ACCELERATOR NEUTRINO PHYSICS

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# CONTENTS

1. **ν-EPISTEMOLOGY**

   1.1 Invention and discovery of the neutrino (1930-1938)  
   1.2 Universality of weak interactions (1949) 
   1.3 V-A: In weak interactions parity is not conserved (1956) 
   1.4 Weak interactions are mediated by intermediate vector-bosons ($W^+$s) (1960) 
   1.5 $\nu_{\mu} \neq \nu_e$ (1961) 
   1.6 Cabibbo angle (1963) 
   1.7 Quarks (1964), partons (1968) and quark-partons (1969) 
   1.8 Neutral currents (1973) 
   1.9 Charm (1974)

2. **ν-TECHNOLOGY**

   2.1 Principle of neutrino experiments 
   2.2 Neutrino beam 
      2.2.1 Production of $\nu$-parents 
      2.2.2 Focusing of $\nu$-parents 
      2.2.3 Decay of $\nu$-parents 
      2.2.4 Shielding 
      2.2.5 Neutrino beam monitoring and spectrum determination 
   2.3 Neutrino detectors 
      2.3.1 The ideal $\nu$-detector 
      2.3.2 Bubble chambers 
      2.3.3 Counters, spark chambers, calorimeters 
      2.3.4 Photographic emulsions 
   2.4 List of neutrino experiments 1961-1976

3. **ν-NUMEROLOGY** (Methods and results of data analysis)

   3.1 Charged current interactions 
      3.1.1 $\nu N \rightarrow \mu N'$ 
      3.1.2 $\nu p \rightarrow \mu + \Lambda$ 
      3.1.3 $\nu N \rightarrow \mu + \text{hadrons}$ 
      3.1.4 $\nu e \rightarrow e + \text{hadrons}$

Page

4 4 4 5 6 10 11 11 12 14 19 21 22 22 23 27 28 32 35 35 36 39 40 40 43 44 44 49 50 57
(ii)

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>Neutral current interactions</td>
<td>58</td>
</tr>
<tr>
<td>3.2.1</td>
<td>$\bar{\nu}_e \rightarrow \bar{\nu}_e$</td>
<td>58</td>
</tr>
<tr>
<td>3.2.2</td>
<td>$\nu_N \rightarrow \nu + \text{hadrons}$</td>
<td>59</td>
</tr>
<tr>
<td>3.3</td>
<td>Dilepton events</td>
<td></td>
</tr>
<tr>
<td>3.3.1</td>
<td>$\nu_N \rightarrow \mu\mu + \text{hadrons (in counter experiments)}$</td>
<td>63</td>
</tr>
<tr>
<td>3.3.2</td>
<td>$\nu_N \rightarrow \mu^-\bar{e}^+ + \text{hadrons (in bubble chamber experiments)}$</td>
<td>66</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS AND OUTLOOK  

APPENDICES  

REFERENCES
INTRODUCTION

The first part is a brief description of the development of the theoretical framework from the discovery of β-decay to charm. This is necessarily incomplete [1], but should show the interplay between experiment and theory to arrive at the main ideas and most popular models used in the analysis of present neutrino physics data. Some concepts, like form factors, structure functions, conserved vector current, will be treated along with cross section analyses in part 3.

The aim of the second part is to familiarize the reader with the technological aspects of accelerator neutrino experiments. It should show what the technical requirements are to obtain reasonable numbers of analysable neutrino interactions.

In the third part, the main numerical results of the recent neutrino experiments will be derived and analyzed in some detail in the context of part 1.
1. **v-EPISTEMOLOGY** (from β-decay to charm)

1.1 **Invention and discovery of the neutrino**

On Sunday afternoon, 1 March 1896, Becquerel discovered that uranium salt had blackened a photographic plate. Many experiments — even underground — were necessary to prove that this effect was independent of external conditions and due to electron (discovered 1897) emission by nuclei \( N \rightarrow N' + e \). Neutrino physics started when Pauli [2] proposed in 1930 "a desperate remedy to save the laws of conservation of energy and statistics, which were in serious danger because of the results of three decades of experiments with the β-decay electrons". These had led to the conclusions:

(a) the electrons are emitted by nuclei, hence are in the nucleus, so e.g. the nitrogen nucleus \( ^{14}N \) would consist of 21 fermions (14 protons and 7 electrons) which is statistically impossible (the \( ^{14}N \)-band spectrum showed Bose-Einstein statistics);

(b) the electrons emitted in the (two-body) decay are not always monoenergetic which is energetically impossible.

The remedy was a fourth elementary particle, the neutrino(*) (in addition to p, e, γ) which should exist in the nucleus, have spin \( \frac{1}{2} \), a mass of the order of the electron mass, and be emitted during β-decay together with an electron.

Taking up this neutrino hypothesis, Fermi [3] described β-decay as the emission of an (ev)-pair by a nucleus analogous to γ emission by an excited atom in radiation theory (fig. 1).

![Fig. 1](image)

(*) which Pauli called neutron, and Fermi neutrino after Chadwick's neutron discovery.
with the matrix element as we would write it today [4] for a vector
current × vector current interaction

\[ M = \frac{G}{\sqrt{2}} \langle \bar{\psi}_p \gamma_\alpha \psi_n \rangle \langle \bar{\psi}_\nu \gamma_\alpha \psi_e \rangle \]  

(1)

where \( G \) is the Fermi coupling constant \((G = 1.4 \times 10^{-49} \text{ erg cm}^3 \text{ or } 10^{-5} \text{ m}^{-2})\).

The neutrino was detected on a cloud chamber picture in 1938 [5]
by the observation of a non-collinear configuration of a \( \beta \)-ray and its
recoil nucleus and in 1953 [6] by the observation of the inverse \( \beta \)-decay
induced by nuclear reactor emitted antineutrinos

\[ \bar{\nu} + N \rightarrow N' + e^+ . \]  

(2)

1.2 Universality of weak interactions

Many different decay interactions like

\[
\begin{array}{l|c}
\text{n} \rightarrow \text{p} + \text{e}^- + \bar{\nu} & 918 \\
14.06^8 \rightarrow 14.7^7 \text{N} + e^+ + \nu & 71.4 \\
\tau \rightarrow \mu + \nu (1947) & 2.6 \times 10^{-8} \\
\mu \rightarrow \nu + e + \nu & 2.2 \times 10^{-6} \\
\end{array}
\]

and also muon capture \((\mu^- + p + n + \nu, \text{ capture rate } \sim 270 \text{ sec}^{-1})\) were
found to have nearly the same coupling constant, the difference in lifetime coming only from phase space:

\[ \tau \sim \frac{G^2}{2} \times |M|^2 \times \text{phase space} . \]

This led to the hypothesis of the universal Fermi interaction pictured
usually by an equilateral triangle [7] (fig. 2).

Fig. 2 Universal Fermi interactions
Each corner is called a (charged) current(*)
\[ J = (e\nu), (\mu\nu) \text{ lepton currents, } J^L \]
\[ J = (NP) \text{ hadron current, } J^H \]

each connection line represents a current-current interaction with the matrix element (**)
\[ M = \frac{G}{\sqrt{2}} \{<J> <J>^+ + \text{herm. conj.}\} \]

The whole weak current (in 1949) is the sum of all corners
\[ J = (e\nu) + (\mu\nu) + (NP) \]
and
\[ M = \frac{G}{\sqrt{2}} \{(e\nu)(\mu\nu) + (e\nu)(PN) + (\mu\nu)(PN) + (e\nu)e\nu + (PN)(PN)\} \]

where term one describes \(\mu\)-decay, term two: \(\beta\)-decay of the neutron \(n \rightarrow p + e + \bar{\nu}\) or \(e\) capture by nuclei \(e^- + p + n + \nu\) or scattering of neutrinos off nucleons, e.g. \(\bar{\nu} + p + n + e^+\), etc. Also each corner (current) can interact with itself: \((e\nu)(e\nu)\) describes neutrino-electron scattering \(\nu + e \rightarrow \nu + e\) (still unobserved, but see sect. 3.3.1) and \((NP)(NP)\) is the weak neutron proton interaction which in principle could be detectable in certain nuclear transitions by circularly polarized photon emission [8].

1.3 V-A: In weak interactions parity is not conserved

Since particle states are described by four-component spinors, the matrix element for an interaction between four particles, like \(p \rightarrow n + e + \nu\), would involve 256 coupling constants:
\[ M = \text{dr} \left[ \Sigma_{ijkl} g_{ijkl} (\psi^*_p \psi_n)(\psi^*_e \psi_\nu) \right] \]

(*) Current means here just transition from initial state of a particle to its final state: \(e \rightarrow \nu\), see eq. (1).

(**) \(1/\sqrt{2}\) historical, to keep \(G\) at its original numerical value after parity violation discovery.
Lorentz invariance (interactions do not depend on orientation or speed of the laboratory) requires that the currents (bilinear spinor combinations) are either

- scalars ($\bar{\psi}\psi$, $\bar{\psi}\gamma_5\psi$ (pseudoscalar))
- vectors ($\bar{\psi}\gamma_\alpha\psi$, $\bar{\psi}\gamma_\alpha\gamma_5\psi$ pseudo (axial, polar) vector)
- tensors ($\bar{\psi}\gamma_\alpha\gamma_\beta\psi$, $\bar{\psi}\gamma_\alpha\gamma_\beta\gamma_5\psi$ (pseudo tensor)),

since the products (contractions) of these give scalars, and thus reduces the 256 to 12 basic coupling constants, or rather 10 (pseudo tensor can be expressed by the other couplings): for instance for neutrino scattering $\nu + n \rightarrow \mu + p$

\[
\begin{align*}
&\bar{\psi}_p \gamma_\alpha \psi_n (\bar{\psi}_\mu \gamma_\alpha \psi_\nu) \\
&\bar{\psi}_p \gamma_5 \psi_n (\bar{\psi}_\mu \gamma_5 \psi_\nu) \\
&\bar{\psi}_p \gamma_\alpha \gamma_5 \psi_n (\bar{\psi}_\mu \gamma_\alpha \gamma_5 \psi_\nu) \\
&\bar{\psi}_p \gamma_\alpha \gamma_\beta \gamma_5 \psi_n (\bar{\psi}_\mu \gamma_\alpha \gamma_\beta \gamma_5 \psi_\nu)
\end{align*}
\]

(Fermi's proposal eq. (1)) (7)

or any linear combination thereof.

In 1956 Lee and Yang [9] questioned Mach's principle, i.e. that no physical process can distinguish between right-handed and left-handed coordinate systems, i.e. all physical processes conserve parity. The experimental reason for this question was the $T=0$ puzzle: one and the same particle ($m_\tau = m_\theta$) decays once in $2\pi$'s (parity +1), once in $3\pi$'s (parity -1). The crucial experiment to test whether parity is violated in weak interactions was to measure the correlation between spin (axial vector) of $\beta$-active nuclei and momentum direction (vector) of the decay electrons, i.e. a pseudoscalar term (which changes sign under space reflection). The experiment [10(a)] done with polarized $^{60}\text{Co}$ nuclei ($^{60}\text{Ni} + e^- + \nu$) showed clearly parity violation: backward emission of electrons is favoured.

The idea of Lee and Yang (and Salam, Landau) was then that the lack of parity conservation was due to the neutrino spinning only in one
direction, and to describe the neutrino by 2-component spinors satisfying
Weyl equations rather than by the 4-component spinors and the Dirac equation.
Starting from the Dirac equation
\[ \gamma_\alpha \frac{\partial \psi}{\partial x^\alpha} = -m\psi, \]  
and defining the chiral projections \( \psi_+ = \frac{1}{2}(1 + \gamma_5)\psi \), one obtains
\[ \gamma_\alpha \frac{\partial \psi_+}{\partial x^\alpha} = -m\psi_+ , \]  
and for \( m = 0 \) and in momentum space the Weyl equation
\[ \left( 1 \pm \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|} \right) \psi_\pm = 0, \quad \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|} = H = \text{helicity}, \]  
i.e. \( \psi_+ \) describes a massless particle with negative helicity \((\sigma \leftrightarrow \vec{p})\) or
an antiparticle with positive helicity \((\sigma \leftrightarrow \vec{p})\); and for \( \psi_- \) the correspondence is reversed. "The neutrino spinning only in one direction" means, it participates in interactions only as \( \psi_+ = \frac{1}{2}(1 + \gamma_5)\psi \) or as \( \psi_- = \frac{1}{2}(1 - \gamma_5)\psi \), corresponding to maximum parity violation. Charge conjugation is also violated (particle and antiparticle solutions are not symmetric) but the combined operation \( CP = (\text{charge conjugation}) \times (\text{spatial inversion}) \) is well defined \( (CP \text{ violation - observed in } K^0 - \text{decays - is not described by this theory!}) \).

The circular photon polarization from the e capture process
\[ ^{152}\text{Eu} + e \rightarrow ^{152}\text{Sm}^* + \nu \rightarrow ^{152}\text{Sm} + \gamma + \nu [10(b)] \]  
fixed the neutrino helicity: neutrinos (all leptons) are left handed (\( \psi_+ \)), antineutrinos (all antileptons) are right handed (\( \psi_- \)).

The next step was to extend this idea to all, also massive (bare) fermions. This was done first by postulating chirality invariance \([11]\) (four fermion interaction invariant if \( \psi \) is transformed into \( \psi \rightarrow \psi = \gamma_5\psi \)), then by reformulating the Dirac equation as Klein-Gordon equations also for massive particles \([8]\), then by applying mass reversal invariance \([12]\). So, all bare fermions are left-handed skrews
\[ \psi \rightarrow \frac{1}{2}(1 + \gamma_5)\psi \]
and the only remaining (non gradient) coupling was
\[ M = \frac{G}{\sqrt{2}} \left( \frac{1}{2}(1 + \gamma_5) \psi_p \gamma_\alpha \frac{1}{2}(1 + \gamma_5) \psi_n \right) \left( \frac{1}{2}(1 + \gamma_5) \psi_e \gamma_\alpha \frac{1}{2}(1 + \gamma_5) \psi_\nu \right) \]
\[ = \frac{G}{\sqrt{2}} \left( \psi_p (1 + \gamma_5) \gamma_\alpha \psi_n \right) \left( \psi_e (1 + \gamma_5) \gamma_\alpha \psi_\nu \right), \tag{9} \]

i.e. vector-axial vector (V-A).

