# B MESON DECAY ASYMMETRY AND B-$\bar{B}$ MIXING*†

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## I. INTRODUCTION

The discovery of $B^0$-$\bar{B}^0$ mixing by the ARGUS group[1] has brought the search for CP violation in the B system from a mere theoretical dream to an experimental challenge. While observation of any CP violation in the B system is exciting, there are several channels for which CP asymmetry is predicted unambiguously within the three family Kobayashi-Maskawa model[2]. Here we shall concentrate on these predictions and give a geometric interpretation of our results[3]. The importance of a triangle in the complex plane, formed by the smallest KM elements, will be stressed[4]. Specific decay asymmetries, b quark lifetimes, $B^0$-$\bar{B}^0$ mixings, and charmless B decays probe different angles or sides of this triangle, allowing a test of the three generation KM model.

## II. ASYMMETRIES

Since there is substantial $B^0$-$\bar{B}^0$ mixing, one can consider two decay chains:

$$\begin{array}{l} B^0 \to B^0 \searrow \\ \qquad\qquad\quad f \\ B^0 \to \bar{B}^0 \nearrow \end{array}$$

where f is a CP eigenstate. The amplitudes for these decay chains can interfere and generate nonzero asymmetries between $\Gamma(B^0(t) \to f)$ and $\Gamma(\bar{B}^0(t) \to f)$. In order to discuss this asymmetry we follow the notation of Ref.3. The time evolution of a meson that was produced as a $B^0(\bar{b}d)$ or $\bar{B}^0(b\bar{d})$ meson at time t=0 is given by

$$\begin{aligned} |B^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{q}{p}\, g_-(t)|\bar{B}^0\rangle \\ |\bar{B}^0(t)\rangle &= \frac{p}{q}\, g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle \end{aligned} \tag{1}$$

where

$$g_\pm(t) = \exp(-\frac{1}{2}\Gamma_1 t)\, \exp(im_1 t)(1 \pm \exp(-\frac{1}{2}\Delta\Gamma t)\exp(i\Delta m t))/2$$

and we have made the definitions:

$$\Delta\Gamma = \Gamma_2 - \Gamma_1;\ \Delta m = m_2 - m_1;\ \frac{q}{p} = \frac{1-\epsilon_B}{1+\epsilon_B} \tag{2}$$

$\Gamma_i$ and $m_i$, i=1,2 are the width and mass, respectively, of the two mass eigenstates $B_i$.

It is commonly accepted (readers who are interested in further details are referred to Ref.2) that $\Delta\Gamma \ll \Gamma$, $|\frac{q}{p}| \approx 1$. Taking these into account, the width for $\Gamma(B^0(t) \to f)$ and $\Gamma(\bar{B}^0(t) \to f)$ can be written as

$$\Gamma(\bar{B}^0(t) \text{ or } B^0(t) \to f) \sim e^{-\Gamma t}\{1 \pm \sin[\Delta m t]\mathrm{Im}(\frac{p}{q}\rho)\} \tag{3}$$

where $\rho = A(B \to f)/A(\bar{B} \to f)$.

The study of CP violation in these modes requires information on the identity of the B, i.e., whether it is a $B^0$ or $\bar{B}^0$ at t=0. Since b and $\bar{b}$ quarks are pair produced, such information can be obtained by tagging the other particle as to its b or $\bar{b}$ content. The observable asymmetry in the case where a $B^+$ or $B^-$ is used as the "tag" is

$$\frac{\Gamma(\bar{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)} = \sin[\Delta m t]\mathrm{Im}(\frac{p}{q}\rho) \tag{4}$$

