WHAT NEUTRINOS CAN TELL US ABOUT PARTONS

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The parton model has been useful in guessing at regularities expected at high energy both for hadron collisions and deep inelastic electron or lepton scattering. The model gets its idea from field theory. According to a field theory a hadron state wave function (an eigenstate of the field Hamiltonian), could be described by giving the amplitude to find various numbers of particles of the basic fields of the theory at various momenta in the state. A quantum of the basic field theory, whose specific properties we do not know, of course, is called a parton. In our present knowledge of these things we shall have to guess both at the kinds of partons there might be as well as the way they are distributed in the hadron state. Both of these things can, of course, ultimately be determined by experiment - and this suggests a program for the future - (provided the framework is correct). It is the purpose of this paper to show how this might be done. The method I shall use is to take, as an example, very specific assumptions on what partons are (quarks) and how they are distributed and to show, by this example, how predictions might be made and how the specific assumptions could be tested by experiment. It should become obvious by analogy how to make testable predictions for some other choice of what partons are and how they act.

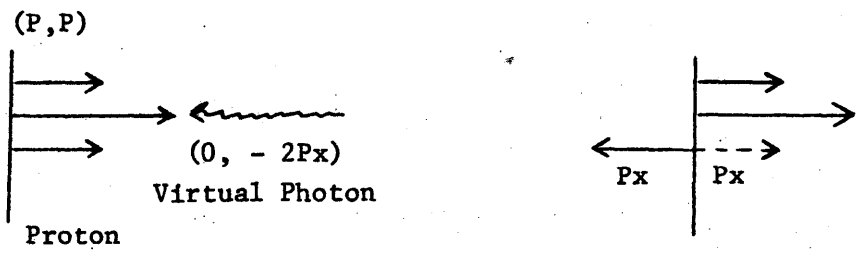
First we give a brief review of the parton idea and show how they have been applied to inclusive electron scattering $ep \to e$ + anything. Then the applications are described to inclusive neutrino scattering. Our assumptions to this point will be consistent with the assumptions of light cone algebra so all the conclusions of that theory can be obtained here too. Finally in the last part of the paper we discuss how, by making additional hypotheses, predictions about the distribution of hadrons in the final state resulting from the collision can be made. In all cases we restrict ourselves to the deep inelastic region.

THE PARTON PICTURE OF DEEP INELASTIC ELECTRON SCATTERING

The wave function for a proton moving with a large momentum P to the right along the z axis, is supposed to be large only when the transverse momenta of the partons are finite (e.g. of order GeV). This is suggested by energetic hadron-hadron collisions. The longitudinal momenta may be finite, or large of order P. In the latter case we write the parton longitudinal momentum as xP and then suppose that as P approaches infinity the amplitude to find partons with various values of x (from 0 to 1) is independent of P. Reasons for these assumptions are discussed more fully in reference 1. These assumptions may or may not be consistent with the quantum field theory which inspired the model in the first place, but we make them anyway, disregarding to some extent the original motivation of the model.

When we scatter a high energy lepton from such a proton it scatters from a particular parton and, from the conservation of energy and momentum we can determine the momentum, or x, of the parton that did the scattering. Thus the spectrum of the scattered lepton determines the distribution in x of the parts inside, in a manner analogous to the way the frequency distribution of radar scattered from a swarm of bees determines the velocity distribution of the bees inside the swarm. To use the conservation of energy, however, we use, to sufficient approximation, the energy of the parton as if free, whereas it is in fact in interaction with the other partons of the proton. We explicitly assume that the interaction between two partons of large relative momentum is not similarly large, so if we do this experiment with sufficient momentum, this interaction makes relatively little error.

Let q represent the change in momentum of the lepton - therefore the momentum of the virtual photon in electron scattering. We shall review very briefly electron scattering and then go on to our main subject, neutrino scattering. Let p be the momentum of the proton, $p \cdot q = M\nu$ where ν is the energy loss of the lepton in the lab. Now we take $-q^2 = 2M\nu x$ and let $\nu \to \infty$ keeping x fixed. A good system in which to visualize things is the one in which the virtual photon momentum q is spacelike say $(0, -2Px)$ and the proton has large momentum P, energy practically P also $p = (P,P)$. Hence $P^2 = M^2\nu^2/(-q^2)$ and the deep inelastic region corresponds to $P \to \infty$. We picture the proton as a group of partons of which a typical one has momentum $P\xi$ (Figure 1)



(a) Before Interaction

(b) Immediately After Interaction

FIGURE 1

When the virtual photon interacts with one parton its momentum is changed by -2Px but the magnitude of the momentum must not be changed (the approximate conservation of energy we mentioned above) so that only the parton with momentum Px can interact - it gets its momentum reversed to -Px. The state of the partons just after the event is illustrated in Figure 1(b); there is one parton moving to the left with momentum -Px and the original set that were in the proton, less the parton at Px that was scattered away, going to the right. The total momentum of this set is now P(1-x).

The probability that something happens is proportional to the probability of finding a parton with momentum x times its charge (the coupling to the photon) squared.

PARTONS AS QUARKS IN ELECTRON SCATTERING

To illustrate this we shall take as an example (as we will throughout this paper) the case that the charged partons are quarks or antiquarks. We then may characterize the proton by six functions. Let u(x) be the number of up quarks (charge + 2/3, isospin + 1/2, zero strangeness) with momentum fraction x per dx in the proton. Let d(x), s(x) be the corresponding number of down (isospin - 1/2) and strange (isospin 0) quarks respectively; and $\bar{u}(x)$, $\bar{d}(x)$, $\bar{s}(x)$ the corresponding number of antiquarks.