This was for some time in disagreement with some experiments, especially with the electron neutrino angular correlation measured in $^6$He, which favoured scalar-tensor coupling. But the experiments were re-done and the corrected results agreed with the V-A picture.

Now cross sections for scattering of two point like fermions via the weak interaction can be derived from eq. (9). Using the rules for Feynman graphs [4] and $\gamma$-matrix trace calculations, e.g. for $\nu + e \rightarrow \nu + e$ and $\bar{\nu} + e \rightarrow \bar{\nu} + e$ (a process not yet detected, see however sect. 1.8 and 3.3.1) one finds:

\[ \frac{d\sigma}{d \cos \theta} = \frac{1}{32\pi s} |M|^2 (\sqrt{s} = \text{total center of mass energy}) \]

with:

<table>
<thead>
<tr>
<th>$\nu e \rightarrow \nu e$ (s-wave)</th>
<th>$\bar{\nu} e \rightarrow \bar{\nu} e$ (p-wave)</th>
</tr>
</thead>
<tbody>
<tr>
<td>left-handed</td>
<td>left-handed</td>
</tr>
<tr>
<td>$</td>
<td>M</td>
</tr>
<tr>
<td>[ \frac{d\sigma}{d \cos \theta} ]</td>
<td>[ \frac{G^2}{\pi} s ]</td>
</tr>
<tr>
<td>[ \frac{d\sigma}{dy} ]</td>
<td>[ \frac{G^2}{\pi} s ]</td>
</tr>
<tr>
<td>[ \sigma_{\text{total}} ]</td>
<td>[ \frac{G^2}{\pi} s \frac{2G_m^2}{E_\nu} ]</td>
</tr>
</tbody>
</table>

\[ \sigma_{\text{total}} \]

* \[ y = \frac{1 - \cos \theta}{2} = \frac{p_2 q}{p_1 p_2} = \frac{E_1 - E_3}{E_1}, q = p_1 - p_3; s = (p_1 + p_2)^2 = 4 (\frac{m^2 + k^2}{4}) \approx 4k^2 \approx 2mE_1, u = (p_1 + p_3)^2 = -2k^2 (1+\cos \theta), k = \text{c.o.m. mom. of } \nu. \]
1.4 Weak interactions are mediated by intermediate vector bosons (W's)

In order to prevent the cross section eq. (10) from becoming infinite with increasing $E$, the exchange of a weak interaction quantum $W$ of charge $\pm 1$ (charged current interaction, $v$ turns into $e^-$), spin 1 (weak interactions change spin), large mass (weak interactions have short range) and vector type (it has to couple 4-component currents) has been postulated [13] in complete analogy to QED:

\[
M = e^2 (e^- \gamma e) \frac{1}{q^2} (e^- \gamma e)
\]

\[
M = g^2 (e^- \gamma (1 + \gamma_5)) \frac{m_W}{q^2 - m_W^2} (e^- \gamma (1 + \gamma_5) e)
\]

with \((q^2 \to 0) \frac{m_W}{\sqrt{2}} = \frac{g}{\sqrt{2}}
\]

However, 2-boson exchange still leads to quadratic divergences due to the longitudinal polarization of the massive $W$ [14], which is not the case in QED:

\[
\delta + \int \frac{d^4 q}{q^6} \to \text{finite} \quad \quad \delta + \int \frac{d^4 q}{q^2} \to \text{infinite} \quad (11)
\]
In neutrino experiments the intermediate vector boson W could in principle be discovered by direct production of W's through a process similar to electromagnetic Bremsstrahlung. From the fact that no such production has been observed so far, one concludes that the W mass must be greater than 4 GeV/c$^2$.

1.5 $\nu_\mu \neq \nu_e$ (1962)

The most important result of the first accelerator neutrino experiment (performed at Brookhaven [15]) was the fact that a beam of neutrinos from $\pi \rightarrow \mu \nu$ decay produced only events with $\mu^-$, and not equal numbers of events with $e^-$ and $\mu^-$. This proved the existence of muonic neutrinos, $\nu_\mu$ (coupled to muons, e.g. in $\pi \rightarrow \mu \nu$, $K \rightarrow \mu \nu$) as distinct from leptonic neutrinos, $\nu_e$ (coupled to electrons, e.g. in $n \rightarrow p e \nu_e$, $K \rightarrow \pi e \nu_e$), described by different lepton numbers $L$:

\begin{align*}
L_\mu & \quad 0 & 0 & 1 & -1 & 0 \\
L_e & \quad 1 & -1 & 0 & 0 & 0
\end{align*}

which are separately conserved [16].

1.6 Cabibbo angle (1963)

From comparison of the decay rates

\begin{align*}
\Lambda \rightarrow P e \nu_e, & \quad K \rightarrow \mu \nu_\mu \\
N \rightarrow P e \nu_e, & \quad \pi \rightarrow \mu \nu_\mu,
\end{align*}

it was clear that the effective coupling constants in strangeness changing ($\Delta S \neq 0$) weak interactions were $\sim \sqrt{20}$ smaller than in $\Delta S = 0$ weak interactions, thus breaking the weak interaction universality eq. (1) and (2). The universality was rescued by redefining [17] the weak hadron current eq. (3b):
\[
J^H = \cos \theta_c \cdot J^{H,\Delta S=0} + \sin \theta_c \cdot J^{H,\Delta S=1} \\
= \left(\frac{\bar{\nu}_\alpha (1 + \gamma_5) N}{N'}\right) \cdot \cos \theta_c + \left(\frac{\bar{\nu}_\alpha (1 + \gamma_5) N}{N'}\right) \sin \theta_c. 
\]

The universality triangle now becomes:

![Universality Triangle Diagram]

with \( N' = N \cos \theta_c + \Lambda \sin \theta_c \) (\( \theta \) = "Cabibbo angle"). The cross section ratio \( \sigma^{\Delta S=1} / \sigma^{\Delta S=0} = \tan^2 \theta_c = 0.056 \) and the empirical selection rules

- \( \Delta S = 1, \Delta I = \frac{1}{2}, \)
- \( \Delta S = 0, \Delta I = 1, \)

and \( \Delta Q = \Delta S, \Delta S \leq 1 \) have been confirmed in many decay interactions. The test of this scheme in neutrino experiments is difficult, since existing neutrino detectors (sect. 2.3) can identify strange particles only with small efficiency: \( K^- \) are undistinguishable from \( \pi^- \), \( K_L^0 \) and the \( K_S^0 \) and \( \Lambda^- \) decays into neutrals are rarely visible. In addition, the \( \Delta Q = \Delta S \) rule, which forbids e.g.

\[
\nu + n \rightarrow \mu^- + \Lambda + \pi^+ (Q_n - Q_{\Lambda \pi^+} = -1, S_n - S_{\Lambda \pi^+} = +1),
\]

is expected to be apparently violated in the charm model (sect. 1.9). However, single \( \Lambda^- \) production by antineutrinos, \( \bar{\nu} + p \rightarrow \mu^+ + \Lambda \) [18], has been observed (sect. 3.1.2).

1.7 Quarks (1964), partons (1968) and quark partons

Similarities between strange and non-strange particles in strong interactions had been raised to a symmetry principle: strong interactions are invariant under strangeness change, and in the same way as isospin invariance puts hadrons into isomultiplets ((p,n), (\( \pi^+\pi^0\pi^- \)), (\( \Sigma^+\Sigma^0\Sigma^- \)), ...), strangeness invariance puts hadrons into \( SU_3 \) -multiplets (with \( B = \) baryon number, \( S = \) strangeness, \( I_3 = 3 \). Isospin component), two of which are shown in fig. 3.
All physical particles are postulated \([19]\) to be built from three hypothetical fundamental constituents\(^(*)\) - called quarks (spin \(\frac{1}{2}\)) - being the basic representation of the SU\(_3\) symmetry group (fig. 4).

A proton is then:

\[
p = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}
\]
a neutron

\[
N = \begin{pmatrix} \downarrow \\ \downarrow \end{pmatrix}
\]
a lambda

\[
\Lambda = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}
\]

Weak interactions violate strangeness; a p-quark does not know a priori whether it should turn into an n-quark or into a \(\lambda\)-quark. Hence, for weak interactions we may define "Cabibbo rotated" quarks

\[
n' = n \cos\theta_c + \lambda \sin\theta_c
\]  \hspace{1cm} (13a)

and, by analogy

\(^(*)\) However, a 3-quark state is anti-symmetric, whereas baryon wave functions are symmetric; hence at least for spin statistics reasons, one more degree of freedom is needed \([20]\): each quark in three colours, and hadrons mix the colours such that they are colour singlets.
\[ \lambda' = -n \sin \theta_c + \lambda \cos \theta_c, \quad \text{(13b)} \]

but \( \lambda' \) does not seem to participate in weak interactions. All weak interactions (fig. 5), seem to happen really between 4 leptons and 3 quarks, for example, \( \Lambda \)-production by \( \bar{\nu}_\mu \) - \( P \)-scattering might be pictured as in fig. 6:

![Fig. 5](image1.png)  
![Fig. 6](image2.png)

Fig. 5  
Fig. 6

When it became evident in 1968 that electrons [21] and neutrinos [22] scatter off nucleons - in high energy and momentum transfer collisions - as if the nucleon was made of points - partons [21] - namely by the large cross section for high momentum transfers \(^(*)\), the idea suggested itself that these partons are the above quarks (quark-parton model). Then, total neutrino cross sections should just be the sum of neutrino quark cross sections and hence have the same behaviour as \( \nu_e \) scattering cross sections eq. (10a,b). That this is nearly so, will be seen in sect. 3.1.3.

1.8 Neutral currents (discovery 1973)

The electromagnetic current is a neutral current, the electromagnetic interaction being due to (neutral) photon exchange

\[ \text{(\#)} \quad \text{A situation similar to Rutherford's } \alpha \text{-scattering experiments showing that atoms have hard nuclei.} \]
The weak current was, prior to 1973, supposed to be charged, the weak interaction being due to (charged) W exchange:

\[ \nu_e \rightarrow e \]

\[ W^+ \]

\[ n \rightarrow p \]

This was experimentally confirmed by the absence \((< 10^{-5})\) of neutral current interactions like \(K^+ \rightarrow \pi^0 \pi^0 \mu^+ \mu^-\), and others.

In 1973, however, on the Gargamelle\((^\ast)\) neutrino film tracks of single electrons were discovered which could not be interpreted other than by muonic \(\nu\) scattering off atomic electrons \([23]\)

\[ \bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^- \]

which can only happen through a weak neutral current interaction:

\[ \bar{\nu}_\mu \rightarrow \bar{\nu}_\mu \]

\[ Z^0 \]

\[ e^- \rightarrow e^- \]

assumed to happen via the exchange of a neutral massive vector-boson \(Z^0\).

On the same film \([24]\) and also in counter experiments \([25]\) neutrino interactions were discovered which had only hadrons in the final state and which could - with high probability - only be interpreted as due to muonic neutrinos scattering off nucleons without turning into muons (sect. 3.2.3)

\[ \nu_\mu + N \rightarrow \nu_\mu + \text{hadrons}, \]

i.e. by a neutral current interaction

\[ \nu_\mu \rightarrow \nu_\mu \]

\[ Z^0 \]

\[ \text{hadrons} \]

\[ N \]

\(^\ast\) Heavy liquid bubble chamber, see sect. 2.3.2.
The \( Z^0 \) allowed two things highly desired by theorists:

(a) renormalization of the weak interaction theory, i.e. overcoming the difficulty of infinite cross sections eq. (11) and simultaneously

(b) unification of electromagnetic and weak interactions.

There is extensive [26] and didactic [27] literature showing this in detail. Here, only the main steps will be enumerated.

The non-renormalizability of weak interaction theory comes from the longitudinal polarization of the massive \( W \). The renormalizability of the electromagnetic theory (QED) is due to gauge invariance (cf. table I (A)).

**TABLE I**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic field</td>
<td>Yang-Mills field</td>
</tr>
<tr>
<td>Lagrangian of a free fermion field ( L_0 ): ( \psi(x) \rightarrow e^{ie\alpha} \psi(x) )</td>
<td>( \tilde{\psi}(x) (i\not{\delta} - m)\psi(x) ) is invariant if ( \psi(x) \rightarrow e^{i\frac{1}{2}ig(\vec{\tau} \cdot \not{B})(x)} \psi(x) )</td>
</tr>
<tr>
<td>which is a phase transformation, same rotation everywhere in space.</td>
<td>(( \tau ) = Pauli matrix), which is isospin rotation, same everywhere in space.</td>
</tr>
<tr>
<td>Lorentz invariance (( \rightarrow ) charge conservation)</td>
<td>Isospin invariance (( \rightarrow ))</td>
</tr>
</tbody>
</table>

| If rotation different in every space point (fixed locally): \( \psi(x) \rightarrow e^{ie\alpha(x)} \psi(x) \) | If rotation different in every space point: \( \psi(x) \rightarrow e^{i\frac{1}{2}ig(\vec{\tau} \cdot \not{B})(x)} \psi(x) \), |
| then \( L \) still invariant, if \( \partial_\mu A_\mu = \partial_\mu - ieA_\mu \) (covariant derivative, \( A_\mu \) is massless gauge field) and \( A_\mu + A_\mu + \partial_\mu \alpha(x) \) | then \( L \) still invariant, if \( \partial_\mu + D_\mu = \partial_\mu - i\frac{1}{2}g(\vec{\tau} \cdot \not{B})_\mu \) and if the isospin carrying \( B \) field is transformed like \( \vec{B}_\mu + \vec{B}_\mu + g(\vec{B}_\mu \times \not{B}) + \partial_\mu \not{B}(x) \) |

Yang and Mills have (already in 1954) shown that by introducing isospin multiplets of vector-bosons local gauge invariance can be generalized to isospin rotation (table I (B)). However, invariance requires, as in the photon case, massless bosons, and one fact which is certainly known
about the intermediate vector bosons is that they are heavy ($m > 4$ GeV/$c^2$), from the short range, even pointlike behaviour, of the weak interactions. The next step is then to generate masses by the Higgs-mechanism (1964) of spontaneous gauge symmetry breaking. How this is done is outlined in Appendix A, following closely the lectures of Iliopoulos. In order to demonstrate the concept of spontaneous symmetry breaking the phenomenon of ferromagnetism is often used as an example (e.g. S. Weinberg, Sci. Am. July 1974):

"... The equations governing the electrons and iron nuclei in a bar of iron obey rotational symmetry, so that the free energy of the bar is the same whether one end is made the north pole by magnetization or the south. At high temperatures the curve of energy versus magnetization has a simple $U$ shape that has the same rotational symmetry as the underlying equations [see illustration below]. The equilibrium state, the state of lowest energy at the bottom of the $U$, is also a state of zero magnetization, which shares this symmetry. On the other hand, when the temperature is lowered, the lowest point on the $U$-shaped curve humps upward so that the curve resembles a $W$ with rounded corners. The curve still has the same rotational symmetry as the underlying equations, but now the equilibrium state has a definite nonzero magnetization, which can be either north or south but which in either case no longer exhibits the rotational symmetry of the equations. We say in such cases that the symmetry is spontaneously broken. A tiny physicist living inside the magnet might not even know that the equations of the system have an underlying rotational symmetry, although we, with our superior perspective, find this easy to recognize. Reasoning by analogy, we see that a symmetry principle might thus be exactly true in a fundamental sense and yet not be visible at all in a table of elementary-particle masses..."
Weinberg (and independently Salam) suggested [28] such a broken symmetry group which contains the unbroken gauge symmetry group of electromagnetism with the massless photon and - as the photon's relatives - the massive charged (W±) and neutral (Z0) vector bosons associated with broken symmetries. And in 1971 t'Hooft [29] demonstrated that this theory is renormalizable.

