_0-term dominates the gauge, C_i-terms, causing a rapid decrease in $g_t^2(t)$ until the derivative equilibrates and $g_t^2(t)$ remains fairly constant. This leads to the strong focussing in fig.1b and a unique mass prediction. Eventually $g_3^2(t)$ grows appreciable for small t, fig.1a, dominating $g_t^2(t)$'s trajectories[2] which then evolve in fixed ratio to $g_3^2(t)$, independent of $g_t^2(t_X)$.

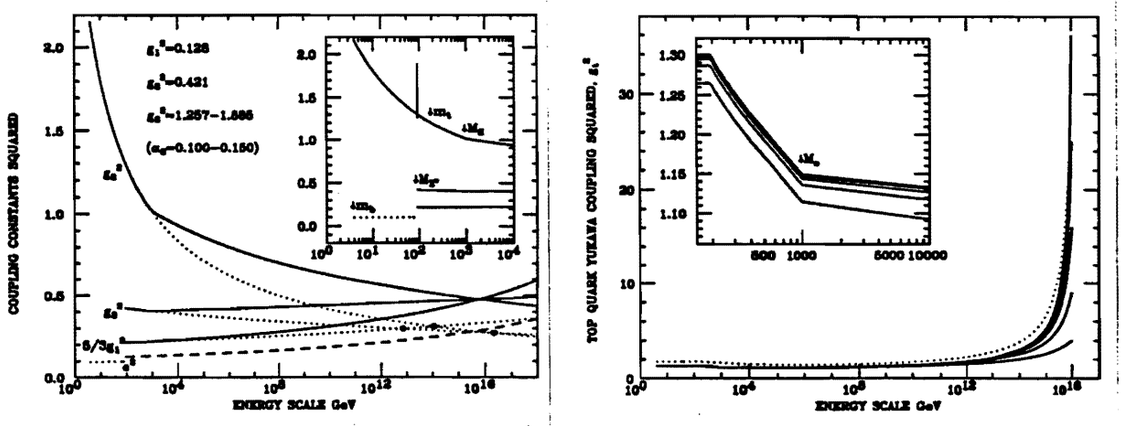

Figure 1a: Evolution of gauge and **1b** top quark Yukawa couplings squared.

The top/bottom system can also be analyzed; above M_S the RGEs are ($g_\tau^2 = 0$):

$$16\pi^2 \frac{dg_t^2}{dt} = 2g_t^2\left[6g_t^2 + g_b^2 - f_t(t)\right]; \qquad 16\pi^2 \frac{dg_b^2}{dt} = 2g_b^2\left[g_t^2 + 6g_b^2 - f_b(t)\right] \tag{5}$$

Assuming the combinations of gauge couplings $f_{t,b}(t)$ to be fixed, the solution's global structure is easily understood. First $f_t \approx f_b$ so a high degree of symmetry is expected. The axes and dashed lines in fig.2 denote where $\dot{g}_t^2 = 0$ and $\dot{g}_b^2 = 0$ for $(f_t, f_b)|_{t=t_S} = (7.639, 7.551)$. In the four sectors thus created the gradients for the g_t^2, g_b^2 evolution have fixed sign: e.g. in the lower right triangle g_t^2 decreases and g_b^2 increase. Finally four 'fixed points' can be identified and locally analysed as: $(0,0)$ unstable; $(0, f_b/6)$, $(f_t/6, 0)$ saddle points; and $((6f_t - f_b)/35, (6f_b - f_t)/35)$ stable. Fig.1b gives the evolution along the g_t^2-axis of fig.2.

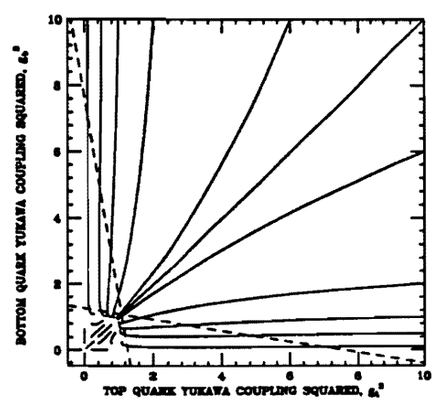

Figure 2: Full supersymmetric evolution of g_t^2 and g_b^2 from $M_X = 10^{16}$ to $M_S = 10^3$ GeV/c^2.

I now give our results, based on assigning $\{g_t,\ g_b,\ g_\tau\}$ at M_X to an integer valued grid and numerically[3] solving the RGEs. From the resulting scatter of final values 'fixed point' Yukawa couplings are defined: either (A) as central value and half-range; or (B) mean and standard deviation. VEV v_2 is then iteratively tuned so that $\langle g_\tau \rangle v/\sqrt{2} = m_\tau$, whence m_t, m_b and M_{h^0} are predicted. The results are compiled below, where unless otherwise stated $\alpha_s(M_{Z^0}) = 0.125$ ($g_3 = 1.253$) and $M_S = 1$ TeV/c^2.

