Fermi National Accelerator Laboratory

FERMILAB-FN-572

Compensation in Calorimeters for Jets and Hadrons

D. Green, A. Para and W. Wu

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

September 1991

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

COMPENSATION IN CALORIMETERS

FOR

JETS AND HADRONS

Dan Green, Adam Para, Weimin Wu
Fermi National Accelerator Laboratory
Batavia, IL 60510

1. INTRODUCTION

The treatment of the subject of "compensation" in calorimetry is extensive [1]. It is also rather confusing to the amateur, in whose ranks the authors are firmly ensconced. In an effort to see more clearly what physical effects are important, some simple minded approximations are explored. The hope is that they will yield some guidance and intuition beyond that which is afforded by massive Monte Carlo packages.

2. STOCHASTIC ERRORS FOR JETS AND HADRONS

Suppose a single hadron is incident on a uniform calorimeter as shown in Fig. 1a. It is assumed that the response of the detector can be approximated to be a "stochastic" term, a, due to statistical fluctuations in the shower/detection process and a "constant" term, 1b, due to non-uniformities in the construction of the detector. For example, the constant term could be due to transverse variation in the thickness of the absorber plate or detector plate. These latter errors lead to fractional energy errors which are independent of the incident particle energy. A calibration procedure is assumed, which connects the detected signal, Eo, to the incoming energy E.

$$
\begin{aligned}
&dE = a\sqrt{E} \oplus bE \\
&Eo(h) = \varepsilon_H E,\ \varepsilon_H = 1 \\
&dEo(h) = dE \qquad\qquad 1)
\end{aligned}
$$

In Eq. 1, a is the stochastic coefficient, b is the constant coefficient, and ε_H is the calibration constant. Clearly, for a single incident hadron, the detected error, dEo(h) is equal to the parameterized error, dE. This is the operational definition of how the error is measured, in any case. In the low energy regime the stochastic term dominates, so that dEo(h) ~ $a\sqrt{E}$.

What about the situation shown in Fig 1b.? In that case, an ensemble of hadrons of total energy E and energy fraction zi impinges on the uniform calorimeter.

$$\begin{aligned} &\Sigma zi = 1, \Sigma ki = E \\ &Eo(J) = \Sigma ki \\ &dEo(J) = \sqrt{\Sigma (dki)^2} \\ &\quad \sim a\sqrt{E} \end{aligned} \qquad 2)$$

The calibration is as in Eq. 1, so that Eo(J) is just the sum of the incoming energies. Assuming statistical independence of the measurements, the error on the ensemble energy, dEo(J), is just what one would expect if the ensemble were a single hadron of energy E, if the stochastic errors dominate the constant errors. In this sense the ensemble of particles acts, with respect to errors, as a single parent particle, of energy E.

Note that this result is completely independent of the "fragmentation function" which determines how the zi are distributed. That is as it should be since the stochastic error is related to the measurement of, and the associated error in, the total number of particles in the calorimeter shower. This total number should be independent of how it is subdivided. Thus, in this limit, the "jet" can be considered to be a single particle w.r.t stochastic errors.

3. CONSTANT TERMS FOR JETS AND HADRONS

Suppose instead that one is in the high energy regime where the stochastic term has died off and only the constant term remains. In that case for a single particle dEo(h) ~ bE. One is no longer measuring properties of the particles, but rather the inhomogenities inherent in the detecting medium.

$$\begin{aligned} &dEo(J) = \sqrt{\Sigma (bziE)^2} \\ &\quad = bE\sqrt{\Sigma zi^2} \\ &\quad \sim bE\, z1 \end{aligned} \qquad 3)$$

In Eq. 3, the approximation is made that the sum of the squares of zi in quadrature retains only the "leading particle" term, the largest zi, which is defined to be z1. This means that the error on E(J), dEo(J), is less than one would ascribe to a particle with a total energy equal to the sum of the energies of the ensemble. This conclusion seems to be an inescapable consequence of straightforward error propagation. Since zi < 1, one will do better in general by measuring the individual particles. In the special case of n particles fragmenting equally, zi = 1/n. The exact result is that dEo(J) = $bE/\sqrt{n}$. Clearly, one does better for n > 1. In the case of a fragmentation distribution where one particle "leads" the fragmentation chain, z1 > zi, i > 1, then the approximation made in Eq. 3. applies.