This model consists of (with Aμ = photon field, Zμ = Z0 field)

1 doublet of left-handed leptons \( \frac{1}{2}(1 + \gamma_5) \left( \begin{array}{c} \nu_e \\ -e \end{array} \right) = : \psi_L \)

1 singlet of a right-handed lepton \( \frac{1}{2}(1 - \gamma_5) e = : \psi_R \)

1 isospin triplet \( \tilde{W}^± = \{ \tilde{W}^+, \tilde{W}^-, Z^- \} \mu \cos \theta + A_\mu \sin \theta \)

1 isospin singlet \( B^-_\mu = \{-Z^- \mu \sin \theta + A_\mu \cos \theta \} \)

with the following interaction between leptons and vector bosons:

\[
L = \bar{\psi}_L \gamma_\mu (g \cdot \gamma_5 \cdot \tilde{W}^-_\mu) \psi_L - g'' \bar{\psi}_R \gamma_\mu \psi_R B^-_\mu .
\] (14)

From the condition that the electromagnetic interaction for electrons

\[
(e_L \gamma_\mu e_L) A_\mu (-g \sin \theta - g' \cos \theta) - (e_R \gamma_\mu e_R) A_\mu g'' \cos \theta
\]

must be equal to \( e(e_L \gamma_\mu e_L + e_R \gamma_\mu e_R) A_\mu \),

follows: \( g \sin \theta + g' \cos \theta = g'' \cos \theta = -e \).

From the condition that the neutrino makes (in 1. order) no electromagnetic interactions

\[ g \sin \theta - g' \cos \theta = 0 \]

follows: \( g'/g = \tan \theta \)

and hence \( e = -2g \sin \theta = -\frac{2gg'}{\sqrt{g^2 + g'^2}} \). (15)

The Weinberg angle \( \theta \) is the only free parameter in this model and can be determined from the observed rate of \( \bar{\nu}_\mu e \) scattering (sect. 3.2.1) and after extension of the model to hadrons (implying further assumptions) also from \( \bar{\nu}_\mu + N \rightarrow \nu_\mu + \text{hadrons} \) (sect. 3.2.2).
1.9 Charm (1974)

Now we have weak neutral currents (and a nice model describing them), so in addition to the charged hadron currents eq. (3b) $J^+ = (\bar{p}n')$, $J^- = (n'\bar{p})$ with $W^\pm$ exchange, we also have

$$J^0_{\text{H}} = (pp) - (n'n')$$

with $Z^0$ exchange; however, the current

$$(n'n') = (nn) + (\lambda\lambda) + (n\lambda) + (\lambda n)(*)$$

contains also transitions between neutron and $\lambda$-quarks (eq. (13a)) which are strangeness changing ($\Delta S = 1$) neutral ($\Delta Q = 0$) currents, and these do not exist in nature (cf. page 15). The way out of this dilemma is the introduction of a fourth quark, $c$, called charmed quark [30] and to complete the universality triangle to a square (fig. 7) in which also the somewhat lost $\lambda'$-quark eq. (13b) finds its place. In this model the production of charmed particles by neutrinos is predicted which would be observable by the leptonic and/or hadronic decay of these particles into strange particles ($\Delta C = \Delta S$).

Now, the neutral current has still the terms $(pp)$, $(n'n')$, but in addition $(cc)$ and $(\lambda'\lambda')$

(*) $J^+ = (\bar{p}n')$ is short for the isospin raising current (see eq. (12))

$$J^+_a = \bar{p} \gamma_\alpha (1 + \gamma_5) n \cos \theta_c + \bar{p} \gamma_\alpha (1 + \gamma_5) \lambda \sin \theta_c$$

and

$$J^0_a = \bar{p} \gamma_\alpha (1 + \gamma_5) p - \cos^2 \theta_c n \gamma_\alpha (1 + \gamma_5) n - \sin^2 \theta_c \lambda \gamma_\alpha (1 + \gamma_5) \lambda - \sin \theta_c \cos \theta_c \left( n \gamma_\alpha (1 + \gamma_5) \lambda + \bar{p} \gamma_\alpha (1 + \gamma_5) n \right).$$
\[ J^0 = (pp) + (cc) - (n'n') - (\lambda'\lambda') \]
\[ \quad \downarrow (nn) + (\lambda\lambda) - (n\lambda) - (\lambda n) \]
\[ \quad (nn) + (\lambda\lambda) + (n\lambda) + (\lambda n) \]
\[ = (pp) + (cc) + 2(nn) + 2(\lambda\lambda) \]

with the effect that the strangeness changing transitions \((n\lambda)\) cancel (because of the minus sign in \(\lambda'\), see eq. (13b)).

As we know, in 1974 in a Brookhaven experiment studying \(p + Be \rightarrow e^+ + e^- + \text{anything} \) and in a SLAC experiment studying \(e^+ + e^- \rightarrow e^+ + e^- + \text{anything} \), peaks were discovered (Koch and Krammer lectures of this School) interpreted as \((cc)\) states \((J/\psi)\), and in 1976 at SLAC and DESY \(K^- \pi^+ \pi^+\) and \(K^- \pi^+\) peaks were interpreted as the charmed mesons \(D^+\) and \(D^0\). Also in 1974, the FNAL neutrino experiment reported di-muon events (sect. 3.3.1) interpreted as the production and subsequent leptonic decay of charmed particles

\[ \nu + N \rightarrow \mu^- + C + ... \]
\[ \quad \downarrow \mu^+ + \nu + ... \]

and bubble chamber neutrino experiments observed interactions (sect. 3.3.2) compatible with the same scheme but the charmed particle decaying into \(e^+ + \nu + ...\).

The four quarks have the following quantum numbers for charge, baryon number, isospin, strangeness and charm - all quarks appearing in three colours (cf. footnote p. 13) -

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>B</th>
<th>I</th>
<th>S</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>2/3</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>p</td>
<td>2/3</td>
<td>1/3</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>n</td>
<td>-1/3</td>
<td>1/3</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>\lambda</td>
<td>-1/3</td>
<td>1/3</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

and the production of a charmed baryon by neutrinos might be pictured as shown in fig. 8.
Production of a charmed baryon.

with the subsequent decay (immediately, \( \tau < 10^{-12} \) sec) of the charmed particle into a strange particle \((\Delta C = \Delta S)\) and \((l \nu_e)\) or hadrons.

In Appendix B a list of possible charmed particles is given, copied from ref. [31].

2. \(\nu\)-TECHNOLOGY

2.1 Principle of neutrino experiments

Because of the small neutrino cross section \(\sigma^\nu \approx 3\sigma^\bar{\nu} \approx 10^{-38} \) E(\(\text{GeV}\)) cm\(^2\) (page 9 and sect. 2.3.2) neutrino beams of high intensity and neutrino detectors of large mass are necessary to obtain reasonable event rates. For instance, in a detector of 1 ton \((N_N = 6 \times 10^{29}\) nucleons) traversed per accelerator pulse by \(10^{10}\) neutrinos \((\nu, 10^6 \) \(\nu\)'s/cm\(^2\)) of 1 GeV energy

\[
N_{\nu \text{ev}} = \sigma \cdot N_\nu \cdot N_N = 10^{-38} \cdot 10^6 \cdot 6 \cdot 10^{29} = .006 \text{ events,}
\]

i.e. every 170 pulses, a neutrino interaction would occur.

The source of neutrinos in accelerator neutrino beams is the weak decay of pions and kaons \((\pi + \mu + \nu_\mu, K + \mu + \nu_\mu, \pi + \mu(e) + \nu_\mu \nu_e)\) produced by protons in external targets (sect. 2.2.1) and focused by magnetic fields (sect. 2.2.2). The neutrinos are separated from the other particles produced in the target, from the parents which have not decayed and from the muons by means of several 1000 t of absorber material (shielding, sect. 2.2.4). The neutrino interactions are observed in large bubble chambers (sect. 2.3.2), counter arrangements and spark chambers (sect. 2.3.3),
or in nuclear emulsions (sect. 2.3.4), depending on the aim of the experiment. In addition, neutrino experiments require a good knowledge of the neutrino spectrum (absolute intensity as a function of energy) since the incident neutrino is unobservable.

In so called wide-band neutrino beams, essentially all parents produced in the target are used requiring special focusing devices with large momentum and angle acceptance (magnetic horn) and yielding neutrino spectra from lowest (several 100 MeV) to highest energy (close to proton energy). In so-called narrow-band "dichromatic" neutrino beams the parents are momentum selected ($\Delta p/p \sim 5\%$) by quadrupole lenses yielding a neutrino spectrum of two peaks, one from pions, the other one from kaons, as will become clear in sect. 2.2.3.

A schematical layout of neutrino experiments is shown in fig. 9.

![Fig. 9 Principle of neutrino experiments.](image)

### 2.2 Neutrino beams

#### 2.2.1 Production of neutrino parents

Pions and kaons are produced - along with other particles - in proton nucleus collisions in external targets onto which the ejected proton beam is focused, with nearly exponential distribution in momentum and angle.
Extensive measurements were done at 24 GeV proton energy [32] (and will have to be done at 400 GeV) in order to determine this distribution as accurately as possible.

In order to obtain maximum particle output, target shape and material must be optimized. It must be two to three proton absorption lengths long and its diameter must be fitted to the proton beam shape (accounting for multiple scattering of the protons), in order to keep π and K reabsorption small. Intra-nuclear absorption can be reduced by using material with small nuclei. In the CERN wide-band neutrino experiments 90 to 130 cm long boron carbide (B$_4$C) and beryllium targets of 3-8 mm diameter were used.

2.2.2 Focusing of v-parents

(a) Wide-band beams

Mesons are produced with a mean transverse momentum of about $p_T \approx 0.3$ GeV/c. Hence the natural divergence of a beam of 4-15 GeV pions (from 26 GeV protons) is 75 to 20 mrad and the parent beam width after about 100 m (length necessary for decay and shielding) is 2-8 m; in addition, the neutrino intensity is further reduced by the decay angle distribution. Since neutrino detectors - for practical reasons - cannot be much bigger than 1 or 2 m in radius, it is necessary to focus the particles produced in the target. This is achieved with rotationally symmetric magnetic fields [33], "magnetic horns", (principle shown in fig. 10) the

![Fig. 10 Principle of magnetic horn](image-url)
inner conductor of which can be shaped such that most of the production angles and momenta over the entire target length are accepted and bent towards the detector. Focusing efficiency is increased by using a second (and sometimes a third) lens of this type [34].

Since transverse momenta up to ~800 MeV/c must be focused, and

\[ B \ell = 0.033 p_t, \quad (B \text{ in kG, } \ell \text{ in m, } p_t = \text{ in MeV/c}), \]

\[ B = 0.2 \frac{I}{r}, \quad (I \text{ in kA, } r \text{ in cm}), \]

currents of several 100 kA are required. Hence these devices must be pulsed (fig. 11) and mechanically strong (pressure between end plates = 6.3 \((I/d)^2 \sim 13 \text{ t/m}^2\)). Figs 12(a) and (b) show one of the CERN magnetic horns.

\[ I \sim B \]

\[ I_0 \sim 340 \text{ kA} \]

\[ \text{Beam passage} \]

0 100 200 \(t/\mu\text{sec}\)

**Fig. 11** Typical horn current form

(b) **Narrow-band beams**

In principle, magnetic horns can also be shaped such that they focus only a small momentum band, but a cleaner selection and better adjustable angular divergence is achieved by using quadrupole beams. If the ejected proton beam is not pointing towards the detector (fig. 13) – as in the present narrow-band beam at the CERN SPS, wide-band beam background flux from parents decaying before the momentum selection is further reduced.
Fig. 12(a) Magnetic horn
Fig. 12(b) Horn in neutrino tunnel, target and its mounting on the righthand side.
2.2.3 *Decay of $\nu$-parents and $\nu$-spectra*

The exponential drop of neutrino spectra with energy is caused by the energy distribution of the parents of the target. The "fine structure" of neutrino spectra is given by the decay kinematics (fig. 14) and the relative abundance of parents and their decay branching ratios. Pions have a Q-value of 29 MeV and populate the lower energy region, kaons (Q-value 239 MeV) the higher energy region (whereas muons from both decays have energies up to the parent energy).

$$E_\nu = \frac{m^2 \cos^2 \theta_\nu}{2(E-p)}$$

$$E_\mu = \frac{m^2 \cos^2 \theta_\mu}{2(E-p)}$$

Fig. 13 Principle of narrow-band neutrino beam

Fig. 14 Two-body decay kinematics of $\pi$ and $K$
The three-body decay of the kaon, $K^0_{e3}$, is the high-energy source of electronic neutrinos in the neutrino beam, whereas the decay of muons from $\pi$ (and K) decay, $\mu^\pm \rightarrow e^\pm (\bar{\nu}_e) e^\mp (\nu_\mu)$, contributes the low energy electronic neutrinos. The neutrino spectra due to these decays are calculated, using the measured centre of mass energy ($E^*$) distribution for electrons

$$\frac{dN_e}{dE^*} \propto \frac{(E^* - E^2)}{1 - 2 E^*/m_K}$$

for $K^0_{e3}$ decay

$$E^*_{\text{max}} = \frac{2 m^2 - m^2_0}{2 m_K},$$

as the neutrino energy distribution and transforming it - properly weighted with the parent momentum distribution - into the laboratory system.

