The Effective Gauge Boson Approximation

Wayne W. Repko
Department of Physics and Astronomy, Michigan State University
East Lansing, MI 48824
and
Wu-Ki Tung
Department of Physics, Illinois Institute of Technology
Chicago, IL 60616

Summary

The results of several studies concerned with the range of applicability of the effective gauge boson approximation are presented. These include direct comparisons of the conventional effective gauge boson approximation with exact calculations involving the production of massive final states, assessments of the effect of transverse polarization states on cross section predictions, and a new precise formulation of the method which puts it on a sound theoretical basis by eliminating unnecessary kinematical approximations. The range of applicability of this method is thereby greatly expanded. This body of work helps to establish the effective gauge boson method as a convenient and reliable calculational technique for estimating observables associated with production processes initiated by gauge bosons.

1. Introduction

The goal of the effective gauge boson working group was to clearly establish the range of validity of this technique in applications relevant to production processes at SSC energies. Two complementary approaches were followed. One subgroup was concerned with detailed comparisons between exact parton model calculations and the corresponding effective gauge boson approximations. Another subgroup investigated the theoretical basis of this useful approach in order to clarify the assumptions and to put the method on a sound basis.

On the one hand, the results reported by the first subgroup show that the effective gauge boson approximation, even in its simplest form, is a useful technique for investigating gauge boson fusion processes. In particular, for Higgs searches the estimates obtained by this method are found to be very reliable.

On the other hand, the second subgroup was able to develop a new precise formulation of the effective gauge boson method based on the factorization of the boson-exchange diagram, which obviates the need to introduce any kinematics related approximations. This leads to exactly defined gauge boson distribution functions for use with all applications, and to a definitive understanding of the reliability of the conventional "leading-log" longitudinal and transverse distributions.

In the next section, we briefly review the background of the effective gauge boson approximation approach. This is followed by a review of the work done at Snowmass 86. We conclude with some comments about the status of the method.

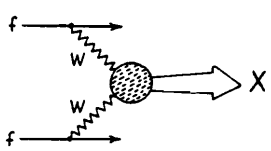

Fig. (1). Illustration of $ff \to ff + X$ by gauge boson fusion.

2. Development of the Effective Gauge Boson Approximation

The effective gauge boson approximation is a natural generalization of the "equivalent photon" or Weizsäcker-Williams approximation[1] for virtual photon processes. This approximation is widely used, for instance, in two-photon physics. It was observed by Kane[2] that quarks in protons at SSC energies can radiate massive gauge bosons, which then initiate physically interesting hard-scattering processes, in much the same way that energetic electrons can initiate photon collisions. This suggests that gauge boson collisions can be studied at the SSC, and that parton model techniques can be applied to describe these collisions. For this to be true, it is necessary that the characteristic energy scale for the hard-scattering is, on the one hand, much larger than the gauge boson mass and, on the other hand, much smaller than the total energy for overall collision. For "two-vector-boson" processes, such as WW scattering, this condition takes the form

$$m_W^2 \ll m_{WW}^2 \ll \sqrt{s} \,,$$

where $\sqrt{s}$ is the collider energy. (Since the W bosons are actually radiated off quarks, the "s" is more appropriately considered as the sub-energy of the quark-quark system.)

In order to implement this idea, it is necessary to write the cross section for the process under consideration in terms of the convolution of a hard-scattering cross section and clearly defined gauge boson distribution functions (inside the quark). In addition, one must be able to derive the distribution functions $\hat{f}_\lambda(x)$, $\lambda = -1, 0, 1$, which represent the probabilities that a fermion (either a lepton or a quark) contains a gauge boson of a particular helicity λ. The features which distinguish the (massive) effective vector boson approach from photon case are: (i) the important role played by the longitudinal polarization in high energy collisions, and (ii) the existence of distinct left-handed and right-handed polarizations due to parity violation. Point (i) is significant because in the Standard Model the couplings associated with the longitudinal degree of freedom frequently are dominant at high energies. This was first observed in the Feynman diagram calculation of Higgs production by gauge boson fusion,[3] and has since been demonstrated to be true in other production processes as well.