[2] However as the fig.1b inset shows $g_t^2(t)$ decouples (at 190 GeV/c^2) somewhat before reaching this limit.
[3] Equivalent results were given by Adams-Moulton and fourth order Runge-Kutta integration schemes.

Table 1: Fermion and Higgs mass predictions using three integer valued starting grids and two methods of tuning v_2.

Case	M_{h^o} GeV/c^2	m_t GeV/c^2	m_b GeV/c^2	m_τ GeV/c^2	v_2 GeV/c^2
1-6 *A*	119.2 ± 9.1	179.3 ± 14.7	4.20 ± 0.83	1.78 ± 0.79	4.25
B	**121.8 ± 4.3**	**184.3 ± 6.8**	**4.07 ± 0.33**	**1.78 ± 0.33**	**3.86**
3-6 *A*	123.1 ± 1.6	186.6 ± 2.4	4.00 ± 0.15	1.78 ± 0.18	3.74
B	123.2 ± 0.7	186.8 ± 1.1	3.97 ± 0.07	1.78 ± 0.08	3.69
1-3 *A*	119.4 ± 6.9	180.2 ± 11.1	4.21 ± 0.54	1.78 ± 0.54	4.10
B	120.4 ± 4.2	182.2 ± 6.7	4.16 ± 0.29	1.78 ± 0.30	3.96

As might be anticipated method (*A*) gives larger errors than (*B*) which is dominated by the quasi-fixed point; using the (3-6) starting grid further enhances this sensitivity. As illustrated by the fig.1b inset the spread in final values is governed by the smaller initial values; thus the (1-6) and (1-3) grids give comparable errors. The dependence on g_3 and M_S is shown below, with v_2 fixed using scheme (*B*).

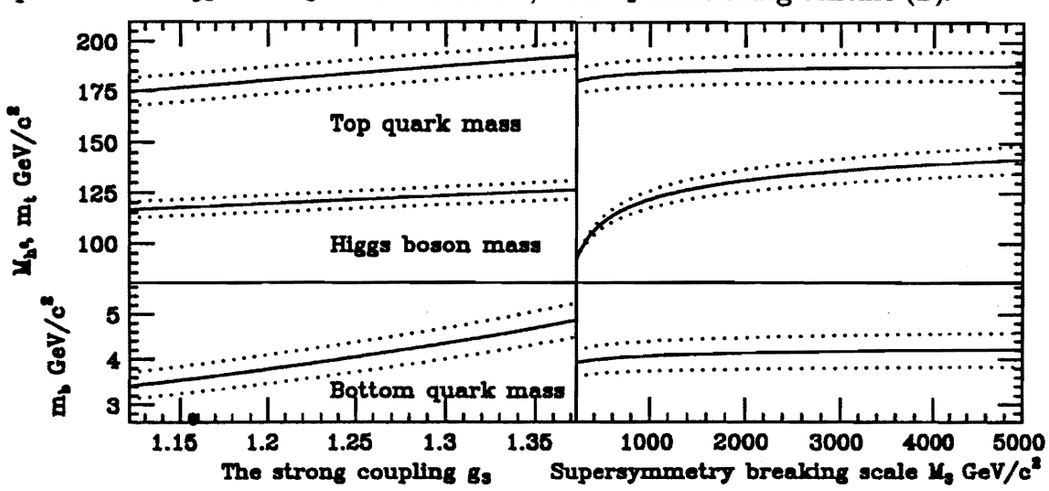

Figure 3: The strong coupling and SUSY breaking scale dependence of the mass predictions.

The $g_3(M_Z)$ dependence of both m_t and M_{h_o} is moderate and approximately linear; this is not the case for m_b which shows a marked change. Note that $g_3(t)$ grows for small t with differences due to changes in $g_3(t_{Z^o})$ scaling as $g_3^3(t)$. With respect to the M_S scale, m_t and m_b show a mild logarithmic dependence; however M_{h^o} falls significantly faster as M_S is decreased, it also has M_S dependent errors.

In summary: those ansatzes for high scale Yukawa matrices involving large initial couplings give corresponding low energy predictions determined primarily by the RGEs and so will be common to many specific models. In particular an acceptable value of the ratio m_b/m_τ doesn't require $m_b(t_X) = m_\tau(t_X)$ but arises even for essentially random starting values. Adopting a large value of $\tan\beta$ we find: $m_t = 184.3 \pm 6.8$ GeV/c^2, $m_b = 4.07 \pm 0.33$ GeV/c^2, $m_\tau = 1.78 \pm 0.33$ GeV/c^2 (input) and a light Higgs mass $M_{h^o} = 121.8 \pm 4.3$ GeV/c^2, using $\alpha_s(M_z) = 0.125$ and $M_S = 1$ TeV/c^2.

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1. C.D. Froggatt, R.G. Moorhouse and I.G. Knowles, Argonne pre-print: ANL-HEP-PR-92-62, to appear in *Phys. Lett.* B.