We do not assume there are only three quarks in the proton, two ups and one down, as in the low energy quark model. There may be many quark pairs in addition, but the total charge + 1, and total isospin + 1/2, and strangeness 0, of the proton does require that the net number of up quarks be 2, net down be 1, and net strange be 0;

$$\int_0^1 \left(u(x) - \bar{u}(x)\right) dx = 2$$

$$\int_0^1 \left(d(x) - \bar{d}(x)\right) dx = 1 \qquad (1)$$

$$\int_0^1 \left(s(x) - \bar{s}(x)\right) dx = 0$$

The probability it was an up quark that scattered the photon is then 4/9 (the square of the charge 2/3) times u(x), the number of up quarks available. Thus the total probability of interaction with virtual photons is expressed in terms of our six functions by

$$f^{ep}(x) = \frac{4}{9}\left(u(x) + \bar{u}(x)\right) + \frac{1}{9}\left(d(x) + \bar{d}(x)\right) + \frac{1}{9}\left(s(x) + \bar{s}(x)\right) \qquad (2)$$

(This $f^{ep}(x)$ is related to experimental quantities in a way described in Kutis' talk[2)] at this meeting. It is the function $2MW_1$ in the scaling limit.) We have been able to disregard the mass and transverse momenta of the partons as we are here only dealing with the leading terms in the results at high momentum.

Properly we should deal separately with three directions of polarization e_μ of the virtual photon which we can take as two transverse; positive helicity, negative helicity, and one "longitudinal" in the t direction (it must be perpendicular to q which is in the z direction). For unpolarized protons the two helicities give the same result of course. For brevity we shall not analyze the polarized proton case here, but it is discussed by Kuti[2)], where a remarkable sum rule for G_A/G_V due to Bjorken, results. If partons are spin zero the coupling, when one changes momentum from p_1 to p_2, is $(p_1 + p_2)\cdot e$. We see this is zero (i.e. of lower order in P) for transverse e_μ, and large for e_μ in the t direction. On the other hand if partons were spin 1/2 the coupling $(\bar{u}_2 \not{e}\, u_1)$ is large for transverse e_μ, but zero for e in the direction of $p_1 + p_2$ (the t direction). In a given experiment with q, ν fixed, as we vary the angle of the electron scattering by which the virtual photon is produced, we vary the proportion of longitudinal and transverse polarization produced by the electron. Thus we can separate the contribution of the longitudinal and transverse polarization. (Hence there are two structure functions which scale, they are combinations of νW_2 and $2MW_1$). Experiments indicate that the longitudinal scattering is small, near 20% of the transverse. We shall, therefore, guess that the charged partons have spin 1/2, the remaining 20% would presumably be the result of not having large enough P in the experiments.

(Thus there should at high energy be only one independent function, we expect $\nu W_2 = x \cdot 2MW_1$ in the limit.) This is our first example of how lepton scattering can say something about the character of the charged partons.

The function $f^{ep}(x)$ is, of course, known from experiment. For small x it goes roughly as .32/x, near x = 1/2 it is about .3 and as x approaches 1 it falls away possibly as $(1-x)^3$ (as suggested by Drell and Yan who relate it to the proton form factor). A fall off near x = 1 is expected for if one parton has nearly all the momentum all the others must be restricted to low momentum and the probability of that is small. Now we must discuss in more detail the region of small x.

WEE PARTON REGION

For a given P if x gets small enough, of order 1 GeV/P, which we call "wee," the momentum of the partons becomes finite and many of our approximations fail there. The formula dx/x probably fails there. We expect that it does not continue to rise in this region, for it must eventually fall toward zero in the x negative wee region. The total number of wee partons is then always finite and the total number of partons grows logarithmically with P (a wave function is not a covariant idea). This is best appreciated by describing things in rapidity space, $y = 1/2\ \ell n \frac{\varepsilon + p_z}{\varepsilon - p_z}$ where p_z is the longitudinal momentum in GeV and $\varepsilon = \sqrt{p_z^2 + p_\perp^2 + m^2}$ where m is some convenient mass (say .3 GeV). Then for finite x, $p_z = Px$ and $y \approx \ell nx + \ell n2P$, whereas wee partons correspond to finite y. The $\frac{dx}{x}$ behavior for small x becomes a long plateau dy from finite y to the region of $y = \ell n2P$. In this variable the distribution of some parton looks like Figure 2.

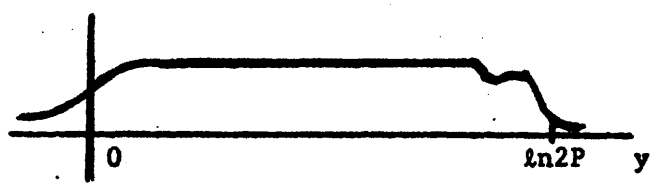


FIGURE 2

Scaling implies that as we increase ℓn2P the upper end moves out, only the plateau region stretches. We shall suppose that the tail near y = 0 stays the same.

This distribution arises from the interactions among partons given by the field Hamiltonian and we shall suppose that it happens in the following way:. Interactions are only important over a finite range of y - the entire range of y gets filled by a cascading produced by the Hamiltonian. Finally, because the scaling character of the equation changes near x = 0, the cascade is terminated in a characteristic way. It is analogous to the cascade of cosmic ray showers. No matter how they start they develop similarly and end (when ionization losses become important and change the equation) in a characteristic way. It is like the wave function for a liquid layer (with y replaced by space) between two surfaces (except the number of molecules is not fixed). At each surface there is a characteristic behavior, and there is a uniform density region in between although interactions are always local. It is the spirit of this paper to make as strong assumptions as possible to generate targets for experiments to shoot down. We shall try the assumption that the wee region (y near 0) and, by continuity, the plateau region (x small) are the same for every hadron, only the y - ℓn2P (larger x) region varies from case to case.