The complete set of gauge boson distribution functions in the leading-log approximation is available in the literature,[4,5,6] and is summarized in Table 1. Using these distribution functions, the cross section for $ff \to ff + X$ by gauge boson fusion, illustrated in Fig. (1), can be written as

$$\frac{d\hat{\sigma}}{d\, m_{WW}^2} = \frac{1}{\hat{s}} \int_{\hat{\tau}}^{1} \hat{f}_\lambda(x)\, \hat{f}_{\lambda'}\left(\frac{\hat{\tau}}{x}\right) \hat{\sigma}_{\lambda\lambda'}(m_{WW}^2) \,, \qquad (1)$$

where $\hat{\sigma}$ is the cross section for $WW \to X$, $\hat{s}$ is the square of the ff center of mass energy and $\hat{\tau} = m_{WW}^2/\hat{s}$.

For SSC applications, this constituent level expression must be folded with the appropriate quark distribution functions, which leads to the expression

$$\frac{d\sigma}{d\,m^2_{WW}} = \frac{1}{s}\int_{m^2_{WW}/s}^{1} \frac{d\tau}{\tau}\int_{-\eta}^{\eta} dy\; q_a(\sqrt{\tau}\; e^{y})\, q_b(\sqrt{\tau}\; e^{-y})$$

$$\times \int_{-\hat{\eta}}^{\hat{\eta}} d\hat{y}\, \hat{f}_{\lambda}(\sqrt{\hat{\tau}}\, e^{\hat{y}})\, f_{\lambda'}(\sqrt{\hat{\tau}}\, e^{-\hat{y}})\, \hat{\sigma}_{\lambda\lambda'}(m^2_{WW}). \quad (2)$$

The various quantities in Eq. (2) are defined by

$$\eta = \ln(1/\sqrt{\tau}), \qquad \hat{\eta} = \ln(1/\sqrt{\hat{\tau}}), \qquad \hat{s} = \tau s\,. \quad (3)$$

Table 1

λ	$\hat{f}_{\lambda}(x)$
± 1	$\frac{1}{16\pi^2 x}\left[(g_V - \lambda g_A)^2 + (1-x)^2(g_V + \lambda g_A)^2\right]\left\{\ln\left[\frac{P_\perp^2 + m_V^2(1-x)}{m_V^2(1-x)}\right] - \frac{P_\perp^2}{P_\perp^2 + m_V^2(1-x)}\right\}$
0	$\frac{(g_V^2 + g_A^2)}{4\pi^2}\;\frac{(1-x)}{x}\;\frac{P_\perp^2}{P_\perp^2 + m_V^2(1-x)}$

The fermion distribution functions above correspond to the fermion-gauge boson coupling $\bar{u}\,\not{\epsilon}^*(g_V + g_A\gamma_5)u$. To obtain the corresponding anti-fermion distribution functions, replace g_A by $-g_A$. $P_\perp$ is the maximum gauge boson transverse momentum, and the "leading-log" approximation is obtained by taking $P_\perp^2 \gg m_V^2$.

In addition to Higgs production in pp collisions,[3] Eq.(2) has also been used to study W pair production due to WW scattering at SSC energies.[7] Figure (2) shows the contribution to W pair production arising from the scattering of longitudinally polarized initial W's compared with the annihilation process $q\bar{q} \to WW$. These results showed for the first time that, within the effective gauge boson approach, the implications of WW scattering on Higgs searches could be studied in a systematic way. What remained to be shown was that this approach revealed the essential physics.