If the "tag" is also a neutral B which can oscillate, the situation is slightly more complicated and oscillation of both $B^0$ and $\bar{B}^0$ must be taken into account. With a common final state f and a semileptonic tag of the associated neutral B, the decays of a $B\bar{B}$ pair in a coherent state of given charge conjugation are

$$BR(B(t)\bar{B}(\bar{t})|_{C=\mp 1} \to f + (D\ell\bar{\nu}X)_{tag})$$

$$\propto e^{-\Gamma(t+\bar{t})}\{1 - \sin[\Delta m(t \mp \bar{t})]\mathrm{Im}(\frac{p}{q}\rho)\} \tag{5a}$$

$$BR(\bar{B}(t)B(t)|_{C=\mp 1} \to f + (\bar{D}\bar{\ell}\nu\bar{X})_{tag})$$

$$\propto e^{-\Gamma(t+\bar{t})}\{1 + \sin[\Delta m(t \mp \bar{t})]\mathrm{Im}(\frac{p}{q}\rho)\} \tag{5b}$$

Note that for C = -1, i.e., $B\bar{B}$ in an odd relative angular momentum state, the potential asymmetry vanishes if the times t and $\bar{t}$ are treated symmetrically.

Several comments about Eq.(4) are in order:

(a) If we write

$$\sin[\Delta m t] = \sin[x_d \tau] \tag{6}$$

where $x_d = \Delta m/\Gamma$ and $\tau = \Gamma t$, we see that for $x_d \sim .7$ the asymmetry has a definite sign for $\tau \lesssim 4$. Thus there is very little chance for the asymmetries for $B_d$ meson decays to "wash away". This is in contrast with $B_s$ meson asymmetries where we expect $x_s$ to be at least as large as $5x_d$.

(b) We have already mentioned that $|\frac{p}{q}| \sim 1$. Since we can show that $|\rho| = 1$ for final states with single isospin channels. We have

$$\frac{p}{q}\,\rho_f = e^{i\phi} \tag{7}$$

(c) Eq.(7) implies a possibility of large asymmetry as there is no obvious dynamical suppression factor in the right-hand side of eq.(4).

(d) In order for the asymmetry to be measurable, the branching ratio for $B \to f$ decay channel has to be substantial.

The main candidates for f, those that are CP eigenstates, are $f = \psi K_S$, $\chi_0 K^*$, $\pi^+\pi^-$, $D^+D^-$, $D^{0*}\bar{D}^0$,+cc and those that have small admixture of both CP eigenstates are $\psi K^*$, $\psi\pi\pi$, $\chi_1 K^*$, $\chi_2 K^*$, $D\bar{D}K_S$. For example, $D\bar{D}K_S$ may be dominated by a channel in which all three particles are in a relative S wave state, so that only one CP eigenstate dominates the decay. It should be noted that the branching ratios for $\psi K_S$, $\chi K^*$ channels do not have any suppression from the KM matrix elements.

(e) For those decay channels in which one CP eigenstate dominates, the RHS of Eq.(4) is totally predictable once the KM matrix element is known. This is in sharp contrast with the standard model prediction for $\epsilon$ and $\epsilon'$ where the predictions depend crucially on unknown hadronic matrix elements.

## III. MAGNITUDE OF ASYMMETRIES AND THE TRIANGLE

We shall now update the prediction of the standard model. As $\Delta m$ is known from experiment, we shall be concerned mainly with $\phi$ given in Eq.(7). To this end, note that,

$$\frac{q}{p} \approx \left(\frac{M_{12}^*}{M_{12}}\right)^{1/2} \approx \left(\frac{U_{td}\, U_{tb}^*}{|U_{td}\, U_{tb}|}\right)^2 \tag{8}$$

where $M_{12} - \frac{i}{2}\Gamma_{12} = \langle B|H|\bar{B}\rangle$ following the notation of Lee and Wu[5], and

$$\rho = \begin{pmatrix} U_{cb}^* U_{cs} \\ U_{cb}^* U_{cd} \\ U_{ub}^* U_{ud} \end{pmatrix}^2 \quad \text{for} \quad \begin{matrix} b\bar{d} \to c\bar{c}s\bar{d} \\ b\bar{d} \to c\bar{c}d\bar{d} \\ b\bar{d} \to u\bar{u}d\bar{d} \end{matrix} \tag{9}$$

Thus we have

$$\phi = 2 \arg \; [U_{tb}U_{td}^*] \begin{pmatrix} U_{cb}^* U_{cs} \\ U_{cb}^* U_{cd} \\ U_{ub}^* U_{ud} \end{pmatrix} \tag{10}$$

for the quark decay channels given in Eq.(9).

