CHROMATIC PERTURBATION AND RESONANCE STRENGTHS IN SSC*

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Abstract

We have investigated the nonlinear effects of the SSC-CDR clustered lattice. In this note, we will give a brief theoretical discussion and an overview of some of our results using program HARMON. Tune diagrams for the structure resonances of the SSC are also included.

I. Introduction

We have investigated the nonlinear effects with the emphasis on nonlinear resonances [1-3]. In this paper we present some of our findings (e.g., the structure resonances; stop bandwidths, etc.) for the SSC-CDR clustered lattice using program HARMON [4]. This program is based on "finding the adverse effects of a particular quadrupole-sextupole configuration and then adjusting the sextupole strengths to minimize these effects". The functions to be minimized are chromatic effects (variation of particle motion with respect to the variation in particle momentums) and non-chromatic effects due to presence of nonlinear elements (chromaticity correction sextupoles). In section II we discuss the general treatment of the nonlinear effects following Guignard [5] and Donald [4]. In section III using program HARMON the nonlinear effects of the SSC-CDR clustered lattice is analyzed and tabulated. These as well as higher order resonances are discussed in references [1 and 6] using program NONLIN (since HARMON is not equipped to do so).

II. Theory

Perturbation theory no longer holds when the system is operating near a resonance. Although, an approximate invariant can be found for system near a single resonance if the contribution from the other resonances are small. The perturbing part of the Hamiltonian near a given resonance can be expressed as:

$$H = \frac{2\pi}{C} eI_x + \sum_{k=1} V_k I_x^{\frac{k}{2}+1} + R\cos\psi_x \qquad (1)$$

where the bandwidth $e = n_x\nu_x + n_z\nu_z - p$; I_x and I_z are the action (proportional to the square root of the emittance (E_x and E_z) and ψ is the conjugate phase. R is the resonance strength; V_k are the stabilizing coefficients; s is the distance along the orbit (the time variable of the Hamiltonian); ν_x, ν_z are the betatron tunes; n_x, n_z and p define a given resonance. Eq. (1) is an approximate invariant as long as the contribution of the other resonances are small in the Hamiltonian.

Following Guignard [5], dynamic properties can be found from invariant of the motion H. The stop bandwidths (Δe) are defined to be the smallest bandwidth such that the action in Eq. (1) is still bounded. This can be done by first considering the fixed points of Eq. (3) which are defined to be the points at which there is no motion. These fixed points are the solutions to the following equations (note I_z can be treated as a constant since it is also an invariant of the motion).

$$0 = \frac{dI_x}{ds} = \frac{-\partial H}{\partial\psi_x} = A\sin\psi_x \qquad (3)$$

$$0 = \frac{d\psi_x}{ds} = \frac{\partial H}{\partial I_x} \qquad (4)$$

$$= \frac{2\pi}{C} e + \sum_{k=1} \left(\frac{k}{2}+1\right) V_k I_x^{k/2} + \frac{\partial R}{\partial I_x}\cos\psi_x$$

Eq. (4) has many solutions, $\psi_x = n\pi$, which leads to two cases, $\cos(\psi_x) = \pm 1$, in Eq. (3). Note that, for the smallest positive value of I_x there is at most one solution for each of these two cases corresponding to stable and/or unstable fixed point(s). The nature of these solutions are determined by the bandwidth (e) and the stabilizing coefficients (V_k).

The stop bandwidth for these two cases can be found by substituting $\cos(\psi_x) = \pm 1$ into Eq. (1) which leads to the following two equations:

$$G_\pm = \frac{2\pi}{C} eI_x + \sum_{k=1} V_k I_x^{\frac{k}{2}+1} \pm R \qquad (5)$$

From these four equations (for the given initial conditions) can be deduced:

$$\begin{aligned} G_\pm(I_{x_o}) - G_\pm(I_x) &= 0 \\ G_\pm(I_{x_o}) - G_\mp(I_x) &= 0 \end{aligned} \qquad (6)$$

We can then obtain the extreme values of I_x and the stop bandwidths from these equations. Eq. (6) can be satisfied for any values of action I if $e<\Delta e$ (Δe is the stop bandwidth). The necessary distance (δe) between the operating tunes (ν_x, ν_z) and the resonance line [5] which must be kept to limit or avoid the relative growth of amplitude or beating of a single particle in a given interval

$$\Lambda = [(I_x/I_{x_o})^{1/2} - 1]$$

is as follows:

For a sum resonance

$$\delta e \geq \frac{\Delta e}{2}\left(1 + \frac{1}{\Lambda}\,\frac{n_x E_z + n_z E_x}{n_x^2 E_z + n_z^2 E_x}\right)$$

and for difference resonance

$$\delta e \geq \frac{\Delta e}{2}\,\frac{1}{\Lambda}\,.$$

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In section III, we give some of our results for the SSC including the stop bandwidths (defined above) using program HARMON.