The composition of all components in wide-band $\nu$ and $\bar{\nu}$-beams is shown in table II. Resulting neutrino spectra for experiments at three proton accelerators (Argonne National Laboratory, CERN and FNAL) are represented in fig. 15.

From fig. 14, follows that the neutrino spectrum from momentum (and sign) selected parent beams (sect. 2.2.2 (b)) of energy $E$, consists of two peaks, one close to $E$ (from $K^0_{\mu 2}$-decay, the other close to $E/2$ (from $\pi^0_{\mu 2}$ decay), for neutrinos which traverse the detector around the beam axis ($\theta_v \approx 0$). Further away from the axis the two peaks shift towards lower energy ($\theta_v$ finite). Their width depends on $\Delta p/p$ and angular divergence of the parent beam.

2.2.4 Neutrino shielding

In order to shield the neutrino detector from all particles other than neutrinos, large amounts of absorber material must be placed between the end of the decay region and the detector. Hadrons (protons not absorbed in the target, pions, kaons not decayed, and other hadrons produced in the target) are absorbed sufficiently in 15-20 absorption lengths (e.g. 2-3 m steel). The ranges of the muons - being attenuated mainly by ionization loss ($\delta$-ray and pair production contribute only above $\sim 100 \text{ GeV}$)
TABLE II
Composition of wideband $\nu$ beam

<table>
<thead>
<tr>
<th>Positive particles focused</th>
<th>Negative particles focused</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parent</td>
<td>$%$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>$\sim 80$</td>
</tr>
<tr>
<td>$K^+$</td>
<td>$\sim 12$</td>
</tr>
<tr>
<td>$\pi^0 + \mu^+ + \nu_\mu$</td>
<td>3.2</td>
</tr>
<tr>
<td>$\pi^0 + e^+ + \nu_e$</td>
<td>4.8</td>
</tr>
<tr>
<td>$\mu^+$</td>
<td>$\sim 10$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Unfocused</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0 \sim 3$</td>
</tr>
<tr>
<td>$\pi^+ + \mu^- + \nu_\mu$</td>
</tr>
<tr>
<td>$\pi^+ + e^- + \nu_e$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Defocused</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-$</td>
</tr>
<tr>
<td>$\mu^-$</td>
</tr>
<tr>
<td>$\pi^0 + \mu^- + \nu_\mu$</td>
</tr>
<tr>
<td>$\pi^0 + e^- + \nu_e$</td>
</tr>
<tr>
<td>$e^- + \nu_\mu + \nu_e$</td>
</tr>
</tbody>
</table>

(*) Varying with energy.
Fig. 15 Examples of neutrino spectra from three accelerators. The insert shows the fraction of $\nu_e$ spectra for the 26 GeV CERN Neutrino Beam.
Fig. 16 Total neutrino event rates above various neutrino energies as a functions of decay and shielding length.
- determine the thickness of the shielding in beam direction, e.g. in steel about 0.7 m/GeV to which 10-15% have to be added to account for straggling (= fluctuations in energy loss).

Because of the finite divergence of neutrino beams it is necessary to use high density material \((dE/dx \sim p)\), in order to maximise the neutrino flux intensity at the detector. This is illustrated in fig. 16, where total event rates above various neutrino energies (assuming the \(\nu\) cross section to rise linearly with energy) are plotted as a function of shielding thickness for three different decay lengths.

The shielding width transverse to the beam direction is dictated by multiple scattering: multiply scattered muons must not escape by the side and back scatter into the neutrino detector. By keeping the decay tunnel diameter small (e.g. 1.2 m at the CERN SPS) - just big enough not to lose parent particles - and lining it with heavy shielding material, (steel), many muons are absorbed in the decay tunnel wall and the main shield need not be wider than 2 to 2.5 times the decay tunnel diameter.

2.2.5 Neutrino beam monitoring and spectrum determination

For neutrino experiments it is important to maximize the flux and to know its quantity and energy spectrum. Both require continuous monitoring of the beam. The proton beam intensity is usually monitored by so called beam current transformers (fig. 17).

Fig. 17 Principle of beam current transformer.
The proton beam position at the target can be monitored by the signals from secondary emission of split foils (or from thermocouples) arranged symmetrically to the target.

Knowing the $\pi$ and $K$-production by protons and the focusing properties of the parent beam, one can then predict in principle the neutrino spectrum in the detector plane. However, experimental factors like unobserved deficiencies in the focusing system, mismatch between proton beam and target, nuclear cascade effects in the target, $\pi$ and $K$ absorption and production in the decay tunnel make these predictions uncertain.

A more direct way to obtain the absolute neutrino spectrum is to measure continuously, throughout the whole neutrino experiment, the flux of muons produced along with the neutrinos by the focused $\pi$ and $K$ parents \cite{35}. These muon fluxes are measured in the shielding - after the hadrons have been absorbed - using ionization detectors (gas ionization chambers or surface barrier solid state detectors) in several depths and radii. The muon flux relative to the incident proton beam monitors the real target efficiency, its spatial distribution monitor beam symmetry, its distribution in depth yields via the momentum range relation, the neutrino energy spectrum and its quantity measures the absolute neutrino spectrum. However, due to the Q-values of $\pi$ and $K$ decay (fig. 14), about 90% of the measurable muons come from $\pi$-decay only, and precise information on the $K/\pi$ production ratio is necessary to obtain the high energy part of the neutrino spectrum (above $\sim$ 20% of the proton beam energy) which is entirely due to $\nu_K$ (fig. 14). In fig. 18 the distribution of such muon fluxes measured and calculated is shown. The calculated muon flux distribution is then fitted to the measured muon fluxes by varying the input pion spectrum (keeping the $K/\pi$ ratio constant). The best estimate for the neutrino spectrum is obtained from the modified $\pi$ and $K$ spectra which make the best fit between measured and calculated muon fluxes.
**Fig. 18** Muon fluxes in the steel shield of the CERN - 26 GeV - neutrino experiment as function of depth and radius. The calculation assumes particle production in a thin target. The fit accounts for thick target effects and other inefficiencies.
2.3 Neutrino detectors

2.3.1 The ideal neutrino detector

A neutrino detector should fulfill the following requirements:

(a) Large mass because of the small $\nu$ cross section, especially for rare processes.

---

Fig. 19  T: $D_2(H_2)$ Bubble chamber
for event production
vertex observation
detection of $\Lambda, K^0, \Sigma^+$, and spectator nucleon in $D_2$
measurement of low momentum large angle secondaries.

I: Charged hadron identifier
for $\pi^+/K^+/p$ separation.

C: Total absorption calorimeter
for detecting and measuring gammas, neutrons and $K^0_L$
identifying and measuring electrons
identifying muons.

M: Magnetic field
for identifying and measuring $K^0$, $\Lambda$, low momentum
electrons and hadrons in T
measuring hadron momenta in T and I
measuring muon momentum in T, I and C.
(b) Free nucleons (H₂) in order to avoid nuclear effects (secondary interaction of neutrino produced particles, binding energy, unknown state of motion of target nucleon due to Fermi momentum).

(c) Complete reconstruction of event since the incident neutrino energy and direction are unknown (even in narrow-band beams); this requires good vertex visibility (detection of Λ, K⁰ decay vertices, and to decide whether particles produced were primary or secondary) and detection, identification and measurement of all particles produced in the event.

Since no single detector technique fulfills all these requirements, such a detector would have to be a separate function hybrid system [36] according to the scheme shown in fig. 19.

2.3.2 Bubble chambers in magnetic fields (table III) [37]

They represent the closest approach to meet above requirements (b) and (c). Large hydrogen (and deuterium) bubble chambers have free (quasi-free) nucleons and good measurability (particle momentum determination from track curvature) but in general cannot distinguish between fast e, μ, π, K, have small γ and n detection probability and relatively small mass (max. 1 t). Possible extensions (partial hybridisation) are:

(i) External Muon Identification (EMI) by e.g. large area multi-wire proportional chambers placed at the downstream end of the bubble chamber after sufficient absorption material to absorb hadrons by strong interactions (fig. 20(a)).

(ii) Track Sensitive Target technique (TST) [38], essentially a H₂ (or D₂) bubble chamber inside a neon bubble chamber (fig. 20(b)), where the neon (due to the short radiation length (table III(b)) allows electron identification, some gamma detection and (due to the larger density) some neutron detection.

Heavy liquid bubble chambers (freon, neon, propane) have larger event rates (proportional to density), good electron identification,
Fig. 20  
(a) FNAL - 15 foot - Bubble Chamber with external muon identifier (EMI).

(b) BEBC (Big European Bubble Chamber) at CERN with Track Sensitive Target (TST) and EMI.
TABLE III

(a) Bubble chambers as ν-detectors

<table>
<thead>
<tr>
<th>Name</th>
<th>Laboratory</th>
<th>Dimensions</th>
<th>Volume $^3$ (fid.) m$^3$</th>
<th>Magn. field kG</th>
<th>liquids used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 HLB</td>
<td>CERN 63-65</td>
<td>1.15 ø x .5 deep</td>
<td>0.5 (0.27)</td>
<td>27</td>
<td>freon (CF$_3$Br)</td>
</tr>
<tr>
<td></td>
<td>CERN 67</td>
<td>1.15 ø x 1 deep</td>
<td>1.13 (0.66)</td>
<td>27</td>
<td>propane (C$_3$H$_8$)</td>
</tr>
<tr>
<td>7 foot</td>
<td>BNL 70</td>
<td>2.1 ø x 2.5 high</td>
<td>8 (?)</td>
<td>18</td>
<td>H$_2$, D$_2$</td>
</tr>
<tr>
<td>12 foot</td>
<td>ANL 72</td>
<td>3.8 ø x 1.9 high</td>
<td>20 (11)</td>
<td>18</td>
<td>H$_2$, D$_2$</td>
</tr>
<tr>
<td>Gargamelle</td>
<td>CERN 71-75</td>
<td>1.9 ø x 4.8 long</td>
<td>12 (6 or 3)</td>
<td>20</td>
<td>freon propane</td>
</tr>
<tr>
<td></td>
<td>(SPS 1977)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 foot</td>
<td>FNAL 74</td>
<td>3.8 ø sphere</td>
<td>30 (19)</td>
<td>30</td>
<td>H$_2$, Ne/H$_2$</td>
</tr>
<tr>
<td>SKAT</td>
<td>Serpukhov 1975</td>
<td>4.5 x 1.5 x 1</td>
<td>6.5 (?)</td>
<td>25</td>
<td>freon propane</td>
</tr>
<tr>
<td>BEBC</td>
<td>CERN/SPS 1976</td>
<td>3.7 ø x 2 high</td>
<td>30 (18)</td>
<td>35</td>
<td>Ne$_2$H$_2$, D$_2$</td>
</tr>
</tbody>
</table>

(b) Properties of bubble chamber liquids

<table>
<thead>
<tr>
<th>Liquid</th>
<th>density g/cm$^3$</th>
<th>radiation length cm</th>
<th>nuclear collision length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$</td>
<td>0.063</td>
<td>1000</td>
<td>~ 690</td>
</tr>
<tr>
<td>D$_2$</td>
<td>0.14</td>
<td>900</td>
<td>~ 322</td>
</tr>
<tr>
<td>Ne (21% at.)/H$_2$</td>
<td>0.27</td>
<td>115</td>
<td>~ 100</td>
</tr>
<tr>
<td>C$_3$H$_8$ (propane)</td>
<td>0.41</td>
<td>111</td>
<td>~ 134</td>
</tr>
<tr>
<td>Ne</td>
<td>1.2</td>
<td>24</td>
<td>~ 54</td>
</tr>
<tr>
<td>CF$_3$Fr (freon)</td>
<td>1.5</td>
<td>11</td>
<td>~ 50</td>
</tr>
</tbody>
</table>
gamma conversion and neutron detection, some muon/hadron distinction due to the shorter hadron interaction length (but EMI also needed) and up to about 1 GeV pion/proton distinction due to the difference in ionization loss, but the neutrinos interact with nucleons bound in nuclei and hence the kinematics of the interaction is obscured.

\[ K^+, K^- \] detection is only possible in case of characteristic decays and hence restricted to low energy kaons. The kinematics of neutrino interactions remain largely undetermined in case of outgoing neutrals (like in neutral current interactions with outgoing neutrinos).

2.3.3 Spark chambers, counter arrangements, calorimeters [39]

Spark chambers (for photographic track registration) or drift or multiwire proportional chambers (for electronic track registration and event analysis on-line) are sandwiched with high density material (iron plates, sometimes magnetized) to form large mass neutrino detectors (30-100 t, c.f. table IV, in the present counter experiment at the CERN SPS 1400 t). These detectors (example in fig. 21) allow depending on their design and trigger logic - large event rates, good muon/hadron distinction, good total energy measurement (calorimeter principle), but poor vertex visibility, no gamma-electron distinction and neutrino interactions only on bound nuclei.

![Fig. 21 FNAL-experiment 1A detector.](image-url)
2.3.4 Nuclear emulsions [40]

Nuclear emulsions have highest spatial resolution (several microns, compared to 100-300 \( \mu \) in bubble chambers) and can be used for the detection of short lived neutral particles (\( \tau \approx 10^{-13} \) s). Only small masses can be processed and scanning for events is tedious.

2.4 List of neutrino experiments 1961-1976

Table IV lists the neutrino experiments at the accelerators of Argonne National Laboratory (ANL) (12 GeV), CERN (26 GeV), Brookhaven National Laboratory (BNL) (30 GeV), Serpukhov (70 GeV) and Fermi National Accelerator Laboratory (FNAL) (400 GeV) since 1961, indicating the main beam features, the detector type (B = bubble chamber, S = spark chamber-counter set-up) and the main topics studied (which will be treated in sect. 3).