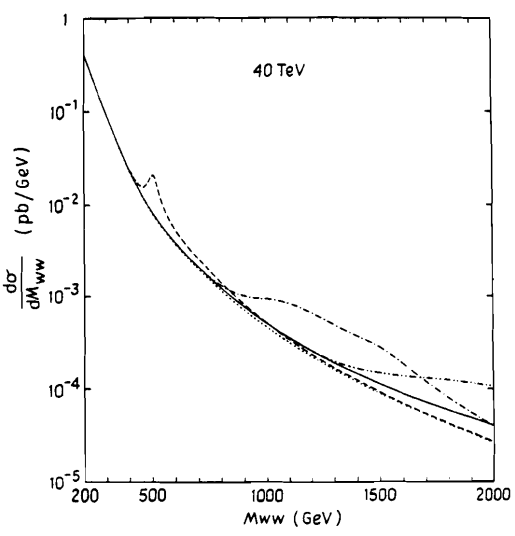

Fig. (2). $d\sigma(pp \to WW + X)/d\,m_{WW}$ is given for $\sqrt{s} = 40$ TeV and Higgs masses of 0.5 (dashed), 1.0 (dash-dot), and 1.5 TeV (dash-dot-dot) as well as the limiting case $m_H \gg \sqrt{s}$ (solid). A rapidity cut of 1.5 is imposed and the dotted line is the $q\bar{q}$ background.

3. Results of Snowmass 86

The initial applications of the effective gauge boson approximation suggested a number of topics to be clarified during the Snowmass workshop. These included:

(a) the effect of transverse luminosity on the calculated cross sections,

(b) additional comparisons between exact and approximate calculations,

(c) the implications of gauge invariance for exact calculations involving virtual gauge bosons as compared to approximate calculations where the gauge bosons are real,

(d) the treatment of t channel photon exchange singularities which are encountered in the effective gauge boson approach,

(e) the renormalization group evolution of the gauge boson structure functions,

(f) rigorous derivations of the effective gauge boson formalism and precise gauge boson distribution functions beyond the leading-log formulas.

Significant progress was achieved in each of these areas, as mentioned in the introduction. The main results will be highlighted here; details can be found in the contributed papers. We begin with the new precise formulation of the effective gauge boson method which provides a useful basis to understand the results on specific tests and comparisons.

General Results

The gauge boson exchange diagram (for a single-boson-exchange process) which underlies the effective gauge boson approach has a natural factorized structure. Thus, it can be formulated precisely as a convolution of a "distribution function" with a "hard-scattering amplitude." Applying simple group-theoretical arguments, it can be shown that only diagonal terms in the boson helicity index contribute to the overall scattering process (integrated, of course, over the direction of the parton radiating the boson). This settles previous doubts about the unknown effects due to off-diagonal matrix elements. Furthermore, under very general conditions, one can establish that the cross section for the overall process can be written in the conventional effective gauge boson form with precisely defined distribution functions. Simple formulas for these functions can be found in Ref. 8 in these Proceedings. When these exact gauge boson helicity distribution functions are used in applications, all errors of the conventional approach arising from kinematic approximations are eliminated.

Comparisons with the leading-log distribution functions of Table 1 reveal that the asymptotic longitudinal function is reasonably accurate, hence explaining the many case-studies in the literature and in these proceedings for Standard Model processes dominated by the longitudinal cross section. On the other hand, the leading-log results for the transverse distribution function are shown to over-estimate the exact ones by a significant factor for most common ranges of variables. (This is due to an unexpected cancellation between the "leading-log" and the "constant" terms in the expansion involving m_V^2/s). This fact further diminishes the contribution of the transverse polarization to various Standard Model processes as compared to previous estimates.

Tests and Comparisons

Since all published applications of the effective gauge boson approximation restricted the polarization of the initial bosons to be longitudinal, the effect of transverse luminosity on observables of interest at SSC energies had to be determined. This effect was checked in the case of W pair production in WW scattering by computing the transverse, as well as the longitudinal, cross sections in the effective W approximation.[9] Near the Higgs peak, the contribution from transversely polarized W's is negligible. As m_{WW} becomes large, this contribution dominates the continuum scattering contribution to W pair production, although it is small relative to $q\bar{q}$ annihilation. This is illustrated in Fig. (3).

