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Construction of $\Delta m^2 - \sin^2 2\theta$ Plots*

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1 A general description

1.1 Introduction

In the two-flavor approximation, the probability for a neutrino to oscillate from one flavor to the other is given by

$$P_{\nu_a \to \nu_b} = \sin^2 2\theta \, \sin^2(1.27\, \Delta m^2 \frac{L}{E_\nu}) \tag{1}$$

where θ is the mixing angle, $\Delta m^2 = |m^2_{\nu_a} - m^2_{\nu_b}|$ is measured in $(eV/c^2)^2$, L, the distance from the source, is measured in km, and E, the beam energy, is measured in GeV [1]. If either Δm^2 or $\sin^2 2\theta$ is zero, there is no oscillation. They might also have small, non-zero values, causing the oscillations to be so small as to be unobservable in a particular experiment. They may also have values which allow us to determine the probability of oscillation, but so far no compelling evidence for oscillation exists. The universal method of portraying what region of parameter space is explored by a neutrino oscillation experiment is to mark off an area on a $\Delta m^2 vs. \sin^2 2\theta$ plot. See Figure 1. Typically, a line is graphed, with the claim that if the experiment finds no evidence of oscillation, one can, for example, be 90% certain that Δm^2 and $\sin^2 2\theta$ lie within the region below and to the left of the line. Since these plots are so widely used, it is useful to understand the process by which they are created.

1.2 Measuring Neutrino Oscillation Parameters

We start with the case where neutrino oscillations are observed. An experiment chooses to measure some variable, x, which is sensitive to neutrino oscillations. Let's say that in the absence of neutrino oscillations, the expected value of the variable is X, and that if neutrinos oscillate, a different value than X would be expected.

$$X \neq x_{meas} = x(P) \tag{2}$$

where P is the probability of oscillation. (Throughout this note, upper case will be used for the expected value of a variable, and lower case for the measured value.) First we consider

*Work supported by the U.S. Department of Energy, Division of High Energy Physics Contract No. W-31-109-ENG-38.

he case of a monoenergetic beam at a fixed distance. We solve equation 2 for P to get $P = P(x_{meas})$ and then invert equation 1

$$\sin^2 2\theta = \frac{P(x_{meas})}{\sin^2(1.27\Delta m^2 \frac{L}{E_\nu})} \tag{3}$$

This equation gives a value for $\sin^2 2\theta$ as a function of Δm^2. The allowed values of these arameters would lie on a curve in the $\Delta m^2 - \sin^2 2\theta$ plane.

Next we consider the case in which there is a beam spectrum, $n_\nu(E) = \phi_\nu(E)\sigma_\nu(E)$ here n_ν is the event rate, ϕ_ν the neutrino flux and σ_ν the neutrino cross section. It ay not be possible to solve equation 2 for a value of P. There may be terms in x like $f(E)P(E)n_\nu(E)dE$ with f a known function of energy. Then equation 2 must be solved umerically. Equation 1 is put into equation 2 wherever P appears. (An example is given in quation 22 below.) At each value of Δm^2, θ can be varied until equation 2 is satisfied.

A simpler procedure is available for several tests in which all of the energy dependence ppears only as a result of equation 1. We consider

$$P = \frac{\int P(E)n_\nu(E)dE}{\int n_\nu(E)dE} = \frac{\sin^2 2\theta \int \sin^2(1.27\Delta m^2 \frac{L}{E})n_\nu(E)dE}{\int n_\nu(E)dE} \tag{4}$$

his P, which we may call $P(x_{meas})$, is the average probability of oscillation of the events in e event energy distribution. This is necessary, in part, because no proposed long baseline r atmospheric experiment can measure E on an event-by-event basis. Often, we will not se a distinct notation to distinguish P from this average $P(x_{meas})$. Equation 4 leads to

$$\sin^2 2\theta = \frac{P(x_{meas}) \int n_\nu(E)dE}{\int \sin^2(1.27\Delta m^2 \frac{L}{E})n_\nu(E)dE} \tag{5}$$

Even when the energy dependence is more complicated than equation 4, it may be useful approximate all energy dependent variables by their average values, and solve for $P(x_{meas})$. the $R_{\mu/\nu}$ test below, for example, this approximation is a good one, even though the event ergy distributions are slightly different. However a test which uses the rate of upward going uons from atmospheric neutrinos, the entire effect comes from the different P's from two ata sets, and the approximation is invalid. Then Equation 2 must be solved numerically.