That implies the plateau is neutral having as many particles of a kind as antiparticles. It is indifferent as to isospin. For example near $x = 0$ we must have the $\frac{1}{x}$ behavior of $u(x)$, $\bar{u}(x)$, $d(x)$, $\bar{d}(x)$ to have the same coefficient a/x, and $s(x)$, $\bar{s}(x)$ to go as $\beta a/x$. SU_3 would imply $\beta = 1$ but we do not have nerve enough to assume that - for after all of the interactions, the SU_3 breaking could produce differences. We know from experiment $a = .24/(1 + \frac{1}{6}(\beta-1))$.

ELECTRON-NEUTRON SCATTERING

For scattering from neutrons we obtain the same formula as (2) except that $u(x)$ would be replaced by the number of up quarks in the neutron etc. However by isospin reflection this is the number of down quarks in the proton, which we have called $d(x)$. So if we do not change the definition of our six functions, so they still refer to the proton, we find

$$f^{en} = \frac{4}{9}\left(d(x) + \bar{d}(x)\right) + \frac{1}{9}\left(u(x) + \bar{u}(x)\right) + \frac{1}{9}\left(s(x) + \bar{s}(x)\right) \qquad (3)$$

Thus we have available two experimental functions to determine our six. Data on the neutron shows that f^{en}/f^{ep} starts at 1 at $x = 0$ (as implied by our assumption of a universal plateau) and falls gradually to perhaps .4 at $x = .8$, data is not available above $x = .8$. Since all the functions $u(x)$, etc., are positive, we see immediately that should the ratio fall below 1/4, the partons could not be quarks. There is no such difficulty as yet.

For x near 1 all the quarks but one must be pushed to low x. Perhaps the difficulty of doing this depends on the total quantum number of the state of these low quarks, e.g., whether $I = 1$ or $I = 0$. If this is so one case would be easiest and dominate. Only the $I = 0$ (and quark number two) state gives f^{en}/f^{ep} below 1, for $x \sim 1$, so we

explicitly assume this state dominates. Then as $x \to 1$, the last quark in the proton is an up quark (in the neutron a down quark) so f^{en}/f^{ep} must approach $\frac{1}{4}$ as $x \to 1$. The ratio of the form factors G_M^P/G_M^N would go to -2 as $q^2 \to -\infty$. We shall also assume the total angular momentum of the slow quarks is zero, so the u spins as does the proton. Otherwise G_M^P would change sign as $-q^2$ varies from zero to large values.

Finally we notice that the total fractions of the proton momentum carried by the charged quarks is $k = \int_0^1 x(u + \bar{u} + d + \bar{d} + s + \bar{s})dx$ while $\int_0^1 x\, f^{ep}dx = .18, \int_0^1 x\, f^{en}dx = .13$. From this we find that unless the strange quarks carry 70% of the momentum of the proton, which is probably absurd, k is less than 1. Hence there must be some kind of neutral partons in addition to quarks (that carry perhaps 40% of the momentum of the proton).

So far no experiment has definitely proved or disproved the reality of the peculiar quantum numbers of quarks. It should be possible soon, either by using polarized protons or by neutrino scattering. We discuss this next.

NEUTRINO SCATTERING

We now turn to neutrino scattering experiments. We use the usual theory of the lepton current coupling with a hadron current J_μ^h. Tests of whether scaling works directly as for point interaction or only after a suitable q^2 dependent factor for "intermediate W-meson propagation" are, of course, of first importance. They do not ultimately effect what we shall say here for we are studying the hadron current factor.

The other factor (from the leptons) can be, without implying anything physical, represented as an external virtual vector meson field W_μ coupled to J_μ^h. Following Cabibbo we shall take this coupling to be

$(\bar{Q}_2\ \gamma_\mu\ (1+i\gamma_5)Q_1)W_\mu$ (plus its complex conjugate) where Q_2 is an up quark and Q_1 is a "Cabibbo quark" that is one which has amplitude $\cos\theta_c$ to be down and $\sin\theta_c$ to be strange. Experimentally $\sin^2\theta_c$ is small (.06) and for clarity and simplicity I shall give all the discussion here neglecting it. It will be obvious how to rederive all the formulas to allow for this generally small correction. Thus a positive virtual W, produced by incoming neutrinos going to μ^-, can convert a d parton in the proton to a u or a $\bar{u}$ to a $\bar{d}$.

This "virtual W meson" will have momentum q and three polarizations helicity +, say, W_+; helicity -, W_-; and longitudinal, W_t. The proportion of these produced by the neutrino depends on the angle of ν,μ^- scattering, therefore three structure functions are needed to describe the data now, (they are νW_2, $2MW_1$ and νW_3). As before the W_t shouldn't couple in the deep region $P \to \infty$, (so νW_2 should be equal to $x \cdot 2MW_1$). Conservation of z-component spin requires that the positive helicity W_+ couple only with a positive helicity parton, sending it back with + helicity.(We neglect mass and transverse momentum.) But the $1 + i\gamma_5$ says that energetic quarks interact in the weak interaction only if these have negative helicity, antiquarks interact only if they have positive helicity. Thus W_+ couples only with antiquarks and hence only with $\bar{u}$, converting it to $\bar{d}$. W_- sends $d \to u$. Using antineutrinos generates anti-W which if they have positive helicity $\bar{W}_+$ convert $\bar{d}$ to $\bar{u}$; $\bar{W}_-$ converts $u \to d$. Thus neutrino and antineutrino scattering can permit us to select the action on one type of quark at a time and permits, for example, separate determination of u(x), d(x), $\bar{u}(x)$ and $\bar{d}(x)$.(Because of the smallness of $\sin^2\theta_c$ our handle on s(x) or $\bar{s}(x)$ is too weak to be useful.) We describe below the one arm structure functions (e.g. $f_+^{\nu p}$ is for scattering of neutrinos on