Fig. 22 shows the layout of the CERN SPS and the location of the next series of \( \nu \)-experiments at 400 GeV having just started.
Table IV
Accelerator ν-experiments until 1976

<table>
<thead>
<tr>
<th>Accelerator</th>
<th>Year</th>
<th>Primary proton beam energy intensity total Nr (GeV) (10^{11} ppp) (in 10^{17})</th>
<th>Neutrino beam π,K focusing spectrum determin.</th>
<th>Detector type mass material</th>
<th>TOTAL NUMBER OF EVENTS</th>
<th>Main results</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNL</td>
<td>61</td>
<td>15 3 3.5</td>
<td>- from π,K</td>
<td>S 10 Al</td>
<td>50 v</td>
<td>v_{\mu} + v_e</td>
</tr>
<tr>
<td></td>
<td>63</td>
<td>30 3 1.2</td>
<td>1 horn</td>
<td>S 67 Al</td>
<td></td>
<td>1 \pi</td>
</tr>
<tr>
<td>CERN</td>
<td>63/4</td>
<td>25 3-6 7.6</td>
<td>&quot; &quot;</td>
<td>B 0.75 freon</td>
<td>450 v</td>
<td>\sigma(\nu_n + \mu^- p), M_A</td>
</tr>
<tr>
<td>CERN</td>
<td>63-4</td>
<td>25 3-6 3.9/3.7</td>
<td>&quot; &quot;</td>
<td>S 65/90 Al, brass</td>
<td>10^4 v</td>
<td>m_w &gt; 2 GeV</td>
</tr>
<tr>
<td>CERN</td>
<td>65</td>
<td>25 7 3</td>
<td>&quot; &quot;</td>
<td>B 1.8 freon</td>
<td>25 v</td>
<td>\Delta s=1 &lt; \gamma_Cabibbo</td>
</tr>
<tr>
<td>ANL</td>
<td>65</td>
<td>12 10 3</td>
<td>&quot; &quot;</td>
<td>S 33 freon</td>
<td></td>
<td>\sigma(\nu_n + \mu^- p), M_A</td>
</tr>
<tr>
<td>CERN</td>
<td>67</td>
<td>21 7 7</td>
<td>horn + 2 refl. from \mu flux</td>
<td>B 0.5 C_3H_8</td>
<td>450 v</td>
<td>\sigma(\nu_\nu, M^{+++})</td>
</tr>
<tr>
<td></td>
<td>67</td>
<td>21 7 7</td>
<td>&quot; &quot;</td>
<td>S 13 C, Al, Fe, Pb</td>
<td>10^4 v</td>
<td>\sigma_{tot} \gamma E</td>
</tr>
<tr>
<td>ANL</td>
<td>71</td>
<td>12 11 8</td>
<td>1 horn</td>
<td>B 1.6 H_2</td>
<td>3.2 D_2</td>
<td>L_{conservation}</td>
</tr>
<tr>
<td>CERN</td>
<td>71-75</td>
<td>26 10-70 ~ 20 (\approx 50)</td>
<td>1 horn</td>
<td>B 12 freon</td>
<td>10^4 v</td>
<td>\sigma_{el}, NC</td>
</tr>
<tr>
<td>CERN</td>
<td>74/5</td>
<td>26 40-70</td>
<td>&quot; &quot;</td>
<td>B 4 C_3H_8</td>
<td>\nu, \overline{\nu}</td>
<td>\sigma_{tot}, NC, \mu^- e^+</td>
</tr>
<tr>
<td></td>
<td>74/5</td>
<td>26 40-70</td>
<td>&quot; &quot;</td>
<td>S 30 Al</td>
<td>4.10^4 \nu</td>
<td>\sigma_{tot}, NC, \nu^- e^- + \nu^+ e^-</td>
</tr>
<tr>
<td>FNAL-WB</td>
<td>72-75</td>
<td>300-400 20-130</td>
<td>1 horn</td>
<td>S \approx 90 liqu. scin. and steel</td>
<td>\nu, \overline{\nu}</td>
<td>\nu, NC, \sigma_{tot}</td>
</tr>
<tr>
<td>FNAL-NB</td>
<td>72-75</td>
<td>&quot;</td>
<td>quadrupoles</td>
<td>S \approx 30 steel</td>
<td>\nu, \overline{\nu}</td>
<td>\nu, NC, \sigma_{tot}</td>
</tr>
<tr>
<td>FNAL-15'</td>
<td>74-75</td>
<td>&quot;</td>
<td>1 &amp; 2 horns</td>
<td>B 1 H_2</td>
<td>1500 \nu, \overline{\nu}</td>
<td>\nu, NC</td>
</tr>
<tr>
<td>FNAL-15'</td>
<td>75</td>
<td>300 ~ 10 (\approx 10)</td>
<td>1 horn</td>
<td>B 5 N_/H_2</td>
<td>5000 v</td>
<td>\mu^- e^+, in progress</td>
</tr>
<tr>
<td>FNAL-15'</td>
<td>76</td>
<td>400</td>
<td>2 horns</td>
<td>B 12 Ne</td>
<td>\nu</td>
<td>in progress</td>
</tr>
<tr>
<td>SERP.</td>
<td>75-76</td>
<td>70 10</td>
<td>&quot; &quot;</td>
<td></td>
<td></td>
<td>\nu</td>
</tr>
</tbody>
</table>
3. \( \nu \)-NUMEROLOGY

Any event induced by neutral (non visible) particles in a detector exposed to a neutrino beam is a candidate for a neutrino interaction. There is strong evidence for these events to be neutrino events from their spatial and (in time recording electronic detectors) time distribution: the radial distribution is as expected from beam calculations, the flat longitudinal distribution corresponds to an infinitely long interaction length, the time distribution reflects the proton beam time structure.

From counting the neutrino events of certain topologies found on the bubble chamber or spark chamber film as a function of the event energy and knowing the neutrino spectrum one obtains cross sections for various interaction types (table V):

<table>
<thead>
<tr>
<th>( \nu )-beam leptons</th>
<th>( \nu )-beam hadrons</th>
<th>( \bar{\nu} )-beam leptons</th>
<th>( \bar{\nu} )-beam hadrons</th>
<th>Numbers to measure, hypotheses to check</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu^- )</td>
<td>( p )</td>
<td>( \mu^+ )</td>
<td>( n )</td>
<td>Quasi-elastic ( \nu )-scattering ( \nu + N \rightarrow \mu + N' ), nucleon is not a point as assumed in eq. 9, but has a structure measured as a form factor.</td>
</tr>
<tr>
<td>( \mu^- )</td>
<td>( \Lambda \pi^+ )</td>
<td>( \mu^+ )</td>
<td>( \Lambda )</td>
<td>(</td>
</tr>
<tr>
<td>( \mu^- )</td>
<td>any</td>
<td>( \mu^+ )</td>
<td>any</td>
<td>( \mu \Lambda \pi^+ ) checks ( \Delta S = \Delta Q ) rule (p. 12)</td>
</tr>
<tr>
<td>e</td>
<td>any</td>
<td>e^+</td>
<td>any</td>
<td>Total cross section, checks quark parton model (p. 14)</td>
</tr>
<tr>
<td>e</td>
<td>0</td>
<td>e^+</td>
<td>0</td>
<td>Excess over beam expected rate (table II) checks lepton number conservation (p. 11)</td>
</tr>
<tr>
<td>0</td>
<td>p</td>
<td>0</td>
<td>p</td>
<td>Elastic neutral current interaction with nucleon ( \nu p \rightarrow \nu p, \nu N \rightarrow \nu N )</td>
</tr>
<tr>
<td>0</td>
<td>any</td>
<td>0</td>
<td>any</td>
<td>Inelastic neutral current interaction ( \nu N \rightarrow \nu N' )</td>
</tr>
<tr>
<td>( \mu^- \mu^+ )</td>
<td>any</td>
<td>( \mu^+ \mu^- )</td>
<td>any</td>
<td>Checks presence of charmed quark (sect. 1.9, p. 20) or of new leptonic particles</td>
</tr>
<tr>
<td>( \mu^- \mu^+ )</td>
<td>any</td>
<td>( \mu^+ \mu^- )</td>
<td>any</td>
<td>Checks presence of charmed quark (sect. 1.9, p. 20) or of new leptonic particles</td>
</tr>
<tr>
<td>( \mu^- \mu^+ )</td>
<td>any</td>
<td>( \mu^+ \mu^- )</td>
<td>any</td>
<td>Checks presence of charmed quark (sect. 1.9, p. 20) or of new leptonic particles</td>
</tr>
<tr>
<td>( \mu^- \mu^+ )</td>
<td>any</td>
<td>&gt; 2</td>
<td>any</td>
<td>More quarks? Heavy leptons?</td>
</tr>
</tbody>
</table>
How to single out the various types of interactions from other interactions and from background and how to extract the numbers necessary for the development of theory, is the subject of this section.

3.1 Charged current interactions ($\nu + \mu^\pm$)

3.1.1 Quasi-elastic interactions $\nu + N \rightarrow \mu + N'$

Events are supposed to correspond to

$v + n \rightarrow \mu^- + p$ in $v$ beams,

$\bar{v} + p \rightarrow \mu^+ + n$ in $\bar{v}$ beams,

if they have the following topology:

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$\bar{v}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>one $\mu^-$ candidate</td>
<td>one $\mu^+$ candidate</td>
</tr>
<tr>
<td>one &quot;fast&quot; proton</td>
<td>one &quot;fast&quot; neutron</td>
</tr>
<tr>
<td>(kinetic energy $T &gt; 30$ MeV)</td>
<td></td>
</tr>
</tbody>
</table>

any number of low energy nucleons, $T < 30$ MeV, if the target nucleon is bound in heavy nuclei.

A muon candidate is a particle leaving the detector without having undergone a visible strong interaction. Neutrino energy and four-momentum transfer $q$ (fig. 23) are determined from the energy of all final state particles (from curvature or range measurement).

\[
\begin{align*}
E_v &= E_\mu + E_{N'} \\
q^2 &= (p_\nu - p_\mu)^2 \\
&= 2(E_{\nu}\mu - p_\nu p_\mu \cos \theta - m_\mu^2) \\
&\approx 4E_{\nu}\mu \sin^2(\theta/2)
\end{align*}
\]

Fig. 23 Kinematics of neutrino interaction, if Fermi motion of the target nucleon is neglected.
In order to reduce background events in the selected sample, cuts are applied: the events must be inside a well measurable (fiducial) volume, have more than a certain minimum energy (also because of larger spectrum uncertainty at low energy), and more than a minimum total forward momentum $p_X$ (charged hadrons entering the detector from the side simulating $\nu$-events have usually small $p_X$ (fig. 24). Other background sources like $\nu N + \nu \pi^- p$, where $\pi^-$ leaves the detector without interaction or $\nu N + \mu N' \pi$, where the $\pi$ is absorbed in the nucleus, must be statistically corrected for (few per cent). Fermi motion of the target nucleon and the Pauli exclusion principle in the target nucleus complicate further the analysis of this interaction in heavy liquid bubble chambers [41] or in spark chambers [42]. In the 12' bubble chamber at ANL the quasi-elastic interactions were analysed in $H_2$ and $D_2$ [43]. The resulting cross section as a function of neutrino energy $E_\nu$ is shown in fig. 25.

![Figure 24](image)

**Fig. 24** (a) Quasi-elastic neutrino event.
(b) Incoming $\pi^+$ scattering, direction of flight signed by delta rays.
(c) Incoming $\pi^+$, direction not signed.

![Figure 25](image)

**Fig. 25** Elastic cross sections $\sigma(\nu + n \rightarrow \mu^- + p)$ and $\sigma(\bar{\nu} + p + \mu^+ + n)$ as determined at ANL and CERN.
The fact that it does not show the linear increase with energy as given by eq. (10) is due to the nucleon structure.

In order to calculate the theoretical cross section for this process, the (V-A)–matrix element (9) has to be extended to its most general Lorentz invariant expression, which might be written like this [44]:

\[
\mathcal{M} = \frac{G}{\sqrt{2}} \left\{ \mu - \gamma_\alpha (1+\gamma_5) v_\mu \right\} \left\{ \bar{p}(v_\alpha + \gamma_5 A_\alpha)n \right\}, \tag{17(a)}
\]

where

\[
v_\alpha = \gamma_\alpha g_V + i(p + p_n) f_V + iq h_V \tag{17(b)}
\]

\[
A_\alpha = \gamma_\alpha g_A + i(p + p_n) f_A + iq h_A \tag{17(c)}
\]

are the vector and axial vector part of the weak hadronic current and the six form-factors \(g_{V,A} f_{V,A}\) and \(h_{V,A}\) contain the dynamics of strong interactions. We can now use some hypotheses about the hadronic weak current to reduce the number of form factors to be measured to one: (i) assuming time reversal invariance they must be all real; (ii) G-parity conservation (invariance under \(G = C e^{i\pi I_2}\), \(C\) = charge conjugation operator and \(I_2 = \) rotation by \(180^\circ\) around second axis of isospin) implies existence of so-called first class currents only, namely those parts of \(V^\alpha\) transforming like \(GV G^{-1} = V^\alpha\) and of \(A^\alpha\) transforming like \(GA G^{-1} = -A^\alpha\), which in turn implies \(f_A = h_V = 0\); (iii) hypothesis of conserved isovector current. In sect. 1.2 we saw that weak interactions are universal, i.e. have the same coupling constant; consider especially the coupling constant for the pure vector transition \(1^+_0 \rightarrow 1^+_N\) which is equal to that of \(\mu\)-decay; this reminds of the remarkable fact that the absolute value of the proton charge is equal to the electron charge which corresponds to the conservation of the electromagnetic current and leads to the hypothesis that the weak hadronic vector current \(V^\alpha\) is conserved. And since \(V^\alpha\) does nothing else but turn a proton into a neutron \((1^+_0 \rightarrow 1^+_N, \Delta I = 1)\), it must be an \(I = 1\) object: \(V^\alpha = (V_{\alpha 1}, V_{\alpha 2}, V_{\alpha 3})\) with \(V_{\alpha 1}^2 p = V_{\alpha 2}^2 n = 0\) and \(V_{\alpha 3}^2 p = -V_{\alpha 3}^2 n\). Then, \(V_{\alpha 3}\) is a conserved isovector neutral current connecting \(p\) with \(p\) and \(n\) with \(n\), which is the well known electromagnetic current. So, having the electromagnetic current and the weak hadronic current as members of the same
isotriplet, allows one to assume that also the form factors of the weak hadron current are the same as those of the electromagnetic hadron current and hence, are related to the electric and magnetic form factors $F_Q$ and $F_M$ in the Rosenbluth-formula by

$$g_V = F_Q(q^2) + (\mu_p - \mu_n) F_M(q^2)$$
$$f_V = \frac{\mu_p - \mu_n}{2m} F_M(q^2),$$

whence the term "weak magnetism" for $f_V$. $\mu_p = 1.52$ and $\mu_n = -1.79$ are the magnetic moments of proton and neutron, respectively, in Bohr magnetons.