It is interesting to note that $U_{tb}U_{td}^*U_{cb}^*U_{cd}$ and $U_{tb}U_{td}^*U_{ub}^*U_{ud}$ are two members of the rephasing invariants as noted by many authors[6] and studied extensively by Bjorken and Dunietz[7], Branco and Lavoura[8], and Hamzaoui[9]. The first combination in Eq.(10), $U_{tb}U_{td}^*U_{cb}^*U_{cs}$, is not a rephasing invariant. It should be noted that we restrict ourselves to the final states which contain $K_S$. Any phase redefinition $s \to e^{i\alpha}s$, $d \to e^{i\beta}d$ which changes $U_{tb}U_{td}^*U_{cb}^*U_{cs}$ by a phase leads to a compensating change in the $K_S$ meson wave function $\langle\bar{d}s|K_S\rangle \to e^{-i(\alpha-\beta)}\langle\bar{d}s|K_S\rangle$, keeping the experimentally measurable asymmetry invariant as it should be. With this in mind we might study $\arg(U_{tb}U_{td}^*U_{cb}^*U_{cs}U_{us}^*U_{ud})$ which is a rephasing invariant. It can then be represented by the product of $U_{tb}U_{cb}^*U_{cs}U_{ts}^*$ and $U_{td}^*U_{us}^*U_{ud}U_{ts}$ which are both rephasing invariants.

One interesting characteristic of these rephasing invariants is

$$\mathrm{Im}[U_{ij}U_{ik}^*U_{lk}U_{lj}^*] = \pm\,\delta \tag{11}$$

where $\delta$ is a unique number independent of any indices.[6]

Now, consider a unitarity relation

$$U_{td}U^*_{tb} + U_{cd}U^*_{cb} + U_{ud}U^*_{ub} = 0 \tag{12}$$

With an approximation and a choice of phase, we can choose $U_{tb} \approx 1$, $U_{ud} \approx 1$, $U_{cd} \approx -\sin\theta_c$ and real $U_{cb}$, and obtain[4]

$$U_{td} + U^*_{ub} = \sin\theta_c U_{cb} \tag{13}$$

which leads to a representation of the approximate unitarity relation of Eq. (13) by a triangle shown below in Fig.1.

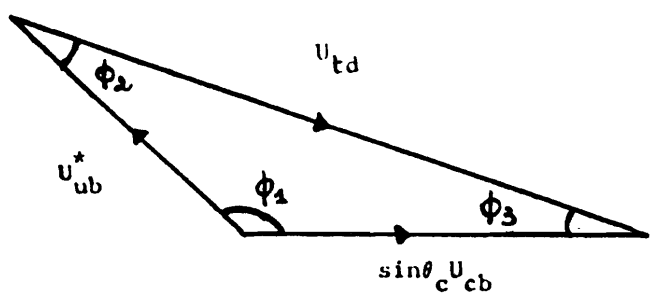


Figure 1.

Figure 1: Geometrical representation of the unitarity relation between the three complex numbers $U_{cd}U^*_{cb} \approx -\sin\theta_c U^*_{cb}$, $U_{ud}U^*_{ub} \approx U^*_{ub}$ and $U_{td}U^*_{tb} \approx U_{td}$. Unitarity constrains these three complex numbers to form a triangle in the $\rho$-$\eta$ complex plane whose area is equal to $\frac{1}{2}|\delta|$.