protons positive helicity part, hence done via W_+ etc.) for each case, give their expressions in terms of the conventional structure functions ($f_1 = 2MW_1$, $f_3 = \nu W_3$) and give their theoretical expression in terms of u(x) etc., for $\sin^2\theta_c = 0$:

$$
\begin{aligned}
W_+, \quad \bar{u} \to \bar{d} : \quad & f_+^{\nu p} = \frac{1}{4}\,(f_1^{\nu p} + f_3^{\nu p}) = \bar{u}(x) \\
W_-, \quad d \to u : \quad & f_-^{\nu p} = \frac{1}{4}\,(f_1^{\nu p} - f_3^{\nu p}) = d(x) \\
\bar{W}_+, \quad \bar{d} \to \bar{u} : \quad & f_+^{\bar{\nu} p} = \frac{1}{4}\,(f_1^{\bar{\nu} p} + f_3^{\bar{\nu} p}) = \bar{d}(x) \\
\bar{W}_-, \quad u \to d : \quad & f_-^{\bar{\nu} p} = \frac{1}{4}\,(f_1^{\bar{\nu} p} - f_3^{\bar{\nu} p}) = u(x)
\end{aligned}
\tag{4}
$$

Having the possibility of determining these four functions individually now leads to a lot of predictions obtained upon substituting these into our previous formulas. (We note that f_1 will be much easier to measure than f_3.) First we get two sum rules by substituting into equation (1); we write them separating the f_1 and f_3 parts:

$$
\begin{aligned}
\int_0^1 (f_1^{\bar{\nu} p} - f_1^{\nu p})\,dx &= 2 \\
\int_0^1 (f_3^{\nu p} + f_3^{\bar{\nu} p})\,dx &= -6
\end{aligned}
\tag{5}
$$

The first was discovered by Adler, derived from the equal time commutation rules of Gell-Mann. It is of central importance to check it. It does not check the specific assumptions of the quark model as well as would the second relation in (5), due to Llewellyn Smith. The -6 here is a special consequence of quark quantum numbers.

The difference of f^{ep} (Eq.(2)) and f^{en}(Eq.(3)) is $\frac{1}{3}(u+\bar{u}) - \frac{1}{3}(d+\bar{d})$ and can be expressed via (4) as

$$f_3^{\nu p} - f_3^{\bar{\nu} p} = 6\,(f^{ep} - f^{en}) \tag{6}$$

This relation, also due to Llewellyn Smith, is to be valid at every value of x and is definitely quark dependent. It's verification would represent a fundamental demonstration of the reality of quark quantum numbers. Unfortunately its verification requires measuring the difficult quantities f_3. If only f_1 is available, we note

$$f^{ep} + f^{en} = \frac{5}{9}(u + \bar{u} + d + \bar{d}) + \frac{2}{9}(s + \bar{s}) = \frac{10}{9}(f_1^{\nu p} + f_1^{\bar{\nu} p}) + \frac{2}{9}(s + \bar{s}) \tag{7}$$

and we do have the possibility of seeing the theory is wrong right away because the $s + \bar{s}$ term must be positive (an inequality) and probably a fairly small fraction (e.g. less than 20%) of the term preceding it.[5]

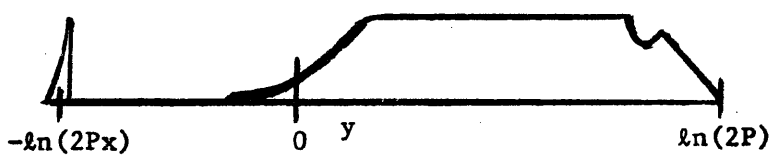
For small x all four functions in (4) should become equal to a/x, so f_3 should not go as 1/x for small x.

For $x \to 1$ where we expect only u(x) to survive, $f_-^{\bar{\nu} p}$ is the largest, and should ultimately become equal to (9/4) $f^{ep}(x)$.

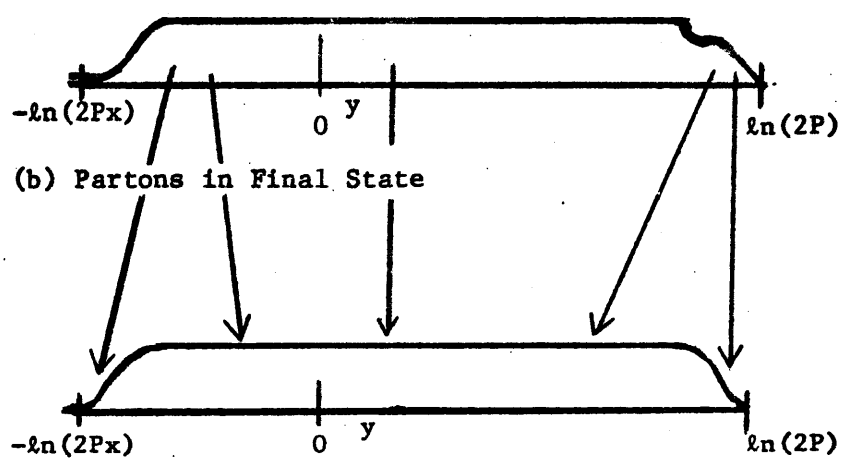
For scattering from neutrons, for example $f_-^{\bar{\nu} n}$ should give scattering from the up quarks in the neutron, equal in number to the down quarks of the proton or $f_-^{\nu p}$. We find in general antineutrino on neutron is neutrino on proton and vice versa. This should work only if $\sin^2\theta_c = 0$. If data ever becomes accurate enough, neutron data can be used along with proton to eliminate some $\sin^2\theta_c$ uncertain terms in other tests. One experiment, neutrino on deuterium, would directly measure $f_1^{\nu p} + f_1^{\bar{\nu} p}$ needed to test inequality (7) where it is compared to electron on deuterium.[6]