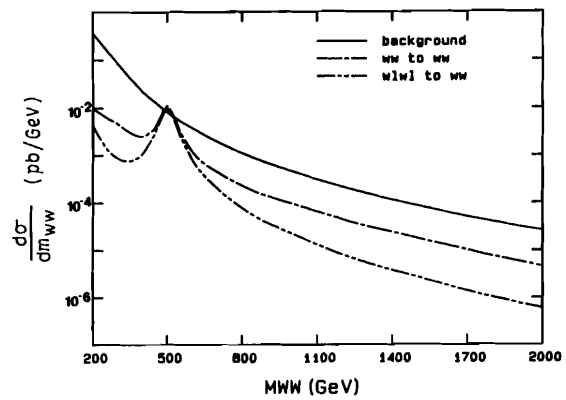

Fig. (3). Cross section for $pp \to W^+W^-$ at $\sqrt{s} = 40$ TeV. Here, both W's have rapidity <1.5 and $m_H = 0.5$ TeV. The solid curve labeled background is $q\bar{q} \to W^+W^-$, the dash-dot curve is the contribution to WW scattering from all polarizations of initial W's, and the dash-dot-dot curve is the contribution from longitudinal initial W's only.

Prior to Snowmass 86, the only direct comparison between a Feynman diagram calculation and the related effective gauge boson approximation was limited to the process $ff \to ff$ + Higgs. This situation has improved with the completion of two additional comparisons, one involving heavy fermion production and the other involving the complete gauge invariant set of diagrams associated with $ff \to ff + W^+W^-$.

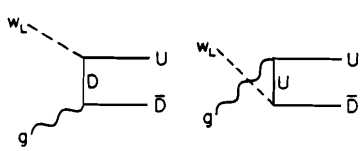

Fig. (4). Illustration of heavy quark production by the W-gluon fusion mechanism.

An exact calculation[10] of heavy quark production by the W-gluon fusion mechanism illustrated in Fig. (4) was compared with the effective gauge boson approximation[11] using only the longitudinal luminosity of the W. As seen in Fig. (5), the agreement between the exact and approximate calculations is quite satisfactory. Also reported were effective gauge boson calculations of the heavy lepton production by W^+W^-, ZW, and ZZ fusion.[12] These results indicate that the gauge boson fusion process is not likely to be a copious source of heavy lepton production.

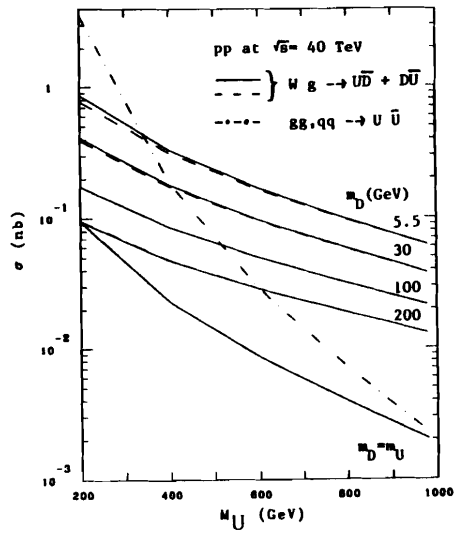

Fig. (5). Cross section for $pp \to U\bar{D}X + D\bar{U}X$ at $\sqrt{s} = 40$ TeV as a function of the U quark mass for a number of D quark masses. The dashed lines are the exact calculation, the solid lines are the effective gauge boson result and the dash-dot line is the background from $gg \to U\bar{U}$.

Although the comparisons between exact and effective gauge boson approximation calculations mentioned above yielded encouraging results, it was essential to check the method against a complete diagram calculation involving WW scattering. Only such a comparison would address several critical differences between the exact and approximate calculations. Chief among these differences are:

(i) the effect of the diagrams in Fig. (6), which in addition to those implied by Fig. (1), are required by gauge invariance to ensure that the virtual W's of the Feynman diagram calculation satisfy unitarity,

(ii) the appropriate treatment of the Coulomb singularity which occurs in real WW scattering and therefore enters the effective gauge boson approximation,

(iii) the contribution of transverse initial gauge bosons to the cross section.

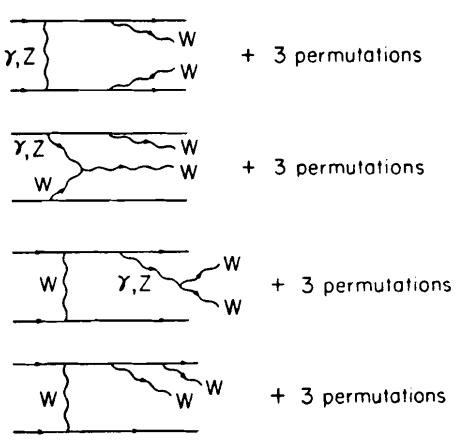

Fig. (6). Additional Feynman diagrams needed to form a complete gauge invariant set for the process $us \to dc + W^+W^-$.