.3 Limit Curves

the more general case, x_{meas} is not measured exactly, and may not be significantly enough ifferent from X to claim neutrino oscillations. Also in the absence of neutrino oscillations, uctuations around the mean value of X are expected. We want to know what minimum alue of $x_0 \neq X$ can be taken as significant evidence of oscillation. (Here we consider the case here x follows a gaussian distribution, but this need not be the case. It is straightforward generalize the procedure for any distribution.) The measured value of x will depend upon , the probability of oscillation.

$$x(P) - X = s\sigma_x \tag{6}$$

here s is a constant that depends on the required confidence level (CL). For a one sided aussian distribution, and 90% CL, s = 1.29. In other words, one expects a 1.29 sigma or

more excess 10% of the time. The goal, then, is to find that value of x(P), and hence P for which

$$x(P) - X = 1.29\sigma_x \tag{7}$$

If there were in fact no oscillations, and the true $P = 0$, then we would expect $x_{meas} \geq x_0$ less than 10% of the time. Thus if we were to measure $x_{meas} \geq x_0$, we would be 90% confident that the true value of P was less than the value implied by equation 7. We denote the solution to equation 7 as P_{min}, and this is analogous for a limit to the $P(x_{meas})$ in the last section. As before, in the more complicated examples, we cannot solve for P_{min}. Then we view x(P) in equation 7 as $x(\sin^2 2\theta, \Delta m^2)$ and solve numerically.

Again, if the only energy dependence in x comes from P, we can consider an average value, and solve as before to get:

$$\sin^2 2\theta = \frac{P_{min} \int n_\nu(E)dE}{\int \sin^2(1.27\Delta m^2 \frac{L}{E}) n_\nu(E)dE} \tag{8}$$

2 Long Baseline $R_{nc/cc}$ test for $\nu_\mu \rightarrow \nu_e$

As a first example, consider searching for $\nu_\mu \rightarrow \nu_e$ oscillation by measuring the $r_{nc/cc}$, the ratio of the number of neutral current interactions (defined here as interactions which do not produce a muon) to the number of charged current interactions (defined here as interactions which do produce a muon). If no oscillation occurs, the number of neutral current interactions

$$N_{nc} = N_{tot}(\frac{R}{1+R}) \tag{9}$$

where N_{tot} is the total number of neutrino interactions and R is the expected value for the ratio in the absence of oscillations. (In this paper, wherever R appears without a subscript, it refers to the expected neutral current to charged current ratio.) The number of charged current interactions

$$N_{cc} = N_{tot}(\frac{1}{1+R}). \tag{10}$$

Suppose, now, that there is probability P of oscillation. All of the ν_e interactions will be counted as neutral current interactions, since none of them will produce a muon. Thus, the number of apparent neutral current interactions will increase to

$$n_{nc} = \frac{N_{tot}R}{1+R} + \frac{PN_{tot}}{1+R} \tag{11}$$

and the number of apparent charged current interactions will decrease

$$n_{cc} = \frac{N_{tot}}{1+R}(1-P). \tag{12}$$

Combining these two expressions, we find the measured value of the ratio

$$r = \frac{R+P}{1-P}. \tag{13}$$

Solving for P,

$$P = \frac{r - R}{1 + r} \tag{14}$$

This corresponds to $P(x_{meas})$ or P_{min} as the case may be. In the former case, we use equation 5 to solve for θ. In the latter we need to calculate σ_r, the error on r. Equation 7 can be written

$$r(P) - R = s\sigma_r = s(\frac{n_{nc}}{n_{cc}})\sqrt{\frac{1}{n_{nc}} + \frac{1}{n_{cc}}} \tag{15}$$

Combining with equation 14, this can be partially solved

$$P_{min} = \frac{s\sigma_r}{1 + r}. \tag{16}$$

Note the P_{min} depends on the measured ratio $r_{nc/cc}$. However, this problem can be sidestepped by first setting n_{nc} and n_{cc} equal to N_{nc} and N_{cc}, the values one would expect in the absence of oscillations. One then obtains a value $r = R - s\sigma_r(N_{nc}, N_{cc})$, which one uses in equation 14. Having found a value for the probability of oscillation, one can use this probability in equations 11 and 12 to find more accurate values for n_{nc} and n_{cc}. This process can be iterated, and P_{min} converges quickly.