As a concrete example, suppose one has a calorimeter with a = 0.3 and b = 0.05. A 10,000 GeV jet fragments into 4 hadrons sharing the energy equally. These fragments have errors dominated by the constant term. For a single particle, the expected error is ~ 500 GeV. However, the hadrons in the jet sum to 10,000 GeV with an error of 250 GeV, half the single particle error.

4. SIMPLE MODEL FOR A SEGMENTED CALORIMETER

One can now go ahead and look at a simple minded model for a segmented calorimeter. Consider a detector with 2 compartments, an electromagnetic, EM, followed by an hadronic, HAD. There are calibration constants for electromagnetic showers, ε_E and ε'_E for the EM and HAD sections. In addition, the HAD section has a separate calibration constant, ε'_H for incident hadrons. The use of these constants will be discussed in what follows. A schematic diagram of this model is given in Fig. 2a.

Consider an incident neutral π°. The EM section is calibrated so that $\varepsilon_E = 1$. The errors are assumed to be describable by a stochastic term, a_E, and a constant term b_E.

$$
\begin{aligned}
&Eo(\pi^\circ) = \varepsilon_E E \\
&\varepsilon_E = 1 \\
&dEo(\pi^\circ) = a_E\sqrt{E} \oplus b_E E
\end{aligned}
\qquad 4)
$$

Now consider a hadron, h, striking the calorimeter. The hadron interacts in the HAD section, and has a fraction ,fo, which goes into neutral particle production. We approximate its statistical error, dfo, as due to the fluctuation in the first interaction initiating the shower. Further fluctuations deeper in the shower for higher generations are ignored. This is similar to the situation in a photomultiplier tube. The photocathode statistical error roughly defines the statistical error in the output pulse height. One calibrates so that, on the average, the HAD response to h is the same as the EM response to neutral pions. The errors are assumed to be covered by a_H and b_H, which are analogous to the quantities defined for the EM compartment.

$$
\begin{aligned}
&Eo(h) = fo\varepsilon'_E E + \left(1 - fo\right)\varepsilon'_H E = \\
&E\varepsilon'_E\left(fo + (1 - fo)\varepsilon'_H / \varepsilon'_E\right) \\
&< fo > \varepsilon'_E + \left(1 - < fo >\right)\varepsilon'_H = 1 \\
&dEo(h) = a_H\sqrt{E} \oplus b_H E
\end{aligned}
\qquad 5)
$$

There is an additional source of error for the HAD section. The fact that the response of the calorimeter to charged and neutral pions may be different means that one has a nonuniform medium. This nonuniformity means that one will induce a "constant term". The calibration scheme insures that $< Eo(h) > = E$, but as seen from Eq. 5, fluctuations in fo, dfo, lead to errors in Eo(h), dEo(h), if $\varepsilon'_H/\varepsilon'_E$ is not = 1. If $\varepsilon'_E = \varepsilon'_H$, the calorimeter is called "compensating" and no extra error is induced, since (see Eq. 5) $Eo(h) = [\ fo + (1\text{-}fo)]\ \varepsilon'_E E = E$, independent of fo.

If the compensation condition is not met, then there is an additional error, $[dEo(h)]_{e/h}$.

$$\frac{[dEo(h)]_{e/h}}{E} = dfo\left(1-\varepsilon'_H/\varepsilon'_E\right)\varepsilon'_E$$
$$\sim dfo\left(1-\varepsilon'_H/\varepsilon'_E\right) \qquad 6)$$

The function fo is the fraction of neutral pions in the shower. The statistical variation in that number should be defined by the number of neutrals produced in the first interaction. The resulting constant term due to "e/h", b(e/h), can be read off from Eq. 6.

$$dfo/fo \sim 1/\sqrt{<n_{\pi o}>}$$
$$b(e/h) \sim \frac{fo(1-\varepsilon'_H/\varepsilon'_E)}{\sqrt{<n_{\pi o}>}} \qquad 7)$$

This equation for b(e/h) is an approximation to the celebrated "constant term" [1]. It is numerically about 0.17 for the factor which converts noncompensation, $\varepsilon'_H/\varepsilon'_E$ not = 1, to a constant error, b(e/h). Clearly, in this simple model b(e/h) cannot be a constant, since the statistical variation in the number of neutral secondaries decreases as the square root of the logarithm of the c.m. energy squared which is proportional to E [2].