We point out that if we normalize the side of the triangle with length $|\sin\theta_c U_{cb}|$ to length 1, we obtain an example of the triangle shown in Fig.2. Parametrizing $U^*_{ub}/(\sin\theta_c U^*_{cb}) = \rho + i\eta$, we find that the base of the triangle rests between (0,0) and (1,0) on the $\rho$-$\eta$ plane and one corner of the triangle rests on the ($\rho$-$\eta$) value chosen by nature. From figure 1, we see that

$$
\begin{aligned}
&2 \arg (U_{tb}U^*_{td}U^*_{cb}U_{cd}) \\
&\approx 2 \arg (U_{tb}U^*_{td}U_{cs}U^*_{cb}U^*_{us}U_{ud}) \\
&\approx 2 \arg (U^*_{td}U^*_{cb}) = -2\phi_3
\end{aligned}
\tag{14}
$$

and

$$
\begin{aligned}
&2 \arg (U_{tb}U^*_{td}U^*_{ub}U_{ud}) \\
&\approx 2 \arg (U^*_{td}U^*_{ub}) = 2(\pi - \phi_2)
\end{aligned}
\tag{15}
$$

## IV. CONSTRAINTS ON THE TRIANGLE

The understanding of expected CP asymmetries is now reduced to understanding the triangle given in Fig.1. Another interesting point is that

$$
\begin{aligned}
\delta &= |U_{td}||U_{cb}| \sin\theta_c \sin\phi_3 \\
&= |U_{td}||U_{bu}| \sin\phi_2 \\
&\text{etc.}
\end{aligned}
\tag{16}
$$

which leads to a geometrical interpretation for $\delta$ - two times the area of the triangle. Furthermore, elementary geometry provides us with a relation between $\phi_3$ and $\phi_2$:

$$
\frac{\sin\phi_3}{\sin\phi_2} = \frac{|U_{ub}|}{|U_{cb}|} \frac{1}{\sin\theta_c} \tag{17}
$$

Using

$$
0.07 \le \frac{|U_{ub}|}{|U_{cb}|} \le 0.2 \tag{18}
$$

leads to

$$
0.32 \le \frac{\sin\phi_3}{\sin\phi_2} \le 0.91 \tag{19}
$$

We now determine the experimental constraints on the triangle. For this purpose, it is useful to consider the Wolfenstein representation[10] of the KM matrix:

$$
U_{KM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix} \tag{20}
$$

The magnitude of $\eta$ is inferred from $\epsilon_K$:

$$B_K\eta \approx \frac{|\epsilon_K|}{4.33A^2} \{f_3S(x_c, x_t) - f_1S(x_c) + 2.34 \times 10^{-3}A^2(1-\rho)f_2S(x_t)\}^{-1} \quad (21)$$

$f_i$ denote the QCD radiative corrections; $S(x_i)$ and $S(x_i, x_j)$ are the various quark box contributions with $x_i = \frac{m_i^2}{M_W^2}$; $B_K$ is the bag constant.

For 30 GeV $\leq m_t \leq$ 180 GeV one can employ a much simpler approximate expression[11]:

$$B_K\eta \approx \frac{0.53}{A^2} \{0.99x_t^{0.1105} - 0.3 + A^2(1-\rho)x_t^{0.8363}\}^{-1} \quad (22)$$

where we have used $m_c \approx 1.5$ GeV; $\eta$ thus drops fairly quickly with increasing $m_t$. In Eq.(21), we have ignored the long distance correction to $\epsilon_K$. In view of the smallness of $\epsilon'/\epsilon$,

$$\frac{\epsilon'}{\epsilon} = (3.5 \pm .7 \pm .4 \pm 1.2) \times 10^{-3} \quad (23)$$

a preliminary result from the NA31 Collaboration[12], we do not expect the long distance effect to change our result substantially.

The QCD corrections to Eq.(21) are[13] $f_1 = 0.85$, $f_2 = 0.61$, $f_3 = 0.38$. We do not expect the final result to be very sensitive to the variation of these corrections in the top quark mass range, 50 GeV $< m_t <$ 180 GeV. The allowed contours in the $\rho$-$\eta$ plane obtained from $\epsilon_K$ are shown in Fig. 2 for the case where $B_K = 0.7$[14] and $m_t$= 50 GeV and 100 GeV. Since the values of $U_{cb}$ and $U_{ub}$ extracted from experiment depend on the assumed model for strong interactions[15], there exists many cases consistent with the allowed region in the $\rho$-$\eta$ plane. However, using the allowed range of the ratio $\frac{|U_{ub}|}{|U_{cb}|}$ given by Eq.(18) and the value of A estimated from the B lifetime, A $\approx$ 1, one can reduce the study to one analysis which includes all these models[15].

