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Do we live in a wormhole?

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Abstract

It is shown that all important closed Robertson-Walker-Friedmann universe models have an analytic continuation into the Euclidean region with wormhole-type topology.

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1. Classical wormholes usually are Euclidean metrices which consist of two large regions of spacetime connected by a throat. They are instantons, solutions of the Euclidean Einstein equations and can be obtained also as an analytic continuation of the Lorentzian solutions to the imaginary time: $t \to -it$. They play important role in quantum gravity [1-2] and should effect on physical constants, particularly the cosmological constant (see e.g. numerous references concerning this problem in [3]). In connection with these problems the microscopic wormholes which have Planck size throat were investigated mainly [1]. But the global wormholes with the size of the throat equals to the maximum size of the universe can exist also and they should define the global topology of the Euclidean sector of our universe. In our paper we shall show that closed Friedmann universes have wormhole type Euclidean continuation for most important cases and we think all closed models of universe should have this property.

2. All known now exact classical wormhole solutions are reduced usually to the three main cases:

a). Axionic instantons.
The scale factor in this case has form

$$a = a_1 \cosh^{1/2} 2\eta, \tag{1}$$

where η is Euclidean conformal "time" and Robertson-Walker-Friedmann (RWF) metric is taken (usually if it is not stressed separately we consider the 4-dimensional spacetime manifold). This Euclidean Einstein equations solution was obtained for the first time in [4] in the case of a third-rank antisymmetric tensor (axion) as a matter source. That is why it has the name of axionic instanton. Generalization of this model was performed in [5,6].

Recently it was shown [7-9] that instanton with exactly the same geometrical structure take place for the RWF universe filled by massless scalar field minimally coupled to the gravity. In [8] this model is generalized to the case of multidimensional universe and the scale factor corresponding to this instanton reads

$$a = a_1^{\frac{2}{d-2}} [\cosh(d-2)\eta]^{\frac{1}{d-2}}, \tag{1.a}$$

where $d \geq 3$ is the dimension of the Euclidean region and the formula (1.a) is reduced to (1) for $d = 4$.

b.) Conformal scalar feild instantons.
In this case the scale factor is

$$a = a_2 \cosh \eta. \tag{2}$$

This type of wormholes was described first in [10] and in [11] was shown that this wormhole corresponds to the solution of the Euclidean Einstein equations with a conformally coupled scalar field. The same type of wormholes was obtained in [12] with the help of cutoff in the scale factor. It gives the possibility to avoid the negative values of the effective gravitational constant problem in the model with conformal scalar field.

It is easy to see that the scale factor in (1.a) for $d = 3$ has the same behaviour as in (2) although the result (2) was obtained in the case of 4-dimensional ($d = 4$) RWF spacetime.

c.) Minimum resolution distance instantons.
For this model the wormhole solution is

$$a = a_3(1 + \cosh \eta) \tag{3}$$

and this result was obtained in [13] due to an extra quantum term arising from

the insertion of a minimum resolution distance in the background theory. As a background theory the RWF universe was taken.

We should stress here again that all these instantons were considered as microscopic ones mainly [1-5,12,13] and the throat sizes $a_i (i = 1, 2, 3)$ in formulas (1)- (3) have Planck scales: $a_i \sim L_{Pl}$. They can be obtained also as an analytic continuation of the corresponding Lorentzian solutions for closed RWF metrics. The wormhole type solutions take place if the models possess the conserved charge which is not changed during the transition from classically allowed Lorentzian region of space-time to the classically forbidden Euclidean region of space [3,8,11]. In the case of minimally coupled massless scalar field (case a.) the momentum p_φ conjugated to the scalar field φ is constant and this quantity is regarded as conserved charge [3,8]. The square of p_φ is proportional to the scalar field energy and play the role of conserved parameter in the Einstein equations [8]. The constancy of momentum leads to the imaginary values of scalar field in the Euclidean region [3,8,9]. In the case of the conformal scalar field (case b.) there is analogous of the conserved quantity also. It is the energy of the conformally transformed scalar field [11] and this quantity is not changed during transition from Lorentzian region to Euclidean.

3. We want to show now that for the most well-known models of the RWF universes there are wormhole type continuation into the Euclidean region and all these solutions usually have the form of (1)-(3).

For the closed RWF metric the master equation which define the time-dependence of the scale factor is [14]

$$\frac{1}{a^4}\left(\frac{da}{d\eta}\right)^2 = -\frac{1}{a^2} + \frac{8\pi}{3}\rho = -\frac{1}{a^2} + \frac{\epsilon_u}{a^6} + \frac{\epsilon_r}{a^4} + \frac{\epsilon_m}{a^3} \tag{4}$$

where η is Lorentzian conformal time. The first term in (4) has its origin

in the curvature of the 3-dimensional hypersphere and the three other terms define the mass-energy density (all numerical constants are included into ϵ and each of ϵ is up to numerical factors conserved energy of the corresponding type of the matter). The first of them is the energy density of the ultra-stiff matter with the equation of the state [15] $P = \rho$. The second one represents the energy-density of the radiation with the equation of state $P = \frac{1}{3}\rho$. And the third term is the energy-mass density of the dust with the equation of state $P = 0$. Let us consider all three cases separately.

a.) Ultra-stiff matter filled universe.
The solution of the (4) in this case is

$$a = \epsilon_u^{\frac{1}{4}}(\cos 2\eta)^{\frac{1}{2}}, \qquad |\eta| \leq \frac{\pi}{4}. \tag{5}$$

We should make an important remark here. The point is we are looking for the Lorentzian solutions which should have wormhole type continuation into Euclidean region. The wormholes usually are symmetric with respect to the throat. Thus our Lorentzian solutions should be also symmetric in respect of the origin of the time axis. This condition defines the constant of the integration of master equation.