FINAL STATE HADRONS FROM DEEP INELASTIC COLLISIONS

We now come to discuss two arm experiments on $\nu + p \rightarrow \mu^- +$ one hadron + other hadrons in which some attempt is made to study the hadron products of the collision. To do this we shall have to make further theoretical suggestions. We shall suppose that immediately after a collision at a given x the partons in rapidity space appear as in Figure 3a (corresponding to Figure 1b).



(a) Partons Immediately After Interaction



(c) Hadrons in Final State

FIGURE 3

This is like our original distribution of Figure 2 with an additional parton at $p_z = -Px$ and one missing at Px. Now this is an initial state and, with time, the Hamiltonian operator forms a cascade of partons

producing a distribution like Figure 3b. There is a long plateau reaching from -2Px to 0. Whether this really fits smoothly at 0 to the previous pleateau or there is a jog in level there I do not now know. It will not effect what I am going to say, for I will again suppose that the plateau (to the left of zero at least) is universal and does not depend on the character of the parton which produced it. This final state is then realized in nature as a set of outgoing hadron particles with finite transverse momenta $p_\perp$ and with rapidities ($y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$ with $E^2 = p_z^2 + p_\perp^2 + M^2$ where M is the hadron mass) spread over the former range. But the character of each hadron will be determined, I shall suppose, by the partons within a finite range Δy of its own rapidity. (This may not be physically directly true, but things farther away on the plateau are universal, so product hadrons in two experiments will differ only if nearby partons are expected to be different in the two experiments.) It is seen that the thrust of these ideas is that, back in the coordinate system of Figure 1, for a fixed x as P increases, hadrons moving with momentum -Pxz to the left will depend only on z and on what parton is going to the left in Figure 1b; whereas those going to the right at momentum $P\eta$ depend on η, the fact it was a proton collision, what parton was taken out, and, of course, x. Particles with finite p_z in this system come out with a distribution which is universal, and independent of all these variables. We should remark at once that this may only apply in detail at very high energy indeed - scaling for the total one arm experiment may set in much before the products scale precisely - if experience with corresponding non-relativistic models is any guide. But at least the appearance of jets with finite transverse momenta should become clear early.

We shall study here, primarily the products to the left - the fragmentation products of the parton, of the quark. Neutrino experiments permit us to choose an especially simple case for consideration - the case where the left moving quark is purely of one kind. For example consider the products expected from a pure W_-, a W meson of negative helicity, which plucks a d quark from the proton and kicks it out to the left at momentum - Px as a pure up quark. The probability it does anything is d(x) but having done it the probability that a particular hadron type i has longitudinal momentum - Pxz, (i.e. a fraction z of the quark momentum) is a function of z only, appropriate to up quarks, $D_u^i(z)$. If p_i is the four momentum of the product hadron i, p is that of the proton then $z = (p \cdot p_i)/(p \cdot q)$ simply the laboratory energy of the product divided by ν, the energy loss of the lepton. The probability we get such a product using a W_- meson with $q^2 = -2M\nu x$ is $D_u^i(z)d(x)$ This could be tested by seeing if the result was indeed a product - that the z distribution aside from normalization is indeed independent of x.

Theoretically we are led to define a set of distribution functions $D_\alpha^i(z)$ which we could call parton fragmentation functions; the probability that a product i is found to the left with z (in dz) if it is known that a parton α goes to the left. α has six values u, d, $\bar{u}$, $\bar{d}$, s, $\bar{s}$. For four of these, neutrino experiments could, in principle, determine $D_\alpha^i(z)$.

If this is all true we see that we would be getting near to measuring fundamental properties of the hadron system - a limited number of distribution functions having to do with kinds of partons (which in our example are quarks). There are many obvious ways to test these ideas and I shall not attempt to choose among them to find the most easily analyzed for the experiments to be done soonest. Instead I will just give a number of theoretical examples.

For small z the universal plateau idea suggests that $D_\alpha^i(z)$ varies inversly with z like C_i/z with a constant that depends on the product hadron i but not on α.

Relations can be derived from I-spin symmetry like $D_u^{\pi^+} = D_d^{\pi^-}$, or charge conjugation like $D_u^{\pi^+} = D_{\bar{u}}^{\pi^-}$, thus there are only three independent distribution functions for pions:

$$D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{d}}^{\pi^+} = D_{\bar{u}}^{\pi^-}$$

$$D_{\bar{u}}^{\pi^+} = D_{\bar{d}}^{\pi^-} = D_d^{\pi^+} = D_u^{\pi^-}$$

$$D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-} \tag{8}$$

Neutrino experiments have the theoretical advantage that the functions can be separated for pure quarks and hence as a product of a function of z and of x. But if less is measured, for example only the products in νp corresponding to $f_1^{\nu p}$ we have for the number of left-going hadrons i of momentum z in an experiment done at fixed x,

$$N^i(z,x) = 2\, D_{\bar{d}}^i(z)\bar{u}(x) + 2\, D_u^i(z)d(x)$$

normalized to $f_1^{\nu p}(x)$. This is a little more complicated to test for it is in general a combination of two functions depending on z. If certain special questions are asked it will factor again. For example if we ask for the number of π^+ (so $i = \pi^+$) by (8) the functions are equal and we have

$$N^{\pi^+}(z,x) = D_u^{\pi^+}(z)\, f_1^{\nu p}(x)$$

a factorization which should be easy to test, and to determine $D_u^{\pi^+}$. In the same way from $N^{\pi^-}(z,x)$ we can get $D_u^{\pi^-}(z)$.

