Reliability of b-Quark Production Rates at Supercollider Energies

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Summary

We discuss present uncertainties in theoretical estimates for b-quark production within the standard model. Assuming that perturbative QCD gives an accurate subprocess cross section, the primary uncertainty comes from model-dependence of the small-x gluon distributions. We present calculations of a double–differential luminosity $\partial^2\mathcal{L}/\partial\tau\partial y$ for different candidate gluon distributions. For $Q^2 \sim (2m_b)^2$, the different luminosities vary over more than an order of magnitude, while for Q $\sim$ 100GeV the distributions vary over about a factor of two.

Small-x

The past few years have shown a marked improvement in the consistency of perturbative QCD descriptions of heavy quark production in hadronic collisions. By now it is generally accepted[1] that the mass of the b-quark is large enough to suppress most of the problems that have plagued charm production, so that there is some confidence[2] in the predictability of b–quark subprocess cross sections at high energy machines. However, there are new problems in predicting rates at very high energies such as produced by the SSC. In any process where the c.m. energy $\sqrt{s}$ is enormously larger than the invariant mass Q of a subprocess of interest, one needs to know the parton fluxes at very small x to make a prediction. The parton distributions for small x, which we take to mean $x \leq 10^{-2}$, have not been measured. Contrary to current notions, the calculated distributions are model dependent.[3] In this paper we will report on the present theoretical uncertainty in predicting b-quark production cross sections at SSC energies.

The most thorough numerical studies of the small-x parton distributions have been presented by EHLQ.[4] Assuming one-loop renormalization group evolution as formulated in the Altarelli-Parisi(AP) equations[5], the small-x structure functions appear to be *a priori* unpredictable: they depend on unknown small-x initial conditions at fixed $Q^2 = Q_o^2$. However, one of the empirical conclusions of the numerical experiments was that structure functions become more predictable at any fixed small-x value with increasing Q^2. The calculations showed[4] that reasonable deformations of initial conditions, given by comparing

$$a)\ \ xG(x,Q_o^2) \sim const., x < 10^{-2}$$

and

$$b)\ \ xG(x,Q_o^2) \sim 1/\sqrt{x} + const., x < 10^{-2}$$

where $G(x,Q_o^2)$ is the gluon distribution, were mathematically "healed" by the evolution procedure. Thus EHLQ observed a merging of the different predictions, occurring at fixed x as Q^2 was increased to a large value, say, $Q^2 >> 10^3 GeV^2$. The healing phenomenon was explained analytically by studying general solutions to the AP equations in the limit $x << 1$; a brief summary[3] was given at the Madison workshop.. The conclusion of our study was that initial conditions give rise to predictions along lines described by $1/x = const.[\ln Q^2/\Lambda^2]^m$, so that for any small-$x$ one can always choose Q^2 large enough to lead to reliable predictions propagated from real data. Unfortunately, in most cases, Q^2 must be rather large.

In fact, a great deal of the data will be at fixed Q^2 and in a region irrelevant for this trick. Our strategy here will be to present differential luminosities which will illustrate this present situation together with its ambiguities.

The EHLQ calculations give luminosities in terms of parton distributions $f^a(x_1)$, $f^b(x_2)$ by

$$\frac{\partial\mathcal{L}}{\partial\tau} = \int dx_1 dx_2 [f^a(x_1) f^b(x_2)] \delta(x_1 x_2 - \tau) \qquad (1)$$

for hadrons a,b. We have chosen one flavor of parton only for illustration. Here τ will correspond to $\hat{s}/s$ in a subprocess where the partons collide with c.m. energy $\sqrt{\hat{s}}$. Note that $\partial\mathcal{L}/\partial\tau$ does not contain information on the rapidity y of the hard subprocess, given by $y = 1/2\ln(x_1/x_2)$, which has been integrated over. To reveal this information we define

$$\frac{\partial^2\mathcal{L}}{\partial\tau\partial y}(f) = \int dx_1\, dx_2\, [f^a(x_1) f^b(x_2)] \Delta(x_1,x_2)$$

$$\Delta(x_1,x_2) = \delta(x_1 x_2 - \tau)\, \delta(1/2\ \ln(x_1/x_2) - y) \qquad (2)$$

which gives the rapidity distribution for each separate choice of τ. The gluon luminosity is the largest and most important, so we present

$$\mathcal{L}_{\tau y} = \frac{\tau}{\hat{s}} \frac{\partial^2\mathcal{L}}{\partial\tau\partial y}(GG),$$

where $\tau/\hat{s}$ has been introduced to incorporate most of what is needed for a typical cross section except for coupling constants. By multiplying with some simple factors it is also possible to convert the Set 1 $\mathcal{L}_{\tau y}$ into a sea–quark luminosity distribution, using the relations found in Ref. (6),

$$xq_s(x,Q^2) \cong \frac{1}{12} \frac{\partial}{\partial(\ln 1/x)} xG(x,Q^2) \qquad (3)$$

which self-consistently solves the Altarelli-Parisi equations and gives numbers quite close to those of EHLQ. For future reference we note that $\mathcal{L}_{\tau y}$ is just an appropriate product of $xG(x,Q^2) \equiv \bar{G}(x,Q^2)$ distributions.

$$\mathcal{L}_{\tau y} = \frac{1}{\hat{s}} \bar{G}(\sqrt{\tau}e^y, \tau s)\, \bar{G}(\sqrt{\tau}e^{-y}, \tau s). \qquad (4)$$