After analytic continuation into the Euclidean region $\eta \to -i\eta$ the expression (5) takes the form of the axionic instanton (1). It is easy to show that this solution describes the two asymptotically flat Euclidean regions connected with each other by the throat (handle). If we use the transformation [9] from conformal "time" η to new "time" $\tau : ad\eta = [1/(2a)]d\tau$ then

$$a = (\epsilon_u + \tau^2)^{\frac{1}{4}}, \qquad -\infty < \tau < +\infty \tag{6}$$

and asymptotic metric for $|\tau| >> (\epsilon_u)^{\frac{1}{2}}$ is

$$ds^2 = \frac{1}{4|\tau|}(d\tau)^2 + |\tau|[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)]. \tag{7}$$

After transformation $R = |\tau|^{\frac{1}{2}}$ we get

$$ds^2 = (dR)^2 + R^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2]. \tag{8}$$

This metric describes the 4-dimensional flat Euclidean space. The size of the throat is $r_0 = (\epsilon_u)^{\frac{1}{4}}$ which coincides, as it follows from (5), with maximum size of the Lorentzian region of the universe.

b.) Radiation filled universe.
The symmetric (with respect to the time axis origin) solution of the (4) is

$$a = \epsilon_r^{\frac{1}{2}}\cos\eta, \qquad |\eta| \leq \frac{\pi}{2} \tag{9}$$

Analytic continuation into the Euclidean region gives the wormhole with the conformal scalar field instanton geometry (2). In the synchronous Euclidean "time" $t : ad\eta = dt$ this wormhole can be rewritten in the form

$$a = (\epsilon_r + t^2)^{\frac{1}{2}}, \qquad -\infty < t < +\infty \tag{10}$$

and for the large "times" $|t| >> (\epsilon_r)^{\frac{1}{2}}$ we have two 4-dimensional flat Euclidean regions with the asymptotic metric (8). The size of the throat here is $r_0 = (\epsilon_r)^{\frac{1}{2}}$ which again coincides with the maximum size of the Lorentzian region of the universe described by (9).

c.) Dust filled universe.
The solution of the (4) which is symmetric in respect of the time axis origin has the form

$$a = \frac{1}{2}\epsilon_m(1 + \cos\eta), \qquad |\eta| \leq \pi \tag{11}$$

and the analytic continuation into the Euclidean region gives the wormhole with the minimum resolution instanton geometry (3). It is easy to see that

for the "times" $\eta >> 1$ this wormhole takes the form (10) in synchronous "time"

$$a = \left(\frac{1}{4}\epsilon_m^2 + t^2\right)^{\frac{1}{2}} \approx |t| \tag{12}$$

and there are again two flat Euclidean regions which are connected with each other by the troat of the size $r_0 = \epsilon_m$.

It is not difficult to check that all these wormholes (6), (10) and (12) can be obtained directly as solutions of the Euclidean version of the master equation (4) and the energies ϵ play the role of the conserved parameters (charges).

4. With the example of the most important cases of the closed RWF universe we have shown that all of them have the analytic continuation into the Euclidean region and this continuation has the wormhole geometry. We think it is a general property of all RWF closed universes. It is interesting that our universe, if it is closed, has a continuation into the infinite-size region — the region where time is absent — and this region has a non-trivial topology. Of course, this sector of our universe is classically forbidden.

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References

[1] S.W. Hawking, Mod. Phys. Lett. **A5** (1990) 145.

[2] S.W. Hawking, Mod. Phys. Lett., **A5** (1990) 453.

[3] S.W. Hawking and Don N. Page, Phys. Rev., **D42** (1990) 2655.

[4] S. Giddings and A. Strominger, Nucl. Phys. **B306** (1988) 890.

[5] R.C. Myers, Phys. Rev., **D38** (1988) 1327.

[6] P.F. Gonzàles-Diaz, Phys. Lett. **B233** (1989) 85.

[7] Don N. Page, in Proceedings of the FIFTH SEMINAR ON QUANTUM GRAVITY, Moscow 1990, eds. M.A. Markov, V.A. Berezin, V.P. Frolov (World Scientific, Singapore 1991) p. 82.

[8] A. Zhuk, Phys. Rev. **D45** (1992), 1192; Sov. Journ. Nucl. Phys. **55** (1992) 149.

[9] S. Wada, Mod. Phys. Lett. **A7** (1992) 371.

[10] S.W. Hawking, Phys. Rev. **D37** (1988) 904.

[11] J.J. Halliwell and R. Laflamme, Class. Quant. Grav. **6** (1989) 1839.

[12] P.F. Gonzàles-Diaz, Phys. Rev. **D40** (1989) 4184.

[13] P.F. Gonzàles-Diaz, Nuov. Cim. **B106** (1991) 335.

[14] C.W. Misner, K.J. Thorne and J.A. Wheeler, *Gravitation*, (Freeman, San Francisco 1974).

[15] Ya.B. Zel'dovich and I.D. Novikov, *Relativistic Astrophysics, V.2: The Structures and Evolution of the Universe* (Chicago and London: The University of Chicago Press, 1983).