(iv) hypothesis of Partially Conserved Axial vector Current (PCAC). The axial vector current $A_\alpha$, cannot be conserved (otherwise the $\pi$ could not decay: $\delta A_\alpha / \delta x_\alpha = 0$), but partially, i.e. the form factor $h_A$ is dominated by $\pi$ exchange, which leads to a negligible contribution (2-3%) to the neutrino cross section, i.e. $h_A = 0$ (however, in $\mu$ capture it is measurable!).

So, eq. (17) reduces to

$$\mathcal{M} = \frac{G}{\sqrt{2}} \left( \mu \gamma_\alpha (1 + \gamma_5) \nu \right) \left( p [ \gamma_\alpha g_V + (p_\alpha + n_\alpha) f_V + \gamma_\alpha \gamma_5 g_A ] n \right),$$

which leads to [45]

$$\frac{d\sigma^{\nu, \nu}}{dq^2} = \frac{a^2}{4\pi} \left[ A + B \frac{q^2}{E^2} + C \frac{q^2}{E^2} \right]$$

with

$$A = 2(g_A^2 + g_V^2) - (4m^2 + q^2) f_V^2 - 8m g_V f_V$$
$$B = -\frac{1}{m} \left( g_A + g_V \right)^2 - \frac{1}{m} (4m^2 + q^2) f_V^2 + 4 g_V f_V$$
$$C = \frac{1}{2} \left( g_A^2 - g_V^2 \right) + \frac{1}{4m^2} (g_A + g_V)^2 - \frac{1}{2} (4m^2 + q^2) f_V^2 + 2m g_V f_V,$$

where the only unknown quantity is the axial vector form factor $g_A(q^2)$.

It is usually parametrized in the same way as $F_Q$ and $F_M = F(q^2) = \left(1 + q^2/m_V^2\right)^{-2}$, namely
and $g_A(0) = 1.23$ is obtained from neutron-$\beta$-decay. The interference term $\mp g_A g_V$ describes the difference between $\nu$ and $\bar{\nu}$ cross sections (parity violation, corresponding to the pseudo-scalar of eq. (7)).

From fitting eq. (20) to the experimental cross sections (fig. 25) one obtains for $M_A$

<table>
<thead>
<tr>
<th>from $\sigma(E)$</th>
<th>from $d\sigma/dq^2$ (flux independent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>$\bar{\nu}$</td>
</tr>
<tr>
<td>CERN-Gargamelle</td>
<td>0.88 ± .19</td>
</tr>
<tr>
<td>ANL - 12 ft</td>
<td>0.97 ± .16</td>
</tr>
</tbody>
</table>

which might be compared to the eN value for $M_\nu = 0.84$ GeV/c$^2$.

Eq. (20) shows that at $q^2 = 0$ and for $E_\nu \rightarrow \infty$ the $\nu$ and $\bar{\nu}$ elastic cross sections are equal

$$
\frac{d\sigma}{dt} (q^2 = 0) = \frac{G^2}{2\pi} \left( g_A^2 (0) + g_V^2 (0) + 4m^2 f_V^2 - 4m g_V f_V \right)
$$

$$
= \frac{G^2}{2\pi} \left( g_A^2 (0) + 1 \right) = 0.4 \; G^2 \approx 2 \times 10^{-38} \; \text{cm}^2/(\text{GeV/c})^2,
$$

using eq. (18) at $q^2 = 0$

$$
g_V (0) = 1 + \Delta \mu, \; f_V (0) = \Delta \mu/2m
$$

and

$$
g_A (0) = 1.23.
$$

d$\sigma^{\nu, \bar{\nu}}/dt (E \rightarrow \infty) = (G^2/2\pi)A$ and, integrated, assuming the above parametrization for the form factor, $\sigma(\nu N \rightarrow \mu N', E \rightarrow \infty) \approx 7 \times 10^{-38} \; \text{cm}^2$, a constant value, of which about 30% is due to weak magnetism.

A similar analysis of events selected for the topology $\mu^- \pi^+ p$ yields information about the weak transition $p \rightarrow N^{*+} (\Delta 1232)$
\[ \nu + p + \mu^- + N^{*++} \rightarrow p + \pi^+ \]

which is the inverse \( N^{*++} \) \( \beta \)-decay. It is a \( SU_3 \) decuplet-\( SU_3 \) octet transition (fig. 3) and involves eight form factors. For details of the analysis and theoretical interpretation of the data (fig. 26) the reader is referred to the literature [45].

![Graph](image)

**Fig. 26** \( \nu + p + \mu^- + \Delta^{++} \)

cross section.

### 3.1.2 Strangeness changing processes

Some neutrino interactions (a few percent, increasing with energy) show a strange particle in the final state, i.e. a decay of a \( \Lambda + p \pi^- \), or a \( K^0_s + \pi^+ \pi^- \), which are easily recognisable by their \( V \) shape topology. Many of these are due to associated production of two strange particles of opposite strangeness (\( \Delta S = 0 \) process), one of which is not detected (\( K^- , K^+ \) which look like \( \pi^- , \pi^+ \), or \( K^0_L \) which escapes). In the Gargamelle experiment [18] it has been tried to study the quasi-elastic process

\[ \bar{\nu} + p + \mu^+ + \Lambda \]

considering the following topologies:

<table>
<thead>
<tr>
<th></th>
<th>( \mu^+ \Lambda )</th>
<th>( \mu^+ \Lambda + p (&lt;30\text{MeV}) )</th>
<th>( \mu^+ \Lambda + \gamma )</th>
<th>( \mu^+ \Lambda + \pi^+, p )</th>
<th>( \mu^+ \gamma + K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number obs.</td>
<td>10</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Most likely</td>
<td>( \Delta S = 1 )</td>
<td>( \Delta S = 1 ) and</td>
<td>( \Delta S = 1 )</td>
<td>( \Delta S = 1 )</td>
<td>associated</td>
</tr>
<tr>
<td>Interpretation</td>
<td></td>
<td>evaporation</td>
<td>( (\Lambda \gamma = \Xi^0) )</td>
<td>inelastic or</td>
<td>production</td>
</tr>
<tr>
<td></td>
<td></td>
<td>proton</td>
<td></td>
<td>assoc. prod.</td>
<td></td>
</tr>
</tbody>
</table>
Taking the first two columns as quasi-elastic Λ-production, estimating from the last column the associated production background (1.5 ± 1.5 events), correcting for losses due to scanning (ν 10%), short lifetimes (ν 20% decay too close to vertex), unseen decay mode (33% Λ → π°n) one obtains 23+12 −5 single Λ-events. From these and the $\bar{\nu}$ flux a cross section of

$$\sigma(\bar{\nu} + p + \mu^+ + \Lambda) = (1.3^+ .9) = 10^{-40} \text{ cm}^2/\text{proton}$$

is estimated assuming Λ-absorption and $\Sigma^0$-Λ conversion in the nucleus to cancel.

This is not in contradiction to the Cabibbo theory (sect. 1.6) [46]. An analysis using more statistics is in progress. In order to test the selection rules underlying the Cabibbo scheme eq. (12), more detailed experiments will have to be done.

### 3.1.3 Total neutrino cross sections: $\nu + N + \mu +$ anything

These have been measured so far in three experiments (cf. table IV): in Gargamelle at CERN [22,47] from about 4000 $\nu$ and 3000 $\bar{\nu}$ events (between 1 and 15 GeV) and $\nu$ spectra measured via muon fluxes (sect. 2.2.5), in the 15 ft bubble chamber at FNAL [48] from about 1000 $\nu$ and 500 $\bar{\nu}$ events between 10 and 200 GeV and $\nu$ spectra calculated from predicted $\mu$ and K production data, and in spark chamber-calorimeter experiments [49-50] at FNAL. In order to arrive at the final result (October 1975 status is compiled in fig. 27), several corrections have to be applied depending on the type of experiment.

In the Gargamelle experiment, for example, where the event energy is obtained from measuring the...

---

**Fig. 27** Total cross sections for neutrinos and antineutrinos
momentum (by curvature or range) of all tracks, it has to be corrected for missing energy, the final event sample has to be corrected for background events from opposite sign neutrinos and neutral current interactions; in addition, the true energy distribution of events is distorted: due to the neutrino spectrum shape and the energy measurement error (≈ 15%) events are piled into the region of the spectrum bend from lower energies (fig. 28). This effect has to be folded out.

![Graph showing the effect of neutrino spectrum shape and energy measurement error on the event distribution.](image)

**Fig. 28** Effect of neutrino spectrum shape and energy measurement error (≈ 15%) on the event distribution.

Cross sections from the Gargamelle experiment are still being analyzed, and new experiments (200 GeV narrow-band neutrino beam into neon filled BEBC and 1400 ton iron-scintillator calorimeter at CERN) are under way. Good evidence, however, from published results is that

\[
\sigma^{\nu\bar{\nu}}_{\text{tot}} \text{ rise linearly with energy}
\]

and

\[
\sigma^{\nu} = (0.8 \pm 0.1) \times 10^{-38} \text{ E(GeV) cm}^2
\]

\[
\bar{\sigma}^{\nu} = (0.3 \pm 0.05) \times 10^{-38} \text{ E(GeV) cm}^2
\]

\[
R = \frac{\sigma^{\nu}}{\sigma^{\bar{\nu}}} = 0.37.
\]

What can be learnt from these numbers?

The most general expression for the total cross section is
\( \sigma_{\nu \bar{\nu}} \sim G^2 \sum_{\text{spins}} \sum_X \left| \left( \mu \gamma_\alpha (1 + \gamma_5) \nu \right) \langle X | J | N \rangle \right|^2 \delta(p_N^+ + p_{\nu}^- - p_{\bar{\nu}}^-) \)

\[ = G^2 \text{ (Lepton tensor } L_{\alpha \beta} \text{)(Hadron tensor } H_{\alpha \beta} \text{)} \tag{24} \]

with the kinematical quantities defined in fig. 29.

\[ q^2 = (p_{\nu}^- - p_{\mu}^-)^2 = 4 E E \mu \sin^2 \theta_\mu \]

\[ \nu = E - E_\mu = \frac{p \cdot q}{m} \]

\[ m = \text{nucleon mass} \]

**Fig. 29** Kinematics of inelastic \( \nu \)-scattering

\( L_{\alpha \beta} \) is known (eq. (9))

\[ = \Sigma_j j^*_\alpha j_\beta \]

\[ = \text{Tr} \not{p} \gamma_\alpha (1 + \gamma_5) \not{p} + m_\mu \gamma_\beta (1 + \gamma_5) \]

\[ = 8 (p_\alpha p'_\beta + p_\beta p'_\alpha - p \cdot p' \delta_{\alpha \beta} + \imath \epsilon_{\alpha \beta \rho \sigma} p_\rho p'_\sigma) \]

\( H_{\alpha \beta} \) is unknown, and its most general Lorentz invariant form is \([44(\text{a})\text{and 51}]\)

\[ = \delta_{\alpha \beta} W_1(q^2, \nu) + p_\alpha p_\beta W_2(q^2, \nu) - \frac{i}{2} \epsilon_{\alpha \beta \rho \sigma} W_3(q^2, \nu) \]

+ terms which after contraction with \( L_{\alpha \beta} \) are of order \( m \).

Both tensors contracted yields

\[ \frac{d^2 \sigma_{\nu \bar{\nu}}}{dq^2 d\nu} = \frac{G^2 E_\mu E}{2\pi m E} \left[ W_1 \cdot 2 \cdot \sin^2 \theta/2 + W_2 \cos^2 \theta/2 + \frac{E + E_\mu}{m} \sin^2 \theta \cdot W_3 \right]. \tag{25} \]
$W_1, 2, 3$ are structure functions and depend on the only two Lorentz scalars $q^2$ and $\nu$. Their relation to the (real) form factors defined in the hadron current (before squaring) in the quasi-elastic case are

$$W_1 = g_A^2 + \frac{q^2}{4m^2} (g_A^2 + g_V^2)$$

$$W_2 = g_A^2 + (g_V^2 - 2m f_V)^2 + q^2 f_V^2$$

$$W_3 = -2 g_A g_V (VA \text{ interference term})$$ (26)

Charge symmetry requires

$$W_i^{\nu p} = W_i^{\nu n}, W_i^{\nu n} = W_i^{\nu p}$$ (27)

Deep inelastic (large $q^2, \nu$) scattering of electrons off nucleons has shown that $\nu W_2$ does not depend on $q^2$ and $\nu$ separately, but only on their ratio; that is scaling invariance (predicted by Bjorken). Defining

$$x = \frac{q^2}{2m\nu}, y = \frac{\nu E}{m}, \text{ i.e. } xy = \frac{2E}{m} \sin^2 \theta/2$$ (28)

and

$$W_1(q^2, \nu) \to F_1(x)$$

$$\frac{\nu W_{2,3}(q^2, \nu)}{m} \to F_{2,3}(x)$$ (29)

one can re-write eq. (25)

$$\frac{d^2\sigma}{dx dy} = \frac{G^2_{\text{ME}}}{\pi} \left[ \frac{y^2}{2} (2x F_1(x)) + (1-y) F_2(x) \pm y\left(1-\frac{y}{2}\right)(x F_3(x)) \right]$$, (30)

which yields by integration a total cross section rising linearly with $E$

$$\sigma_{\nu, \bar{\nu}} = \frac{G^2_{\text{ME}}}{\pi} \left\{ \frac{1}{6} \int 2xF_1(x)dx + \frac{1}{2} \int F_2(x)dx \pm \frac{1}{3} \int xF_3(x)dx \right\}$$

$$= \frac{G^2_{\text{ME}}}{6\pi} \int F_2(x)dx \left[ 3 + A \pm 2B \right]$$.