As a typical example of amplitudes which would exhibit the differences outlined above, those for the process $us \to dc + W^+W^-$ were examined.[13] The corresponding cross section was studied both at the quark level and folded with quark structure functions. By confining the calculation to the charged sector, it was possible to make all the desired comparisons without unnecessary complications.

A comparison between the exact calculation and the approximate calculation of the constituent process is shown in Fig. (7). To assess the sensitivity of each calculation to minimum angular cut θ^*_{min} in the WW center of mass, three comparisons are shown corresponding to $\theta^*_{min} = 10°$, 30° and 60°. The exact calculation is relatively insensitive to this cut, varying by a factor 2. The approximate calculation varies by a factor of 19 for the same series of cuts, and most nearly agrees with the exact calculation for $\theta^*_{min} = 30°$. This variation is diminished by folding the parton level cross sections with quark structure functions as illustrated in Fig. (8), for 40 TeV pp collisions with no rapidity cut on the W's. It does not, however, disappear. Notice that the excess of events in the Higgs peak is about the same in each calculation. However, the absolute number of events given by the effective gauge boson approximation is sensitive to θ^*_{min}.

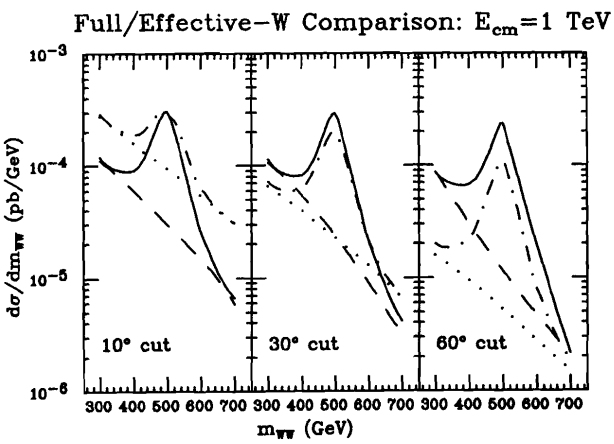

Fig. (7). Results for $d\sigma/dm_{WW}$. As described in the text, different cuts on the WW center of mass scattering angle have been chosen. In all three comparisons spectra for: (i) exact calculation $m_H = 500$ GeV (solid); (ii) exact calculation $m_H = \infty$ (dashes); (iii) effective W calculation $m_H = 500$ GeV (dot-dashes); and (iv) effective W calculation $m_H = \infty$ (dots), are plotted.

Clearly, a better understanding of the minimum angular cut within the effective gauge boson approximation (where the on-shell WW amplitude has a $t = 0$ Coulomb pole) is needed. At the constituent level, this can be studied by comparing neutral sector processes like $ud \to ud\, W^+W^-$ involving W pair production by Z^0's and photons. Presumably, these processes are less sensitive to the θ^*_{min} cut because the corresponding amplitudes have no singularities in the physical region. The sensitivity of the pp results to θ^*_{min} should also be reduced when reasonable rapidity cuts are imposed. Both of these approaches are under investigation. Finally, since Snowmass 86, a complete calculation of $pp \to W^+W^-$ has been published.[14] This calculation was compared with the