III. Results for SSC-Lattice

We consider the SSC-CDR clustered lattice [8] with operating tunes of $\nu_x = 78.265$, $\nu_z = 78.280$ and a periodicity 1.

The results of HARMON are given in the following Tables. For $\beta^* = 0.5$ and $\beta^* = 6$, Tables I and III give the orbit parameters for the SSC as well as the sextupole strengths (SF and SD which is $= B''\ell/B\rho$, where $B\rho$ is the magnetic rigidity, ℓ is the length of the sextupole and B'' is the second derivative of the field). The stop bandwidths for the resonances in the SSC are given in Tables II and IV for $\beta^* = 0.5$ and $\beta^* = 6$ respectively. Note, the columns labeled "MODULUS" give the resonance strengths whereas columns labeled as DE(S) give the stop bandwidth over the full interval about a given resonance.

These and higher order resonances are discussed in References [1 and 6] since HARMON is limited to the calculation of fourth order resonances.

Notation:

HARMON symbol for tune is Q ($\equiv\nu$), where "S" means systematic and "R" means random in the following Tables.

TABLE II

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HARMON - CDR LATTICE FOR SSC WITH BETA*=6.
[INITIAL EMITTANCE:     EX0= EZ0 = 2.5700E-07]

FOURIER ANALYSIS. ORDER OF RESONANCE  = 4

4QX = 313.

  COSINE        SINE        MODULUS      RANDOM       DE(S)
 SF   SD
-2.8860E-04 -4.8607E-05  2.9266E-04  1.6180E+01  2.9053E-09
   DE(R)        DQ(S)        DQ(R)      DQ20(S)      DQ20(R)
 1.6062E-04  7.2633E-10  4.0155E-05  1.0508E-09  5.8093E-05

2QX + 2QZ = 313.

  COSINE        SINE        MODULUS      RANDOM       DE(S)
 SF   SD
-8.6381E-04 -7.1929E-04  1.1241E-03  6.4319E+01  5.5795E-09
   DE(R)        DQ(S)        DQ(R)      DQ20(S)      DQ20(R)
 3.1925E-04  1.9726E-09  1.1287E-04  2.8539E-09  1.6330E-04

4QZ = 313.

  COSINE        SINE        MODULUS      RANDOM       DE(S)
 SF   SD
-1.8434E-04 -2.7677E-04  3.3254E-04  2.5612E+01  3.3012E-09
   DE(R)        DQ(S)        DQ(R)      DQ20(S)      DQ20(R)
 2.5425E-04  8.2530E-10  6.3564E-05  1.1940E-09  9.1960E-05

2QX - 2QZ = 0.

  COSINE        SINE        MODULUS      RANDOM
 SF   SD
-1.8668E+02  0.0000E+00  1.8668E+02  6.4319E+01
```

TABLE I

```
HARMON - CDR LATTICE FOR SSC WITH BETA*=6.
[INITIAL EMITTANCE:   EX0= EZ0 = 2.5700E-07]

TOTAL LENGTH = 82944.          NSUP = 1
QX     =  7.82650E+01   QZ        = 7.82800E+01
BETAX  =  1.11371E+02   BETAZ     = 3.31656E+02
ETAX   =  2.36230E+00

NORMALIZED STRENGTHS
 ID       STRENGTH

 SF    5.12380E-03
 SD   -8.25720E-03

FOURTH ORDER EFFECTS OF SEXTUPOLES
Q SHIFT EFFECTS

            G22000                   DQXDEX       DQX
-3.58180E+02  2.14884E-12 -7.16360E+02 -1.84105E-04
            G00220                   DQZDEZ       DQZ
-2.38812E+02  1.29191E-12 -4.77625E+02 -1.22750E-04
            G11110                   DQXDEZ      DQZDEX
-2.44556E+03  1.02414E-11 -2.44556E+03 -2.44556E+03
                                     DQX          DQZ
                               -6.28510E-04 -6.28510E-04
RESONANCE EFFECTS

[G's are the 2nd order coefs.]
                  COS           SIN           DE
G4000,313     2.5856E-04  -1.0971E-04   1.1549E-09
                              DQ          DQ(20)
                          6.9707E-10   1.0085E-09
                  COS           SIN           DE
G0040,313     1.7267E-05  -1.5545E-04   6.4316E-10
                              DQ          DQ(20)
                          3.8818E-10   5.6159E-10
                  COS           SIN           DE
G2020,313    -2.9871E-04  -5.5839E-04   1.3020E-09
                              DQ          DQ(20)
                          1.1113E-09   1.6078E-09
                  COS           SIN         MODULUS
G2002,  0     6.0136E+02   1.1105E+02   6.1153E+02
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TABLE III

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HARMON - CDR LATTICE FOR SSC WITH BETA*=0.5
[INITIAL EMITTANCE:   EX0= EZ0 = 2.5700E-07]

TOTAL LENGTH = 82944.          NSUP = 1
QX     =  7.82650E+01   QZ        = 7.82800E+01
BETAX  =  1.11371E+02   BETAZ     = 3.31656E+02
ETAX   =  2.36230E+00