For electron proton scattering the expected number of hadron i is a considerable tangle in general,

$$N^i(z,x) = \frac{4}{9}\left(u(x)\ D_u^i(z) + \bar{u}(x)\ D_{\bar{u}}^i(z)\right) + \frac{1}{9}\left(d(x)\ D_d^i(z) + \bar{d}(x)\ D_{\bar{d}}^i(z)\right) +$$

$$+ \frac{1}{9}\left(s(x)\ D_s^i(z) + \bar{s}(x)\ D_{\bar{s}}^i(z)\right) ,$$

but even here we may do a lot to simplify it. For example if we measure the excess of π^+ over π^-, we find again a simple product in virtue of (8)

$$N^{\pi^+}(z,x) - N^{\pi^-}(z,x) = \left[D_u^{\pi^+}(z) - D_u^{\pi^-}(z)\right]\left[\frac{4}{9}\left(u(x) - \bar{u}(x)\right) - \frac{1}{9}\left(d(x) - \bar{d}(x)\right)\right]$$

The last x dependent factor has some obvious sum rule properties in view of (1); its integral over x from 0 to 1 should be 7/9. As another example it turns out that, if we measure the number of charged k's minus the number of neutral k's, it should be a function of z times the function of $x, \frac{4}{9}(u + \bar{u}) - \frac{1}{9}(d + \bar{d})$. Thus measurements on final hadrons in electron scattering could also help in isolating the functions u(x) etc.

We have noted that a proton, when nearly a pure quark (x near 1), is a u quark. This leads us to guess that of all quarks which produce protons near $z \to 1$, u quarks do it most easily. Thus whereas all $D_\alpha^p(z)$ probably fall as a power of (1-z) as $z \to 1$ that power is probably least for $\alpha = u$. Arturo Cisneros (private communication) has suggested by analogy that for a pion or kaon near $z \to 1$, $D_\alpha^i(z)$ is largest for the quark and antiquark that make them up according to the low energy quark model. Thus $D_u^{\pi^+}(z) >> D_{\bar{u}}^{\pi^+}(z)$ as $z \to 1$ (although they both go to zero). If this is true there are still further ways of finding the u(x) etc., from electron experiments, because by going to z near 1 fewer $D_\alpha^i(z)$ functions are involved.

PARTON QUANTUM NUMBERS

There is one point of considerable theoretical interest that should be made. Suppose Q is some additive conserved quantum number like charge, or 3-component isospin or baryon number and Q_i is that number for a particular hadron species i that appears to the left in relative mean number $N_i(z)$. The total $\int_0^1 \sum_i Q_i N_i(z)dz$ we shall call the mean total quantum number Q for all the left-moving particles. We refer not to the value for one particular event, of course, but to the statistical average over events. Near the lower limit z = 0 various $N_i(z)$ are going to infinity as 1/z but in a neutral manner, as many of positive charge as of negative charge, for example, for the y plateau is neutral - so the integral converges, and we do not have to specify precisely where in the y plateau we cut the integral off (near z = 0) in defining the distinction of left and right momenta. Suppose for example we know the reaction is via W_- so we know we have a u quark initially to the left. Then the parton cascading and the eventual conversion to hadrons cannot change the total quantum number provided we have a sufficiently long plateau. For then all kinds of hadrons, strange and baryon, had a reasonable chance to be formed, so that the plateau is fully neutral to all quantum numbers. Under these circumstances the left mean quantum numbers $\int \sum_i Q_i D_u^i(z)dz$ must be those of the quark Q_u, or in general

$$Q_\alpha = \int_0^1 \sum_i Q_i D_\alpha^i(z)dz \tag{9}$$

Thus, in principle, quantum numbers of the partons can be defined directly in terms of experimental quantities. The charge of a parton thus defined need not be integral, like the hadrons, for it is statistically defined. Even were it to turn out that a field theory with parton quarks does not

exist, it might still come out that experiments to determine the sums in (9) from neutrino scattering to select "pure quark" states could give the characteristic third-integral quantum numbers.

However, how high in energy would we have to go to verify this? Probably pions are easier to make than kaons as products, and these are still easier than baryons. Therefore I think the required mean will be approximated most rapidly (lower Px will be sufficient) for 3-isospin, less rapidly for hyperon charge and least easily for baryon number (+ 1/3 for a quark, - 1/3 for antiquark). Therefore charge (which is 3-isospin plus 1/2 hypercharge) will only work when we have enough energy to balance hypercharge. Strangeness requires baryon number be averaged by the plateau.

The easiest to check is isospin, but that is less interesting as the characteristic one-thirds do not yet come in. The easiest place is $f_1^{\nu p}$ neutrino-proton scattering. (There is no need to isolate $f_+^{\nu p}$ from $f_-^{\nu p}$ for they both produce quarks of the same isospin, + 1/2; $\bar{d}$ and u respectively.) We measure; (Number of π^+ - Number of π^-) + 1/2 (Number of k^+ - Number of k^-) - 1/2 (Number of k^o - Number of $\bar{k}^o$) + etc., integrated over all positive z greater than some small number. The total should be + 1/2. Probably the first term alone gives the bulk of the sum.