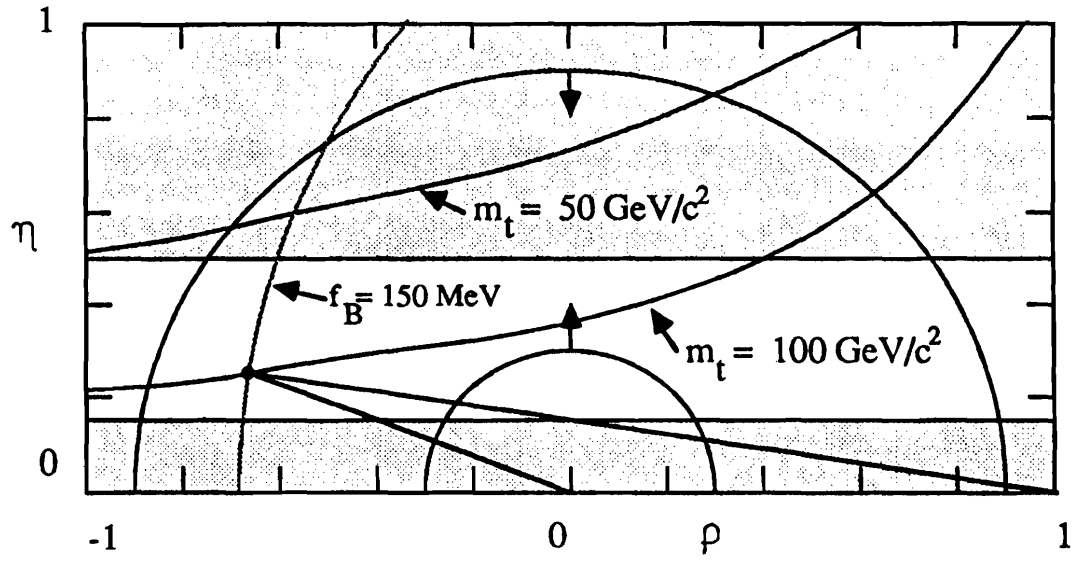


Figure 2: Allowed regions in the $\rho$-$\eta$ plane, obtained from $\epsilon_K$, $\frac{|U_{ub}|}{|U_{cb}|}$, $\frac{\epsilon'}{\epsilon}$, and $x_B$. Solid curves labelled by $m_t$ : $\epsilon_K$ constraint [Eq.(22)]. Solid semicircles: upper and lower bounds on $\frac{|U_{ub}|}{|U_{cb}|}$ [Eq.(18)]. Horizontal unshaded band: $\frac{\epsilon'}{\epsilon}$ (from Refs. 12,16 and 17). Light semicircle: $B^0$-$\bar{B}^0$ mixing. Here we have taken A = 1, $\tau_B = 1.16 \times 10^{-12}$ s, and the illustrative values $f_B$ = 150 MeV, $m_t$ = 100 GeV / $c^2$. The round dot denotes the corresponding apex of the unitarity triangle.

## V. B-$\bar{B}$ MIXING

We now turn to the constraint which can be put on the top quark mass from B-$\bar{B}$ mixing.