For example, $< n_{ch} > = 10$ for c.m. energy of 40 GeV, or E = 800 GeV, while $< n_{ch} > = 4$ for c.m. energy of 10 GeV or E = 50 GeV. Assuming that the mean number of neutrals is 1/2 the mean number of charged pions, and consequently that fo is = 0.33, the "constant" conversion factor is 0.21 and 0.14 for E = 50 and 800 GeV respectively. Clearly, the magnitude is close to that of Ref. 1, and the variation with E, the square root of the logarithm of E, is sufficiently slow that it can be called a "constant" term.

5. MODEL FOR JETS IN A CALORIMETER

The calorimeter response is as defined and discussed in Section 4 above. A schematic of the jet, J, hitting the calorimeter is shown in Fig. 2b. The neutral fraction in the incoming jet is Fo, that for the showering interaction in the HAD section is fo. The jet is an ensemble of particles distributed in momentum as zD(z) with Fo neutrals, zi, and (1-Fo) charged particles, zj. The EM section is calibrated just as before, $\varepsilon_E = 1$.

$$\sum zi = Fo, \sum zj = (1-Fo)$$
$$Eo(\gamma) = \varepsilon_E \sum ki$$
$$= \left(\sum zi\right)E$$
$$= FoE \qquad 8)$$

The hadrons are, likewise, statistically independent objects which interact in the HAD compartment. Calibrating as above for single hadrons, one finds that, on average, the jet energy is correctly measured, < Eo(J) > = E. The weak energy dependence of fo has been ignored, so that it factors out of the energy sum.

$$\mathrm{Eo(ch)} = \left(\mathrm{fo}\varepsilon'_E + (1-\mathrm{fo})\varepsilon'_H\right)\Sigma \mathrm{kj}$$
$$<\mathrm{Eo(ch)}> = \Sigma \mathrm{kj} = (1-\mathrm{Fo})\,\mathrm{E}$$
$$<\mathrm{Eo(J)}> = <\mathrm{Eo(ch)}> + \mathrm{Eo}(\gamma) = \mathrm{E} \qquad 9)$$

This is fine, but what about the fluctuations? The errors on the incident neutral energy can be read off from Eq. 8. They consist of stochastic and constant terms folded in quadrature. The approximation that the sum of the squares of zi is the leading neutral, z1(γ), has been made in Eq. 10.

$$\mathrm{dEo}(\gamma) = \sqrt{\Sigma(\mathrm{dki})^2}$$
$$\sim a_E\sqrt{\mathrm{FoE}} \oplus b_E\,\mathrm{z1}(\gamma)\mathrm{E} \qquad 10)$$

The charged hadrons have statistical errors, errors due to inhomogenities, and the "noncompensation" error. The normal errors come from the dkj^2 term in Eq. 11, while the noncompensation term is proportional to kj^2. The error dEo(ch) can be read off from Eq. 9. As above, energy independence of the calibration constants, and fo, has been assumed. This assumption is not critical to the argument and is merely a convenient approximation.

$$\left(\mathrm{dEo(ch)}\right)^2 = \Sigma\left(\mathrm{dkj}^2 + \mathrm{kj}^2\mathrm{dfo}^2(\varepsilon'_E - \varepsilon'_H)^2\right)$$
$$= \Sigma\,\mathrm{dkj}^2 + \left(\mathrm{dfo}(\varepsilon'_E - \varepsilon'_H)\right)^2 \Sigma\,\mathrm{kj}^2 \qquad 11)$$

In analogy to Eq. 10, the error on the charged hadrons has contributions from the stochastic terms, the constant terms, and the noncompensation terms relevant to the individual hadrons. The leading charged fragment in the jet is labelled by z1(h).

$$\left(\mathrm{dEo(ch)}\right)^2 \sim a_H^2\left(1-\mathrm{Fo}\right)\mathrm{E} + \left(b_H\,\mathrm{z1(h)E}\right)^2$$
$$+\left(\mathrm{b(e/h)\,z1(h)E}\right)^2 \qquad 12)$$

We are now in a position to gather the errors together. The separate neutral and charged stochastic errors are weighted by Fo and (1-Fo) respectively. The neutrals have a constant term contribution, while the charged particles have a similar term plus an additional one due to noncompensation.

$$\frac{\mathrm{dEo(J)}}{\mathrm{E}} \sim \frac{1}{\sqrt{\mathrm{E}}}\left(a_E\sqrt{\mathrm{Fo}} \oplus a_H\sqrt{1-\mathrm{Fo}}\right)$$
$$\oplus\, b_E\,\mathrm{z1}(\gamma) \oplus \mathrm{z1(h)}\left(b_H \oplus \mathrm{b(e/h)}\right) \qquad 13)$$

For a perfectly uniform detector, the stochastic terms make physical sense. If they dominate, then the "jet" has errors which are the same as the sum of the jet parts. In the high energy regime, the constant terms are expected to dominate. The form of these terms for the jet is familiar from a look at Eq. 3. If one has built a very uniform, but noncompensating calorimeter, then, at high energies, the term with the b(e/h) factor will dominate. The factor multiplying b(e/h) means that the effect is reduced with respect to that for a single particle.