It goes without saying that the mean quantum number of the left and the rights together is just that expected from conservation of total quantum number because that is so even for each event alone. For example $\sin^2 \theta_c$ could be measured, in principle, by a W_+ experiment, by the mean total strangeness per collision of all the products right and left at high energy.

We have said little about the products to the right; those moving in the system of Figure 1 to the right with momentum + $P\xi$ for fixed ξ as P goes to ∞.

They are more complicated and less fundamental than those going to the left,, as they depend on all the quarks in the proton less one at x. In the special case x is very small, we can say something. Only the low x partons are disturbed, those at larger x are distributed just as in the proton in hadron-proton collisions where only the wee partons interact. Thus for sufficiently small x and for ξ not too small (not as small as x) the right fragments are unique, independent of x. In fact for small x(and also for $-q^2$ finite, $\nu \to \infty$) the proton fragments in a unique way, the same way as it does in a hadron-proton collision at very high energy. Further, under these circumstances, the lepton also fragments in a characteristic way independent of what hadron was hit, proton or whatever. This is because we assume the wee and small x region is the same for all initial hadrons. (Further details on all these matters may be found in reference 4.)

In our exercise here we have assumed many things, some of which may be inconsistent (e.g. partons as quarks interacting only for low relative momentum yet unable to come apart into real quark states). Even the basis of partons may be quite incorrect. But what the example shows clearly, nevertheless, is that deep inelastic lepton scattering has already told us much that is fundamental about the strong interactions and shows every promise of yielding much more information of an equally basic kind.

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2. J.Kuti, this conference.
3. J.Bjorken, 1971 International Symposium on Electron and Photon Interactions, Cornell University.
4. R.P.Feyman, "Photon Hadron Interactions", W.A.Benjamin Company, (1972), (to be published).
5. For further inequalities see O. Nachtmann CERN Report LPTHE 71/29 (1971).
6. This can be applied to the total cross section of ν and $\bar{\nu}$ on nuclei, for which preliminary results from 2 to 7 GeV have been presented at this meeting.

Let us measure all cross sections in unites of $Gs/2\pi$ where G is the Fermi constant and s the square of the center of mass energy. For nucleons then, our unit is GME/π where E is the laboratory energy.

The total cross section of a neutrino with a spin 1/2 particle is 2. With an antiparticle it is 2/3. Hence on a proton the cross section is

$$\sigma^{\nu p} = 2 \int_0^1 x(d + \frac{1}{3}\bar{u})dx$$

the factor x coming because the cross section varies with s. For neutrons we replace d by u, etc, so the mean neutrino cross section of a nucleon is

$$\sigma = \frac{1}{2}(\sigma^{\nu p} + \sigma^{\nu n}) = \int_0^1 x(d + u + \frac{1}{3}(\bar{d} + \bar{u}))dx$$

The antineutrino cross section is

$$\bar{\sigma} = \frac{1}{2}(\sigma^{\bar{\nu}p} + \sigma^{\bar{\nu}n}) = \int_0^1 x(\bar{d} + \bar{u} + \frac{1}{3}(d+u))dx$$

Since $\bar{d}$, $\bar{u}$ are positive, but undoubtedly less than d,u, we see that $\bar{\sigma}/\sigma$ must be substantially less than 1, but greater than 1/3.

The sum is

$$\sigma + \bar{\sigma} = \frac{4}{3}\int_0^1 x(u + \bar{u} + d + \bar{d})dx$$

However integrating (7) we have

$$\int_0^1 x(f^{ep} + f^{en})dx = \frac{5}{9}\int_0^1 x(u + \bar{u} + d + \bar{d})dx + \frac{2}{9}\int_0^1 x(s + \bar{s})dx$$

Experimentally this integral is 0.31, so if we could forget the integral $\int x(s+\bar{s})dx$ we would have $\sigma+\bar{\sigma} = \frac{4}{3}\cdot\frac{9}{5}(.31) = 0.74$. But $s+\bar{s}$ must surely be less than $d+\bar{d}$ and $u+\bar{u}$ and, when weighted by x, surely much less. It would be hard to manage to make inclusion of the last term produce more than a 10% effect. Thus we have a very stringent test of our parton quark model: $\sigma+\bar{\sigma}$ cannot exceed 0.74 and yet almost surely cannot fall below 0.74 by more than 10%. One can also calculate upper limits for $\sigma^{\nu p} + \sigma^{\bar{\nu}p}$ and $\sigma^{\nu n} + \sigma^{\bar{\nu}n}$ separately (using other proportions of f^{ep} and f^{en}) they are .64 and .84 respectively.

These numerical estimates must be revised by a few percent for we have neglected $\sin^2\theta_c$. They are valid only at asymptotic energy, of course, but T.D.Lee has pointed out that electron data indicate that this should only require a few GeV. These results are those of Bjorken.

DISCUSSION

Moffat: Why have not quarks and partons been produced at SLAC and ISR?