The ARGUS[1] result on B-$\bar{B}$ mixing

$$\frac{N(\ell^{\pm}\ell^{\pm})}{N(\ell^{+}\ell^{-})} = 0.234 \pm 0.067 \pm 0.031 \tag{24}$$

implies that

$$x = \frac{\Delta m}{\Gamma} = 0.73 \pm 0.18 \tag{25}$$

The mixing parameter x in the standard model is given by

$$x = \frac{2|M_{12}|}{\Gamma} = \frac{G_F^2 f_B^2 m_B R_B}{6\pi^2\Gamma} M_W^2 \eta_2^B [A_{tt}] |\lambda^3 A(1-\rho-i\eta)|^2 \tag{26}$$

where $\eta_2^B = 0.86$ is a QCD correction[13], $f_B$ (here we adopt the normalization such that the corresponding decay constant for the pion is $f_\pi = 130$ MeV) and $R_B$ are the decay constant and the bag parameter for the B meson respectively, and $A_{tt}$ is a known function of $m_t$ which can be approximated by $S(\frac{m_t^2}{M_W^2})$, employing the same approximation as that used in Eq.(22).

Thus,

$$x_B = 0.18 \left(\frac{f_B}{150\text{MeV}}\right)^2 R_B \left(\frac{m_t^2}{M_W^2}\right)^{0.8363} A^2 [(1-\rho)^2+\eta^2] \tag{27}$$

If we set

$$r^2 = (1-\rho)^2 + \eta^2 \tag{28}$$

we obtain

$$r^2 = x_B/\gamma^2 A^2 \tag{29}$$

where

$$\gamma^2 = 0.18\left(\frac{f_B}{150\text{MeV}}\right)^2 R_B \left(\frac{m_t^2}{M_W^2}\right)^{0.8363} \tag{30}$$

If the precise value of $\gamma$ is known, the ARGUS measurement of $x_B$ can be represented by a circle centered at $\rho = 1$. There is, however, large uncertainty in the value of $\gamma$ as $m_t$, $f_B$, and $R_B$ are all unknowns. The $\rho$-$\eta$ plane, however leads to a constraint on $\gamma$. The lower limit on $\gamma$ taking the minimum value of $x_B = 0.37$ ($2\sigma$), is found to be $\gamma^2 \gtrsim 0.08$ for all three models of Ref. 15. This puts a limit

$$\sqrt{R_B} f_B \left(\frac{m_t}{M_W}\right)^{0.8363} \gtrsim 0.1 \text{ GeV} \tag{31}$$

In view of the uncertainty in $\sqrt{\bar{R}_B} f_B$, the above result does not translate itself into a useful lower limit on the top quark mass. For example, if one takes $\sqrt{\bar{R}_B} f_B \sim 0.15$ GeV, one obtains

$$m_t \gtrsim 50 \text{ GeV} \tag{32}$$

This lower limit on the top quark mass is consistent with previous analyses[18]. Using the upper limit[19] $m_t \leq 180$ GeV, and assuming $f_B \leq 150$ MeV, one obtains a circle centered at $(\rho,\eta) = (1,0)$ with radius 0.73 as the right-most limit of $(\rho,\eta)$. Thus, negative or only slightly positive values of $\rho$ are preferred on the basis of the ARGUS[1] $B$-$\bar{B}$ result.

## VI. CONCLUSIONS

We have emphasized the importance of a triangle in the complex plane formed by the small elements of the KM matrix. Specific CP-violation signatures, the b-quark lifetime, the rates for charmless B meson decays, and the values of $\Delta m/\Gamma$ for $B_d^0$-$\bar{B}_d^0$ and $B_s^0$-$\bar{B}_s^0$ mixings all probe the geometry of this triangle. The area of the triangle, in particular, is directly correlated with CP violation, and various decay asymmetries are simple functions of its angles. Present data suggest, within wide uncertainties, that this triangle is fairly asymmetric. In terms of the parameters $\rho$ and $\eta$ of Ref. 10, negative values of $\rho$ are preferred by $B_d^0$-$\bar{B}_d^0$ mixing.

Data of increasing precision will eventually be able to specify enough independent parameters to test whether the quantities $U_{ub}^*$, $U_{td}$, and $\sin\theta_c U_{cb}$ in fact form a triangle at all, and thus to test the three-generation Kobayashi-Maskawa model of CP violation in a simple geometric way.

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