3 Long Baseline $R_{nc/cc}$ test for $\nu_\mu \rightarrow \nu_\tau$

As a second example, consider using the same test to look for $\nu_\mu \rightarrow \nu_\tau$ oscillation. Two factors complicate the situation . First, there is 17% chance that a ν_τ charged current interaction will produce a muon. This is the branching ratio B. Second, the ν_τ charged current cross-section is significantly smaller than the ν_μ charged current cross-section. Let $\eta(E)$ be the ratio of ν_τ to ν_μ cross-sections. Taking these two difficulties into account, one finds that the number of neutral current interactions becomes

$$n_{nc} = \frac{N_{tot}R}{1 + R} + \frac{PN_{tot}(1 - B)\eta}{1 + R} \tag{17}$$

and

$$n_{cc} = \frac{N_{tot}(1 - P)}{1 + R} + \frac{\eta BPN_{tot}}{1 + R}. \tag{18}$$

Combining these, one obtains

$$r = \frac{R + P(1 - B)\eta}{(1 - P) + \eta BP} \tag{19}$$

and thus,

$$P = \frac{r - R}{\eta(1 - B) + r(1 - B\eta)}. \tag{20}$$

4 Long Baseline $R_{\mu/\nu}$ test for $\nu_\mu \rightarrow \nu_\tau$

As the next example, consider looking for $\nu_\mu \rightarrow \nu_\tau$ oscillation by measuring the ratio of the number of muons entering the detector from interactions with the rock upstream (n_μ) to the number of neutrinos interaction within the detector (n_ν). If we assume for the moment that $B = 0$ and $\eta = 1$, then $n_\mu = N_\mu(1-P)$, and $n_\nu = N_\nu$, where N_μ and N_ν are the values expected for n_μ and n_ν in the absence of oscillations. If B is still zero, but $\eta \neq 1$, then $m = N_\mu(1-P)$ and $n = N_\nu(1-P) + N_\nu P(\frac{\eta}{1+R} + \frac{R}{1+R})$. Finally taking B into account, one obtains $m = N_\mu(1 - P + P\eta B)$. n remains as before. For a monoenergetic beam, we get,

$$r_{\mu/\nu} = \frac{N_\mu(1 - P + P\eta B)}{N_\nu[1 - P + P(\frac{\eta+R}{1+R})]} \tag{21}$$

For a beam spectrum, however,

$$r_{\mu/\nu} = \frac{\int dE N_\mu(E)[1 - P(E) + P(E)\eta(E)B]}{\int dE N_\nu(E)[1 - P(E) + P(E)(\frac{\eta(E)+R}{1+R})]} \tag{22}$$

$N_\mu(E)$ is the neutrino energy distribution for the rock muon events that we see. It has more high energy events than $N_\nu(E)$ because the range of the muon is proportional to energy. This equation cannot be solved for P, or even for an average P because there is more than one process involved. For a particular observed (or 10% likely) value of $r_{\mu/\nu}$, it can be solved numerically to find $\sin^2 2\theta$ as a function of Δm^2. However, for this process, it is a useful approximation to replace the terms inside the integral with their average values, and proceed.

$$P = \frac{R_{\mu/\nu} - r_{\mu/\nu}}{r_{\mu/\nu}(\frac{\eta-1}{1+R}) - R_{\mu/\nu}(\eta B - 1)} \tag{23}$$

5 Atmospheric Neutrino test using the flavor ratio

Nucleon decay detectors have measured the flavor ratio of the atmospheric neutrino flux as a possible indication of nucleon decay.[2, 3, 4] The ratio of muon neutrinos to electron neutrinos is measured

$$R_{\mu/e} = \frac{n_\mu}{n_e} \tag{24}$$

If there is a probability P of $\nu_\mu \rightarrow \nu_\tau$, then we will measure $r_{\mu/e} = R_{\mu/e}(1-P)$, which we can solve for P,

$$P = \frac{R_{\mu/e} - r_{\mu/e}}{R_{\mu/e}} \tag{25}$$

The error on the measurement of $r_{\mu/e}$ is

$$\sigma_r = \frac{n_\mu}{n_e}\sqrt{\frac{1}{n_\mu} + \frac{1}{n_e}} \tag{26}$$

For this case, the solution to equation 5 involves an integration over both energy and distance:

$$r_{\mu/e} = R_{\mu/e}(1 - \frac{\int\int P(E,L)n_\nu(E)dEdL}{\int\int n_\nu(E)dEdL}) \tag{27}$$

The neutrino energy and distance distributions are uncorrelated, and it is convenient to express the latter as a function of the zenith angle.

$$L = [r^2 1 - (r-d)^2 \sin^2\theta]^{1/2} - (r-d)\cos\theta \tag{28}$$

so that

$$r_{\mu/e} = R_{\mu/e}(1 - \frac{\int_{-\pi}^{\pi}\int P(E,\theta)n_\nu(E)dE\theta d(\cos\theta)}{2\pi\int n_\nu(E)dE} \tag{29}$$

References

[1] Donald H. Perkins, Introduction to High Energy Physics, 3rd ed., (Addison-Wesley Publishing Co.) Reading, MA, 1981. p. 256.