An algebraic realization of the previous work is given in the following example. Consider a jet with a 5000 GeV momentum. Considered as a single particle, one expects that, for a = 0.5 and b = 0.02, the stochastic and constant error is 2.12%. The e/h error, if $\varepsilon'_H/\varepsilon'_E$ = 1/1.3, is 6%, for a total error of 6.4%. The form of Eq. 13 implies that, for z1 = 0.23, [3] there is a 0.84% stochastic and constant error. The b(e/h) error contributes 1.4%, leading to a total estimated error of 1.6%.

The result of the detailed algebra needed to check this approximation is presented in Fig. 3. The momentum fraction for the fastest 20 fragments is shown in Fig. 3a. This method of estimation gives a good feel for the average fragmentation, although the fluctuations are not well represented [3]. In Fig. 3b is shown the constant and stochastic error, dashed line, and the noncompensation error, solid line. The resolution error is 46.7 GeV (0.94%). The noncompensation error is 95.7 GeV (1.9%), while the total error is 106.4 GeV or 2.1%. Clearly, the approximation of the zi^2 sum by $z1^2$ has underestimated by a small factor since Eq. 13 leads to an estimated 1.6% error. A glance at Fig. 3b tells one that the next to fastest fragment is, indeed, a non-negligible correction.

6. MONTE CARLO JET MODEL

More detailed work requires a Monte Carlo model [4]. In particular, fragmentation and its variations leads to additional errors [3]. An optimized cluster cone radius in (η,ϕ) of R = 0.6 for 10 TeV dijets was first established. Stochastic and constant errors for neutrals of a_E = 0.2 and b_E = 0.01 were assumed. Errors for hadrons were assumed to be, a_H = 0.5 and b_H = 0.03. The noncompensation term was taken from Eq. 7. The non-linearity induced by the slow energy variation of fo and dfo was parametrized as in Ref. 5, with Eo = 1 GeV, m = 0.85.

$$e/\pi = \left[\frac{1}{1-(1-\varepsilon'_H/\varepsilon'_E)(E/Eo)^{m-1}}\right] \qquad 14)$$

Note that e/π approaches 1 as E gets very large. This asymptote makes physical sense, since neutral fluctuations become small at high energies. Note that a calibration is implied here. The EM compartment gives a response equal to the hadron compartment at high energy. Note that this constraint is consistent with the calibration procedure which was defined previously. As an example, for E = 100, 1000 GeV and $\varepsilon'_H/\varepsilon'_E$ = 1/1.3 one gets e/π = 0.88 and 0.92 respectively. Two possibilities were explored. First, that the calorimeter was calibrated only at asymptotic energy. In this case the non-linearity defined in Eq. 14 implies an additional error beyond any discussed so far. This is because we assumed previously that the weakly energy dependent calibration implied in Eq. 5 had been performed. That was the second possibility which was considered.

The mass spectrum for 10 TeV dijets is shown in Fig 4. The spectrum of Fig. 4a has clustering errors in addition to calorimeter resolution errors. The spectrum of Fig. 4b is as for Fig. 4a except that $\varepsilon'_H/\varepsilon'_E$ = 1/1.3 and thus an additional error, b(e/h), has been folded in quadrature. The non-linearity of Eq. 14 induces an error of size comparable to that due to b(e/h). The spectrum given in Fig. 4c is as in Fig. 4b, except that the energy dependent calibration is not performed, so that the hadron response is non-linear and reduced by π/e (Eq. 14). The fractional shift down in reconstructed mass from Fig. 4b to Fig. 4c is similar to that expected for the leading fragment, $< z1 > \sim 0.2$, $< k1 > \sim 1000$ GeV, of 0.92. The distribution is only slightly broadened by non-compensation which is as expected from the numerical estimates made above and displayed in Fig. 3.