NORMALIZED STRENGTHS
 ID       STRENGTH

 SF    9.87283E-03
 SD   -1.59109E-02

FOURTH ORDER EFFECTS OF SEXTUPOLES
Q SHIFT EFFECTS

            G22000                   DQXDEX       DQX
-1.32967E+03  1.77976E-11 -2.65934E+03 -6.83449E-04
            G00220                   DQZDEZ       DQZ
-8.86800E+02  3.88949E-11 -1.77360E+03 -4.55815E-04
            G11110                   DQXDEZ      DQZDEX
-9.08039E+03  9.78699E-11 -9.08039E+03 -9.08039E+03
                                     DQX          DQZ
                               -2.33366E-03 -2.33366E-03
RESONANCE EFFECTS

[G's are the 2nd order coefs.]
                  COS           SIN           DE
G4000,313    -2.8384E-04  -6.7087E-05   1.1993E-09
                              DQ          DQ(20)
                          7.2385E-10   1.0472E-09
                  COS           SIN           DE
G0040,313     1.5723E-04   2.0873E-04   1.0745E-09
                              DQ          DQ(20)
                          6.4854E-10   9.3827E-10
                  COS           SIN           DE
G2020,313     5.5679E-04   6.4937E-04   1.7587E-09
                              DQ          DQ(20)
                          1.5011E-09   2.1717E-09
                  COS           SIN         MODULUS
G2002,  0     2.2334E+03   4.1045E+02   2.2708E+03
```

TABLE IV

HARMON - CDR LATTICE FOR SSC WITH BETA*=0.5
[INITIAL EMITTANCE: EX0= EZ0 = 2.5700E-07]

FOURIER ANALYSIS. ORDER OF RESONANCE = 4

4QX = 313.

COSINE SF SD	SINE	MODULUS	RANDOM	DE(S)
1.8933E-04	4.3472E-05	1.9426E-04	3.1176E+01	1.9284E-09
DE(R)	DQ(S)	DQ(R)	DQ20(S)	DQ20(R)
3.0949E-04	4.8211E-10	7.7372E-05	6.9748E-10	1.1194E-04

2QX + 2QZ = 313.

COSINE	SINE	MODULUS	RANDOM	DE(S)
5.0792E-04	3.7613E-04	6.3203E-04	1.2394E+02	3.1371E-09
DE(R)	DQ(S)	DQ(R)	DQ20(S)	DQ20(R)
6.1517E-04	1.1091E-09	2.1750E-04	1.6046E-09	3.1466E-04

4QZ = 313.

COSINE SF SD	SINE	MODULUS	RANDOM	DE(S)
1.2588E-04	1.1795E-04	1.7250E-04	4.9352E+01	1.7125E-09
DE(R)	DQ(S)	DQ(R)	DQ20(S)	DQ20(R)
4.8993E-04	4.2812E-10	1.2248E-04	6.1938E-10	1.7720E-04

2QX - 2QZ = 0.

COSINE SF SD	SINE	MODULUS	RANDOM
-3.5974E+02	0.0000E+00	3.5974E+02	1.2394E+02

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Fig. 1 SSC-CDR Lattice O(78.265, 78.280)

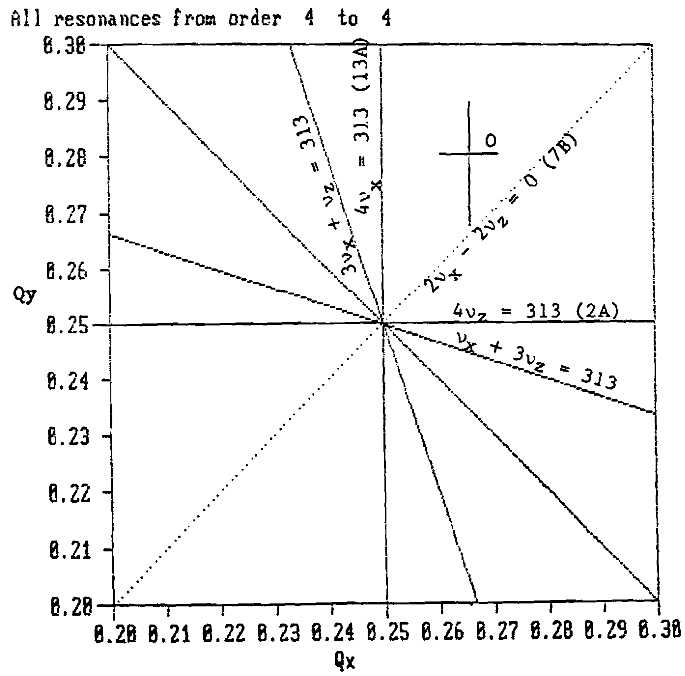

Fig. 2 SSC-CDR Clustered Lattice O(78.265, 78.280)