Using the numerical results of eq. (23) one finds
from $$\sigma^\nu - \sigma^\bar{\nu} = \frac{G^2 mE}{\pi} \cdot \frac{2}{3} \cdot \int xF_3(x)dx$$

$$\int xF_3(x)dx = 0.5 \pm .03 = B \int F_2(x)dx$$

(32)

and from

$$\sigma^\nu + \sigma^\bar{\nu} = \frac{G^2 mE}{\pi} \left( \frac{1}{3} \int 2xF_1 + \int F_2 \right)$$

$$= \frac{G^2 mE}{\pi} \int F_2(x)dx \left( 1 + \frac{1}{3} A \right)$$

(33)

together with

$$R = \frac{\sigma^\nu}{\sigma^\bar{\nu}} = \frac{3 + A - 2B}{3 + A - 2B} = .37, \text{ i.e.}$$

(34)

$$(.9 \pm .04) \leq A \leq 1, \text{ which means}$$

$$\int F_2(x)dx \approx \int 2xF_1(x)dx,$$

(35)

the following bounds for $$\int F_2(x)dx$$

$$(.54 \pm .03) \leq \int F_2(x)dx \leq (.56 \pm .03).$$

(36)

The structure functions obtain a descriptive interpretation in the quark parton model (p. 14), which in its "naive" form states that the nucleon consists of quasi-free point-like constituents of a defined small mass with momentum $$x \cdot p_N$$, which have the quark properties (p. 20). The scaling variable $$x$$ is then the fraction of the nucleon momentum carried by one quark and $$\nu$$ scattering happens on the quarks $$Q_i$$ (fig. 30).

$$\left( xp_N \right)^2 = \left( q + xp_N \right)^2$$

$$= q^2 + 2xpN \cdot q + \left( xp_N \right)^2$$

which yields with $$\nu m = pq$$ (fig. 28)

$$x = q^2/2mv$$

Fig. 30 Interpretation of $$x$$ in quark parton model.
For these interactions the cross section formulae for point-like fermion (p. 9) apply

\[
\frac{d\sigma}{dy} = \frac{G^2}{\pi} s^\nu Q = \frac{G^2}{\pi} x s^\nu p = \frac{2G^2 mE}{\pi} x
\]  

(37)

and the cross sections for \( \nu \)-scattering off nucleons \((p,n)\) become

\[
\frac{d\sigma}{dy} = \frac{G^2}{\pi} s^\nu (1-y)^2 = \frac{G^2}{\pi} x s^\nu p(1-y)^2 = \frac{2G^2 mE}{\pi} x (1-y)^2
\]

(38)

where \( Q_i(x), (\bar{Q}_i(x)) \) are the probabilities to find quarks (antiquarks) of type \( i \) with momentum \( xp_N \) and mass \( m_x \).

On the other hand, eq. (30) becomes with eq. (35)

\[
\frac{d^2\sigma}{dx dy} = \frac{G^2}{2\pi} \left[ (F_2 \pm xF_3) + (F_2 \mp xF_3)(1-y)^2 \right]
\]  

(39)

and by comparison with eq. (38), \( F_2 \) and \( xF_3 \) can be interpreted as

\[
F_2(x) = 2 x Q_i(x) + 2 x \bar{Q}_i(x)
\]

\[
xF_3(x) = 2 x Q_i(x) - 2 x \bar{Q}_i(x)
\]

i.e., as the sum and the difference of parton and antiparton densities.

Without going into further detail the following conclusions can be drawn from the numbers (23) within the context of this picture
(a) From the near equality of eqs (32) and (36)

\[
\frac{Q_i}{Q_i + Q_i} = 0.05 \pm 0.02
\]

i.e. there is nearly no antimatter in the nucleon.

(b) From comparing \(\sigma^v + \sigma^\bar{v} \approx 1.1\) with \(\int F_2 dx = \int (Q + \bar{Q})dx \approx 0.55\), it follows that only half of the nucleon momentum is carried by charged partons. The other half is usually ascribed to quark binding neutral gluons.

(c) From the facts that the equivalent analysis for eN scattering shows the same result (i.e. that only half of the nucleon momentum is carried by active quarks), and electrons should also scatter off strange and charmed quarks (no \(\sin^2\Theta_c\) suppression), it can be concluded that there are also less than 10% \(\lambda\) and \(c\) quarks in matter.

(d) From eqs (37) and (38) and (a), it is expected that

\[
\frac{d\sigma}{dy} \sim \begin{cases} 
\text{const for } v \\
(1-y)^2 \text{ for } \bar{v}.
\end{cases}
\]

However, a recent counter experiment at FNAL indicates deviations from this simple model: the total cross section ratio \(\sigma^\bar{v}/\sigma^v\) seems to become significantly larger than 1/3 at higher energy (fig. 31 (a)) and the distribution in energy transfer from anti-neutrino to hadron does not seem to follow the simple law \(d\sigma/dy \sim (1-y)^2\) at higher energies (fig. 31 (b) and (f)). See, however, ref. [52].

Fig. 31 (a) Total cross section ratio obtained in one of the counter experiments at FNAL (E1A).
3.1.4 Electron neutrino interactions: $\nu_e + N \rightarrow e + \text{anything}$

From table II and fig. 15 it is expected that of the order of 1% of the interactions in a neutrino experiment should be induced by electron neutrinos and - according to sect. 1.5 - lead to electrons ($\nu_e$) or positrons ($\bar{\nu}_e$) in the final state. At present, electrons are best identified in heavy liquid bubble chambers (sect. 2.3.2). In the Gargamelle (freon) neutrino experiments events with electrons (positrons) have been observed and analysed in terms of total cross sections. The first results [53] from about 50 e-events indicate that the total cross section for $\nu_e$ is the same as for $\nu\mu$ (in agreement with $\mu$-e-universality). The number of events also agrees with muon number conservation (sect. 1.5), i.e. $\nu\mu$ do not turn into e. In addition, the e-events at low energy which are mainly induced by $\nu_e$'s from muon decay can be used to check whether the lepton number is conserved additively (allowing only $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$, hence only e -events in neutrino beam) or multiplicatively (allowing also $\mu^+ \rightarrow e^+ + \nu_\mu + \bar{\nu}_e$, hence 50% e - and 50% e + events at low energy). From the small number of events obtained so far complete multiplicativity can be excluded (but not a small admixture).
3.2 Neutral current interactions ($\nu + \bar{\nu}$)

3.2.1 $\bar{\nu}_\mu + e^- + \nu_\mu + e^-$

Two million pictures obtained with the CERN heavy liquid bubble chamber Gargamelle exposed to an antineutrino beam (using $4.7 \times 10^{18}$ protons) were double scanned for events showing nothing but a single electron. Three events were found [23], one of which is shown in fig. 32 (a).

Fig. 32 (a) Single electron corresponding probably to $\bar{\nu}_\mu + e^- + \nu_\mu + e^-$. 
Processes known so far which could produce single electrons, like

\[ \nu_e + N + e^- + p, p \text{ not visible} \]

\[ (\bar{\nu}_e + e^-) \rightarrow (\bar{\nu}_e + e^-) \]

\[ \gamma + e^+ e^- \text{ very asymmetrically (e$^+$ invisible)} \]

\[ \gamma + e^- \rightarrow \gamma + e^- \text{ (Compton electron)} \]

can at most account for 0.44 events. Therefore, and because the lepton number is conserved, it is concluded that the three events are due to the neutral current process ($Z^0$ exchange, p. 15)

\[ \bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^- . \]

The three events, after background subtraction and correction for losses (e.g. electrons radiating so early that they look like converted gammas) correspond to a cross section \( (1.4 \pm 2.1) \times 10^{-41} \text{ cm}^2 \text{/electron} \). This can be analyzed in terms of the Weinberg model (p. 18), which predicts the following matrix element

\[ \mathcal{M} = \frac{G}{\sqrt{2}} \left\{ \bar{\nu}_\alpha (1 + \gamma_5) \nu \right\} \left\{ \bar{e} \gamma_\alpha (g_\nu + g_A \gamma_5) e \right\} \]

and hence

\[ \frac{d\sigma}{dy} = \frac{G_m^2 e^2 \nu}{2\pi} \left[ (g_\nu - g_A)^2 + (1-y)^2 (g_\nu + g_A)^2 \right] \]

with \( g_\nu = -\frac{1}{2} + 2\sin^2 \theta, \ g_A = -\frac{1}{2} \). For the parameter \( \theta \) (Weinberg angle) the following result is found: \( 0.1 < \sin^2 \theta < 0.4 \).

3.2.2 \( \nu + N \rightarrow \nu + \text{hadrons} \)

In order to search for this process events have to be selected which have no charged lepton in the final state. Then, it has to be shown that these events are not induced by incoming neutral hadrons. This was done for the first time in 1973 in the Gargamelle neutrino experiment [24] at the CERN 26 GeV accelerator, and confirmed in a counter-spark chamber experiments [25] at the FNAL 300-400 GeV accelerator.
In heavy liquid bubble chamber the relatively short hadron interaction length (table III, Gargamelle was filled with freon) allows good muon/hadron distinction. About 20% of the interactions observed in the neutrino beam and about 40% in the antineutrino beam had no muon candidates (example in fig. 32 (b). How can one prove that these events are not induced by incoming

Fig. 32 (b) Example of hadronic neutral current interaction.
neutrons? Firstly, by the spatial distribution of the events. Neutron induced events would fall off exponentially, whereas the observed events have distributions similar to the charged current events (fig. 33).

Fig. 33 Distribution in $x$ (beam direction) and $r^2$ ($r =$ distance from beam axis) of the neutral current events relative to the charged current events found in Gargamelle.

Secondly, by the energy distribution of the events. Neutron induced events would mainly come from neutrino produced neutrons, i.e. tertiary particles having a softer energy spectrum than neutrino induced events.

Fig. 34 Distributions of visible energy of the neutral current events and of the hadronic energy of the charged current events.
Fig. 34 demonstrates the similarity between the distribution of total visible energy of events without lepton candidate and that of hadronic energy (= total visible energy minus muon energy) of events with muon candidate. Fig. 34 suggests that in the events without a muon candidate the incoming neutrino transfers about the same energy to the hadronic system as in the events with a muon candidate and that in the first case it comes out of the interaction as a neutrino, in the second case as a muon.

Thirdly, by Monte-Carlo calculations of background from neutrons produced by neutrinos in the material around the bubble chamber, the only possible source of neutrons, since the beam region is well shielded. These calculations show that such background cannot account for more than 15-20% of the events with (hadronic) energy of more than 1 GeV.

The resulting ratios of neutral current (NC) to charged current (CC) interaction rates in neutrino ($R_{\nu}$) and antineutrino ($R_{\bar{\nu}}$) beams can be analyzed in terms of the Weinberg-Salam model (p. 18) which describes the neutral current interaction by the matrix element

$$M = \frac{G}{\sqrt{2}} (\bar{\nu} \gamma_\alpha (1 + \gamma_5) \nu) (j_\alpha^0)$$

and

$$j_\alpha^0 = j_\alpha^3 - 2 \sin^2 \theta_W \cdot j_{1\text{.el.m.}} = A_\alpha^3 + (1 - 2 \sin^2 \theta_W) v_\alpha^3.$$
In order to compare the experimental NC/CC ratios $R_{\nu} = 0.25 \pm 0.04$ for neutrinos and $0.56 \pm 0.10$ for antineutrinos with this theory, corrections must be made for the 1 GeV cut in hadronic energy, and that is model dependent. Assuming scaling, V-A theory for CC, and neutrino scattering off point like, spin-$\frac{1}{2}$ quarks and using the results of sect. 3.2.3, the ratios become $0.26 \pm 0.04$ ($\nu$) and $0.39 \pm 0.06$ ($\bar{\nu}$), without cut in hadronic energy. Table VI shows these results together with those of the two FNAL experiments. From a fit to the theoretical prediction (fig. 35), one obtains for the mixing parameter $\sin^2 \theta W \sim 1/3$, in agreement with the $\bar{\nu}_e \rightarrow \nu_e$ result.

**TABLE VI**

NC/CC rates from various experiments in June 1976 [53]

<table>
<thead>
<tr>
<th>Experiments</th>
<th>$R^\nu$</th>
<th>$R^{\bar{\nu}}$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gargamelle</td>
<td>$0.26 \pm 0.04$</td>
<td>$0.39 \pm 0.06$</td>
<td>$&lt;E&gt; \sim 2 \text{ GeV}, E_h \geq 1 \text{ GeV}$</td>
</tr>
<tr>
<td>HPWF</td>
<td>$0.29 \pm 0.04$</td>
<td>$0.39 \pm 0.10$</td>
<td>$&lt;E^\nu&gt; = 53 \text{ GeV}, E_h \geq 4 \text{ GeV}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$&lt;E^{\bar{\nu}}&gt; \sim 11 \text{ GeV}, E_h \geq 12 \text{ GeV}$</td>
</tr>
<tr>
<td>CITF</td>
<td>$0.24 \pm 0.04$</td>
<td>$0.35 \pm 0.11$</td>
<td>$&lt;E&gt; \sim 50 \text{ GeV}, E_h \geq 12 \text{ GeV}$</td>
</tr>
</tbody>
</table>

3.3 Two lepton events

3.3.1 Di-muon events in counter experiments

In the counter spark chamber arrangements of the FNAL-neutrino experiments in wide-band [55] and narrow-band [56] beams events with two muons in the final state were observed. The muons were identified by their penetration through many meters of iron (fig. 36).

The rate of these events is about 1% of the normal charged current events.
Processes simulating events with direct production of two-muons:

(a) Normal charged current events with one muon and one pion (or kaon) decaying into a muon without visible kink. By comparing the rate of 2-µ-events from regions of the detector with different density (i.e. different relative decay and interaction probabilities) this background can be estimated to be less than 25%.

(b) Neutral current interactions with one of the hadrons being a meson which decays into two muons (e.g. $\rho^0 \rightarrow \mu^+ \mu^-$). This can be excluded because the rate of such process would be much smaller and the invariant mass of the observed two-muon system is varying over several GeV.
Processes with direct production of two muons at the neutrino vertex could be:

(a) Production and leptonic decay of an intermediate vector boson (sect. 1.4):

\[ \text{Fig. 37 (a) Production and decay of an intermediate vector boson.} \]

Then, the $\mu^-$ and $\nu^+$ would be produced with small relative velocity (in their centre of momentum system) and have mean momenta (in the laboratory system) in the ratio of their masses, i.e. $p_{\mu^-} \approx p_{\mu^+}$, whereas the experimentally observed ratio is $p_{\mu^-}/p_{\mu^+} \approx 6$.