We chose three cases to present under the different boundary conditions (a,b) above. The case a) was calculated using

the partonometer method[6] with an improved fit to the boundary conditions of Durand[7], valid to about 10% down to Q = 10 GeV. This produces the curve marked "Set 1". This was compared with the procedure followed by EHLQ[4] in calculating $\partial\mathcal{L}/\partial\tau$, which was to flatten $xG(x,Q^2)$ to its value at $x = 10^{-4}$ for all values of $x < 10^{-4}$, where $G(x,Q^2)$ was not calculated. Such a procedure underestimates the AP evolution since $xG(x,Q^2)$ continues to grow as x gets smaller. For reference, at Q = 10 GeV and $\sqrt{s}$ = 40 TeV, x ranges down to 6 x 10^{-7} for $|y| \leq 6$. The flattening by EHLQ is not as serious a problem for $\partial\mathcal{L}/\partial\tau$ as for $\partial^2\mathcal{L}/\partial\tau\partial y$, since the simple τ distribution is dominated by the central region, but it can be a 40–50% effect for $\partial\mathcal{L}/\partial\tau$.

Finally, these calculations were compared to case b), $xG(x,Q_o^2) \sim 1/\sqrt{x}$, by using the numerical work of EHLQ consistent with another analytic asymptotic estimate valid for this case[3]. To make the calculation for values of x smaller than those reported by EHLQ ($x < 10^{-4}$) one needs to know $xG(x,Q^2) \sim C(Q^2)/\sqrt{x}$. This has been discussed by Collins[8], and it can be shown[3] directly from the AP equations without reference to Regge theory.

The curves show $\mathcal{L}_{\tau y}$ for $\sqrt{\hat{s}}$ = 10, 32 and 100 GeV and $\sqrt{s}$ = 40 TeV. One sees that subprocesses with $\sqrt{\hat{s}} \geq 100$ GeV are becoming predictable, while the situation for typical b-quarks with small transverse momentum is very unclear: a factor of 10 uncertainty is a reasonable assessment for b-quark production cross sections. Although the results we present appear to present the flattened EHLQ estimate as conservative, it is by no means a lower bound. The choice $G(x,Q_o^2) \sim \sqrt{x}$ has also been investigated, and leads to values smaller than any of the results shown. Finally, we emphasize again the caution needed in making rapidity distributions for objects of invariant mass that is small on the SSC scale.

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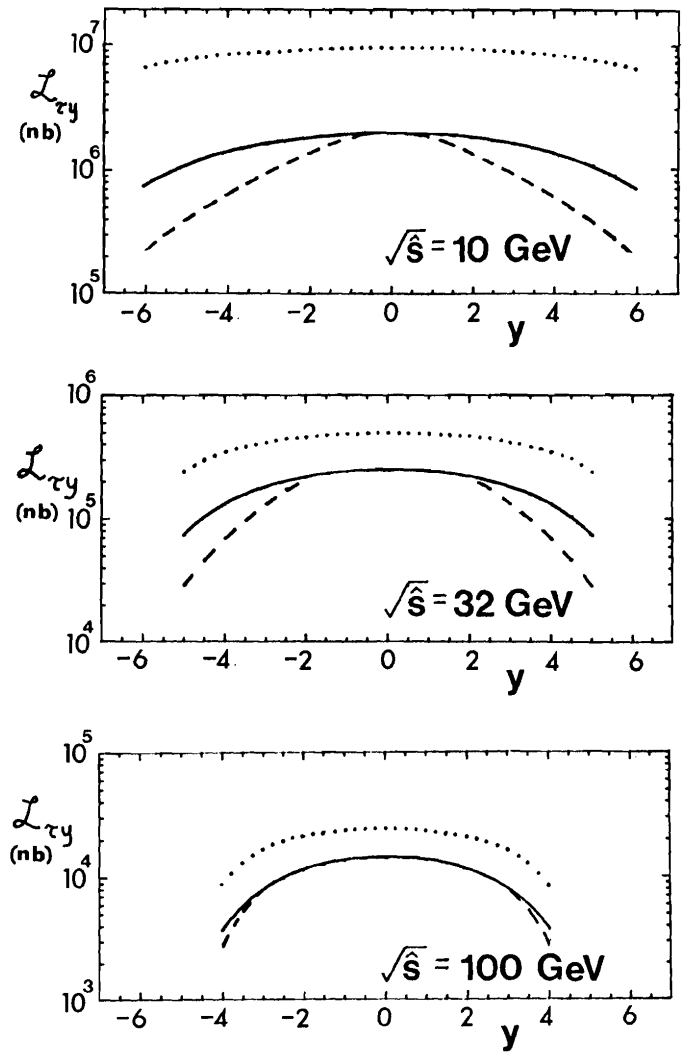

$\mathcal{L}_{\tau y}$, defined in the text, vs y for $\sqrt{\hat{s}}$ = 10, 32 and 100 GeV and $\sqrt{s}$ = 40 TeV. Solid curves are case a), $xG(x,Q_o^2)/\sim$ constant $x < 10^{-2}$; dashed curves are case a) with $xG(x,Q^2)$ "flattened" at its $x = 10^{-4}$ value when $x < 10^{-4}$; the dotted curves are case b), $xG(x,Q_o^2)/\sim 1/\sqrt{x}$, $x < 10^{-2}$.