Feynman. I am so used to the idea that quarks are not produced that I forgot to mention that there is a paradox: is it possible that quarks only have interactions for finite relative momenta and yet they do not get isolated, they cannot get separated? I do not understand that at all, and I am happy with that. I like paradoxes. So what I am trying to do is this: I deduce everything I can from a quark model, except that they should come apart. And in the struggle to be consistent, to have quarks inside which do not come apart, I have to figure all this out and I must say that we might be headed for a paradox. One of the two things can happen: either, one, we find that all this quark stuff, these quark quantum numbers, do not work and then that is very easy: there are no quarks, so they do not come apart; or, two,-mystery of mysteries!- All these predictions / 0.74 for average total cross-section of neutrinos and antineutrinos on nucleons, and so on/ all work, and yet the quarks do not come apart! That will be interesting. I was very interested to notice that if the quarks come apart into a kind of comet tail of hadrons, into a plateau in rapidity space, they can still disintegrate consistently that way, even tho they have non-integral quantum numbers. It is still possible that we do not have any inconsistency. So I am going to assume as long as I can, a paradoxical combination of things, that quarks cannot come apart as free and they are inside of the particles. I do not know how. But that is the fun.

Bell: Part of the trouble seems to come just from the idea that the interactions are restricted over a small interval in the x pace. Is that an essential part of the model, could you say a little where this particular idea comes from?

Feynman: It is essential to the totality of what I said. But it is not essential to everything. Various things that I said do not require that particular assumption, other things definitely do. I got that idea from the fact that transverse momenta were limited. That may be illegal and irrational. I do not know how well based this idea is. The ideas of limiting fragmentation due to Yang et al,/that the fragments going to one side and the other in hadron collisions depend only on the objects that were going in that direction/ if they continue to work, seem to me also to imply that interactions only occur

over a finite range of y space /not x space/ which means relative momentum is finite. Well, I make this assumption I do not know why, and how its based.

Weisskopf: What you said I understand only as an assumption that there is just not much interaction between these particles of relatively high momentum, but not that the interaction is only between particles of almost equal y .

Feynman: You are absolutely right. The only thing I really need is that in the strong interaction the interaction of things with high relative momenta is small. And the only space left for me to put the interaction was at finite relative y.

Kögerler: Is there any theoretical reason that the final state interaction between the partons preserves scaling?

Feynman: If the final state interaction is over a limited range of relative momentum then we come to scaling.

Kögerler: This is an assumption?

Feynman: Yes, I use this all the time. I do not know exactly what I need to get to each place. The Babilonians did geometry by knowing a whole lot of theorems and when they forgot one theorem they proved it by those that they remembered, but did not organize in a way that they started at something called assumptions and everything else was deduced from that. And I got myself into a terrible tangle as I have all these facts. And when I lose one, I remember from an other one. But I cannot remember where I started any more. I start in the middle, I do not have a logical way of doing it. I think it scales; I think the interaction is over a finite range; I think the transverse momenta are finite; I suspect there is a relation in the logic, that all these are not independent assumptions. But I do not know....

Marshak: Do you think that the reason we do not see quarks is they posses very large masses and interact strongly with each other in peculiar ways?

Feynman: This is a completely different direction, but not the direction I am going. If the quark masses are high, then you have strong interactions between them, strong interaction makes the scaling hopeless to understand, transverse momenta become large, the whole thing goes haywire, and the picture is very bad. My quarks have small masses, and they do not came apart because of something I'll tell you about 25 years from now. The masses that I want are so low that we would have absolutely definitely seen them. Thus perhaps the whole thing is nonsense, and that experiment will tell us very soon. Or if it is right, then this is very very exciting, because we are approaching a paradox, and the hope of physics is to find a paradox. This is the real way of making a revolution. We have to find a place where we are shocked. And I think we are getting near to one. I hope we are....

Weisskopf: If somebody detects a quark with a high mass - God forbid.....

Feynman: Right! God forbid!

Weisskopf: Still, we would not be lost, because it may be that the quark has an effective mass inside which is small /with a scalar interaction or something like that/ so it need not be in contradiction with all that you say.

Feynman: I would try,however to answer as you said, by "God forbid": a heavy quark, a real heavy quark is as embarrassing and difficult to understand as no quarks at all. It would not help me much except to confirm the reality of quark quantum numbers, a thing which I believe we can confirm without finding any quarks.

Achiman: In one of your guesses you said that neutral quarks probably exist also. Where did they come from, where can you put these neutral quarks in your theory?

Feynman: I did not say neutral quark, I said neutral parton, some particle other than a quark which is neutral. I have not found any way, by electron scattering or neutrino scattering, to tell us anything more about these neutral partons except their existence, induced by the fact that the conservation of momentum does not work with the charged quarks.

Marshak: I am suggesting a more specific model in analogy to the strong cubic W boson model where we introduce a new quantum number like cubic parity in the strong interaction among the triplet of W's. Let us translate this idea to a triplet of partons. Then you do not have a strong interaction between two partons, but one between the parton and the antiparton to give the pion and also one among the three partons to give the proton. Perhaps a new selection principle can reconcile scaling and large masses for the partons?

Feynman: Your are quite right, I have tried to do things vaguely like that. But at the present time I would rather like to have some kind of information showing quark quantum numbers are real before I have the energy to move on to such definite questions.

Budini: In view of the recent theory of Salam and Weinberg and so on to unify weak and electromagnetic interaction can you tell us what this would do?

Feynman: I believe, if I have "not-understood" Weinberg's theory correctly, this theory does not change the lepton-hadron interaction at all. All I have used here is the lepton-hadron interaction.

Somebody from the audience: What would be the effect of introducing partons with quantum numbers different from those of the normal quarks?

Feynman: Another system of partons with other quantum numbers /such as the triplet quark model and other models/ definitely have a big effect. It changes many of the numerical coefficients and in a paper by Nacthman /CERN Report LPTHE 71/29 /1971// for example, it is shown that already the crude total neutrino-nucleon cross section, if it remain near the present value, is almost ready to eliminate most of the alternatives. And new more accurate measurements of neutrino and antineutrino-nucleon cross section will clarify the situation concerning quark quantum numbers very soon.