(b) Production and leptonic decay of a heavy lepton $L^\circ$ (spin 1/2):

\[ \text{Fig. 37 (b) Production and decay of a heavy lepton.} \]
In that case the average ratio of momenta must be between \( \frac{1}{2} \) and 2 [57], also in disagreement with the observation.

(c) Production and leptonic decay of a new hadron \( (Y) \) with a new hadronic quantum number (charm), in agreement with the scheme outlined in sect. 1.9:

![Diagram of production and decay of a charmed hadron](image)

**Fig. 38** Production and decay of a charmed hadron.

This interpretation is for the time being the most probable one and explains also the recent phenomena observed at \( e^+e^- \) storage ring experiment (\( J/\psi \) and D production at SLAC and DESY).

3.3.2 \( \mu^-e^+ \) events in heavy liquid bubble chambers

Dilepton events have also been searched for in heavy liquid bubble chambers: in Gargamelle at CERN and in the 15' chamber at FNAL. In these experiments the second lepton which is searched for is a positron\(^(*)\), since the short radiation length of heavy liquids allows good electron recognition already at the scanning stage:

![Wave symbols](image)

(a) bremsstrahlung (b) spiraling (c) direct pair-production

**Fig. 39** Electron (positron) identification criteria.

\(^{*}\) Di-muon search in the 15' chamber which has an EMI (sect. 2.3.2, fig. 20(a)) is in progress.
The other advantage of bubble chambers is to search also for strange particles which are expected if the dilepton events are due to charmed particle decay (sect. 1.9). The first event of this type was seen in Gargamelle [58(a)], the final sample [58(b)] is shown in table VII, together with the observations of the two 15' bubble chamber experiments [59]. Fig. 40 shows an example, where the particle X of fig. 38 is a $K_s^0$.

**TABLE VII**

<table>
<thead>
<tr>
<th></th>
<th>Gargamelle freon</th>
<th>15' bubble chamber</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ne (21%)/$H_2$</td>
</tr>
<tr>
<td>No. of photos</td>
<td>2 000 000</td>
<td>70 000</td>
</tr>
<tr>
<td>Charged current events</td>
<td>41 000</td>
<td>5400</td>
</tr>
<tr>
<td>$\mu^+ e^-$ events</td>
<td>20</td>
<td>17</td>
</tr>
<tr>
<td>$E_{e^+}$ cut (GeV)</td>
<td>.2</td>
<td>0.8</td>
</tr>
<tr>
<td>background</td>
<td>7</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>losses due to cuts</td>
<td>&lt; 1</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>$e$ detection efficiency</td>
<td>0.97 ± 0.02</td>
<td>0.36 ± 0.1</td>
</tr>
<tr>
<td>$\mu^+ e^- / \mu^- (%)</td>
<td>0.3 ± 0.1</td>
<td>0.77 ± 0.3</td>
</tr>
<tr>
<td>$\mu^+ e^- V^0 / \mu^+ e^+$</td>
<td>3/20</td>
<td>11/17</td>
</tr>
</tbody>
</table>

In order to arrive at the rates quoted in table VII, detection efficiencies for electrons, background and loss rates have to be estimated. The main background source for $\mu^- e^+$ events are Dalitz pairs ($\pi^0 \rightarrow e^+ e^- \gamma$, 1.15% branching ratio) with one electron branch being so small that single electron production is simulated. Other sources are $K_{e3}$ decays and $\bar{\nu}_e$ interactions. In all these experiments background interactions can only account for a few percent of the $\mu e$-events.

As in the case of the di-muon events the $\mu e$-events are most likely explained in terms of leptonic decay of charmed hadrons $c \rightarrow s + \ell^+ + \nu$. 
$$\nu_\mu \rightarrow \mu^{-}K^{0}e^{+}\nu_{e}\pi^{+}\pi^{0}n$$
The µe-events support further evidence for this hypothesis since they show strange particles at a higher rate than the "normal" charged current events. The only strange particles which are easily identifiable in bubble chambers are \( K^0 \) and \( \Lambda \) by their neutral decay mode (\( \nu^0 \), p. 49). The discrepancy in \( \nu^0 \) rates of the two 15' bubble chamber experiments is not fully understood and must be due to experimental biases and statistics.

The GIM [30] model (p. 19) predicts the production of one charmed particle off valence quarks according to fig. 8 (p. 21)

\[
\nu + n + c + \mu^-.\]

From the matrix element (fig. 7, p. 19)

\[
M = (\mu \nu) (\lambda' c)
= (\mu \nu) [(-n \cdot \sin \theta_c + \lambda \cos \theta_c) c]
\]

it is also predicted that the rate of charmed particle production is suppressed by \( \sin^2 \theta_c \) with respect to the "normal" transition

\[
M = (\mu \nu) (n' p)
= (\mu \nu) [(n \cos \theta_c + \lambda \sin \theta_c) p]
\]

The leptonic decay of a charmed particle is described by

\[
M = (\mu \nu) (\lambda' c), \text{ or } (e \nu) (\lambda' c)
= (e \nu) [(-n \sin \theta_c + \lambda \cos \theta_c) c],
\]

i.e. a charmed particle decays preferentially (\( \nu \cos^2 \theta_c \)) into a strange particle.

Hence - in this model - from the observed di-lepton rates of table VII and p. 68, it can be concluded that the leptonic branching ratio of charmed particles (produced at a rate of \( \sin^2 \theta \sim 5\% \)) is somewhere between 10 and 20%.

However, further studies are necessary - especially of strange particle production in di-lepton events - before conclusions can be drawn about the extent of validity of this model.
CONCLUSIONS AND OUTLOOK

The concept of the left-handed neutrino existing in two types \((\nu_e, \nu_\mu)\) and interacting via \(W^\pm\) and/or \(Z^0\) exchange with pointlike quasi-free three-coloured four-flavoured \((p,n,\lambda,c)\) constituents of (colourless) hadrons allows to describe nearly all phenomena classed as weak interactions, especially high energy neutrino interactions, the subject of these lectures. However, deviations from exact scaling - the structure function \(F_2\), equ. (29), are not functions of \(x\) only but depend also on \(q^2\) [60] - require extension of the theory. The existence of a third lepton type (called \(U\) or, now, \(\tau\)) and lepton-hadron symmetry arguments suggest introduction of more quarks. Also CP violation cannot be accounted for by only four quarks. How many more quarks? One, two or three?

"Wo das Verstehen aufhört, fängt das Zählen an..."

Schopenhauer

(Where understanding ceases, counting starts)
A.1 We start from the Lagrangian density

\[ \mathcal{L}_1 = -\frac{1}{4} F_{\mu\nu}^2 + (\partial_\mu \phi)^2 - V(\phi) \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad D_\mu = \partial_\mu - i e A_\mu \quad V(\phi) = M^2 |\phi|^2 + \lambda |\phi|^4 \]

electromagnetic field covariant derivative classical potential (complex)

\[ \mathcal{L}_1 \text{ is invariant under gauge transformation } \phi \rightarrow e^{i\alpha(x)} \phi, A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x) \]

(a) \( M^2 > 0 \)
\[ \mathcal{L}_1 \text{ describes QED for charged scalar particle.} \]

(b) \( M^2 < 0 \)
\[ \text{perturbation theory around } \phi = 0 \text{ yields unstable solution. Reach stable solution by transforming } \phi \text{ to } \phi' = 1/\sqrt{2} [\alpha + \phi_1(x) + i \phi_2(x)]. \]

A.2 \[ \mathcal{L}_2 = \mathcal{L}_1 + \mathcal{L}_2 = \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} \partial_\mu A_\nu \phi_1 \phi_1 + \cdots + \frac{1}{2} (\partial_\mu \phi_1)^2 - 2 \lambda a \phi_1^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 - e A_\mu \phi_2 \phi_2 + \cdots \]

\[ \mathcal{L}_2 \text{ is still gauge invariant (and is the form used to demonstrate renormalizability).} \]

The \( A_\mu^0 \) term makes the gauge transformation more complicated
\[ \phi_1(x) \rightarrow [a + \phi_1(x)] \cos \theta(x) - \phi_2(x) \sin \theta(x) - a, \phi_2(x) \rightarrow [a + \phi_1(x)] \sin \theta(x) + \phi_2(x) \cos \theta(x), \]
\[ A_\mu \rightarrow A_\mu + 0 \theta(x) \]

and implies a shift of the field.

In order to make the massless, unphysical field \( \phi_2 \) (Goldstone boson) disappear, yet another transformation has to be chosen
\[ \phi' = \frac{1}{\sqrt{2}} [a + \rho(x)] e^{i\chi(x)/a}, A_\mu(x) = B_\mu(x) + \frac{1}{a} \partial_\mu \chi(x) \]

A.3 \[ \mathcal{L}_3 = \mathcal{L}_1 + \mathcal{L}_3 = -\frac{1}{4} [B_{\mu\nu}^2 - e^2 a B_{\mu}^2] + \frac{1}{2} [\partial_\mu \rho^2 - 2 \lambda a \rho^2] - \frac{1}{4} \rho^4 + \frac{1}{2} e^2 B_{\mu}^2 (2 \alpha \rho + \rho^2) \]

\[ \text{massive vector } B_\mu, \text{ massive scalar } \rho \]

\[ \mathcal{L}_3 \text{ is no longer gauge invariant, but it contains only physical particles and their mass spectrum.} \]

The original model of Weinberg (p 18) needs at least four vector bosons, \( W^+, W^- \) for ordinary weak interactions, \( Z^0 \) for the weak neutral currents and one (massless) for the photon (\( A_\mu \)).
APPENDIX B

CHARMED PARTICLE STATES IN THE G.I.M. MODEL (from ref. [31])

| TABLE I. Charmed 1/2+ baryon states. |
|---|---|---|---|
| Label | Quark content | Isospin | Strangeness |
| \( C = 1 \) \( C_1^{++} \) | \( cuu \) | \( T = 1, T_s = 0 \) | \( S = 0 \) |
| \( C_1^+ \) | \( c(ud)^{sym} \) | \( T = 1, T_s = -\frac{1}{2} \) | \( S = 0 \) |
| \( C_0^+ \) | \( cdd \) | \( T = 0 \) | \( S = 0 \) |
| \( C_0^{++} \) | \( c(ud)^{anti} \) | \( T = \frac{1}{2}, T_s = -\frac{1}{2} \) | \( S = 0 \) |
| \( S^+ \) | \( c(su)^{sym} \) | \( T = \frac{1}{2}, T_s = \frac{1}{2} \) | \( S = 0 \) |
| \( A^+ \) | \( c(su)^{anti} \) | \( T = \frac{1}{2}, T_s = -\frac{1}{2} \) | \( S = 0 \) |
| \( A_0^+ \) | \( c(su)^{anti} \) | \( T = 0 \) | \( S = 0 \) |
| \( C = 2 \) \( X_{1/2}^{++} \) | \( ccd \) | \( T = \frac{1}{2}, T_s = \frac{1}{2} \) | \( S = 0 \) |
| \( X_{1/2}^+ \) | \( ccd \) | \( T = 0 \) | \( S = -1 \) |

| TABLE II. Charmed \( 0^- \) mesons. |
|---|---|---|---|
| Label | Quark content | Isospin | Strangeness |
| \( C = 1 \) \( D^0 \) | \( c\bar{d} \) | \( T = \frac{1}{2}, T_s = -\frac{1}{2} \) | \( S = 0 \) |
| \( D^+ \) | \( c\bar{u} \) | \( T = 0 \) | \( S = 0 \) |
| \( D^* \) | \( c\bar{s} \) | \( T = 0 \) | \( S = 0 \) |
| \( C = 0 \) \( \eta' \) | \( \approx (6)^{-1/2}(u\bar{u} + d\bar{d}) \) | \( T = 0 \) | \( S = 0 \) |
| \( \eta \) | \( \approx \frac{1}{2}(u\bar{d} + d\bar{u} + s\bar{s}) \) | \( T = 0 \) | \( S = 0 \) |
| \( \omega' \) | \( \approx (12)^{-1/2}(u\bar{u} + d\bar{d} + s\bar{s}) \) | \( T = 0 \) | \( S = 0 \) |
| \( C = -1 \) \( D^0 \) | \( c\bar{u} \) | \( T = \frac{1}{2}, T_s = -\frac{1}{2} \) | \( S = 0 \) |
| \( D^* \) | \( c\bar{d} \) | \( T = 0 \) | \( S = -1 \) |

| TABLE III. Charmed \( 1^- \) mesons. |
|---|---|---|
| Label | Quark content | \( C = 1 \) |
| \( C = 0 \) \( \omega \) | \( \approx (2)^{-1/2}(u\bar{u} - d\bar{d}) \) | \( \phi \) | \( \phi \) |
| \( \phi \) | \( \approx s\bar{s} \) | \( \phi \) | \( \phi \) |
| \( C = -1 \) | \( D^0 \) | \( D^* \) | \( D^{++} \) |

FIG. 1. Weight diagrams for SU(4). Shaded planes denote multiplets of SU(3) \( \otimes U(1)_c \). (a) The four quarks of SU(4): the conventional SU(3) triplet consisting of \( u \) ("up"), \( d \) ("down") and \( s \) ("strange") with \( C = 0 \), and an SU(3) singlet \( c \) ("charmed") with \( C = 1 \). (b) The three-quark 1/2+ baryons which form a 30-plet of SU(4). The SU(3) multiplets are 8 (\( C = 0 \)), 6 + 3 (\( C = 1 \)) and 3 (\( C = 2 \)). (c) The 15-plet \( \phi \) singlet pseudoscalar. The SU(3) multiplets are 8 (\( C = -1 \)), 8 + 1 (\( C = 0 \)) and 3 (\( C = 1 \)).
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[52] Note added in proofs:
First Analysis of CERN ν-experiments with the SPS narrow-band beam and recent analysis of the CAL-TEC-FNAL experiment show less increase of $\sigma'/\sigma$ and no high $y$ anomaly.

A paper on analysis using more statistics is in preparation.