[2] Kajita, this proceedings

[3] Dye, this proceedings

[4] Roback, this proceedings

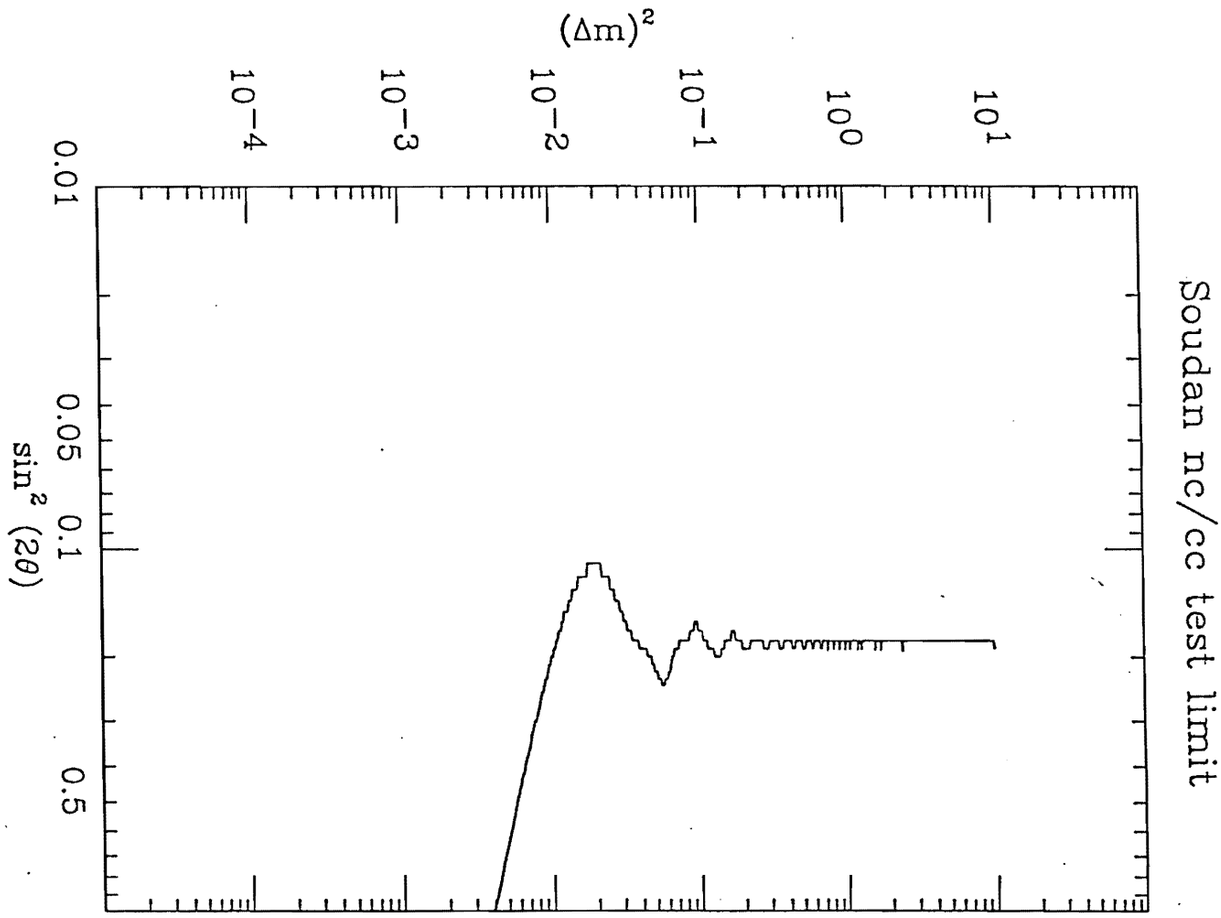
Soudan nc/cc test limit
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$(\Delta m)^2$
0.5
0.1
0.05
0.01
$\sin^2(2\theta)$

Figure 1