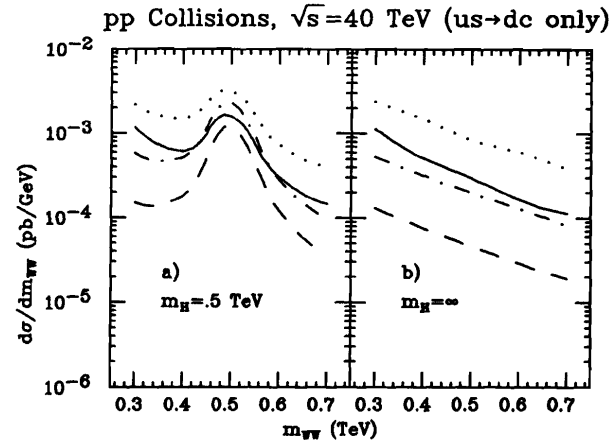

Fig. (8). Cross sections for the exact and effective W calculations in the cases: (a) $m_H = 0.5$ TeV; and (b) $m_H = \infty$ are plotted. The exact result is represented by the solid line, while the effective W $\theta^*_{min} = 60°$, 30°, 10° results are given by the dashed, dot-dashed, and dotted lines respectively.

effective gauge boson treatment of the same process[7] and the agreement between the two is quite reasonable.

Evolution of the Gauge Boson Distribution Functions of a Hadron

In principle, the gauge boson distributions of a hadron should be dependent on a scale parameter, say μ, in addition to x. The numerical importance of the proper choice of scale μ^2 and the q^2 evolution of the distribution functions (a la Altarelli-Parisi) are relevant issues in a complete effective boson formalism. These were examined in Ref. 15. It was noted, in particular, that the V-A nature of the W and Z boson couplings produces a sizable intrinsic polarization of the transverse boson distribution functions. Such large factors could be important in the search for heavy particle "new physics" (where the larger x values are important, and we can not assume a priori that the longitudinal polarizations dominate the production process).

4. Conclusions

The results summarized here provide a more rigorous theoretical understanding of the effective gauge boson approximation and establish the method as a reliable tool for investigating processes involving gauge bosons at SSC energies.

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References and Footnotes

1. C. Weizsäcker, Zeit Phys. 88, 612 (1934); E. Williams, Phys. Rev. 45, 729 (1934); V.M. Budnev, I.F. Ginzburg, G.V. Meledin, and V.S. Serbo, Phys. Rep. 15C, 183 (1974).
2. G.L. Kane in Proceedings of Physics of the XXI Century, (Tucson, Arizona, December, 1983).
3. R.N. Cahn and S. Dawson, Phys. Lett. 136B, 196 (1984); M. Chanowitz and M.K. Gaillard, Phys. Lett. 142B, 85 (1984).
4. G.L. Kane, W.W. Repko, and W.B. Rolnick, Phys. Lett. 148B, 367 (1984).
5. S. Dawson, Nuc. Phys. B249, 42 (1985).
6. J. Lindfors, Zeit. Phys. C28, 427 (1985).
7. M.J. Duncan, G.L. Kane, and W.W. Repko, Nuc. Phys. B272, 517 (1986).
8. P.W. Johnson, F. Olness and W.-K. Tung, "A Precise Formulation of the Effective-Vector-Boson Method for High Energy Collisions," these proceedings.
9. A. Abbasabadi and W.W. Repko, "Transverse Luminosity and the Effective W Approximation," MSU preprint MSU/TH-Sep 86, these proceedings.
10. S. Willenbrock and D.A. Dicus, Phys. Rev. D34, 155 (1986).
11. S. Dawson and S. Willenbrock, "Heavy Fermion Production in the Effective W Approximation," LBL preprint 21914 (July, 1986).
12. S. Dawson, G.L. Kane, C.P. Yuan and S. Willenbrock, "Heavy Lepton Production in the Effective W Approximation," LBL preprint 22087 (September, 1986), these proceedings.
13. J.F. Gunion, J. Kalinowski, A. Tofighi-Niaki, A. Abbasabadi, and W.W. Repko, "Comparison between an Exact $ff \to ffWW$ Calculation for the Charged Current Sector and the Effective-W Approximation," UCD preprint UCD-86-23, these proceedings.
14. D.A. Dicus and R. Vega, Phys. Rev. Lett. 57, 1110 (1986); see also: R. Kleiss and W.J. Stirling, Cern preprint CERN-TH 4451/86, May, 1986.
15. J.P. Ralston and F. Olness, "Intrinsic Polarization of the High Energy W-Boson Structure Functions," ANL preprint ANL-HEP-CP-86-104, these proceedings.