Note that 10 TeV is the highest accessible mass, and thus the most sensitive to b(e/h). Monte Carlo studies were also performed at 0.1 and 1.0 TeV. The results are shown in Fig. 5. Clearly for masses of 1 TeV and less, the effects of $\varepsilon'_E/\varepsilon'_H = 1.3$ or less are barely discernible. Even at 10 TeV mass, the fractional mass resolution is only increased by 30%.

It would appear that the Monte Carlo study confirms the assertion that the errors for a jet, in the high energy limit, are reduced with respect to those for a single particle. This is in accord with Eq. 3, but not in accord with the concept that a jet must be like a particle save that its first interaction is external to the calorimeter. In fact, the first interaction fluctuation, which has been explicitly extracted from the calorimeter, is the fluctuation to which we have ascribed the major error. However, in our simple minded models, if not in the Monte Carlo work, we have not yet allowed the composition of the jet to vary, and thus have evaded the fluctuations in the "first generation". This is clearly not fair and must be explored.

7. "FIRST GENERATION" FRAGMENTATION FLUCTUATIONS

How is it that the composition fluctuations of the jet may be evaded? In Section 4 we argued that an incident hadron interacting in the HAD compartment had first generation fluctuations which caused a substantial error in the case of a noncompensating calorimeter. For example, $\varepsilon'_H/\varepsilon'_E = 1/1.3$ induced a 6% constant term error. What about fluctuations in the fragmentation function D(z) which cause the ensemble in the jet to vary in its neutral fraction Fo?

The calibration constants which have been imposed insure that the error coefficient relating dFo to dEo(J) is zero! Consider again the energy output for the jet sum. Without imposing the calibration constraint the energy output is,

$$\mathrm{Eo(J)} = \sum \mathrm{ki} + \varepsilon'_E \sum \mathrm{kj}\left[\mathrm{fo} + (1-\mathrm{fo})\,\varepsilon'_H/\varepsilon'_E\right]$$

$$\frac{\mathrm{Eo(J)}}{\mathrm{E}} = \mathrm{Fo} + (1-\mathrm{Fo})\,\varepsilon'_E\left[\mathrm{fo} + (1-\mathrm{fo})\,\varepsilon'_H/\varepsilon'_E\right] \qquad 15)$$

Clearly, given the calibration condition for the HAD compartment, Eq. 5, Eo(J) does not depend on Fo. For example, if $\langle fo \rangle = 0.33$, and $\varepsilon'_H/\varepsilon'_E = 1/1.3$, then $\varepsilon_E=1$ and $\varepsilon'_E = 1.2$. If the calibration condition is not imposed, then Eo(J) depends on Fo.

$$\left(\frac{\mathrm{dEo(J)}}{\mathrm{E}}\right)_{\mathrm{dFo}} = \mathrm{dFo}\left[1-\varepsilon'_E\left[\mathrm{fo} + (1-\mathrm{fo})\,\varepsilon'_H/\varepsilon'_E\right]\right]_{\mathrm{fo}=\langle \mathrm{fo}\rangle} \qquad 16)$$

Clearly, if calibrated properly, the calorimeter gives a resultant output which is, on average, not sensitive to dFo. The effect is then second order. For example, if we simultaneously allow dfo, then for dfo = 0.2, the fractional error, dEo(J)/Eo(J) = 1% which should be compared to the 6% effect one expects in the "single particle" jet case. This expectation agrees with the Monte Carlo results shown in Fig. 4 where the jet fragmentation dFo is operational.

Suppose one has a poor calibration, such as $\varepsilon_E = \varepsilon'_E = 1$. This is far from the optimized case, $\varepsilon_E = 1$, $\varepsilon'_E = 1.2$ discussed above. In that case, for $\langle fo \rangle = 0.3$ and $\varepsilon'_H/\varepsilon'_E = 1/1.3$, Eq. 16 yields a fractional jet energy error of 3.2%, which is rather closer to the expected single hadron e/h error, b(e/h) of 6%.

Clearly the measurement process for jets and hadrons is somewhat different. For a single hadron, it will likely interact in a homogenous hadron compartment which is blind to the differences in response of the calorimeter to hadrons and photons. For a jet, one measures the EM fraction in the EM compartment, and independently measures the hadrons in the HAD compartment. This implies that Fo is tagged for a jet, while fo is not known on an event by event basis. The essential ingredient is that the 2 compartments be calibrated such that each give the same response for the same incident energy for incident photons and hadrons respectively. Therefore, since the basic measurement procedure is different, it is perhaps no surprise that the errors for the 2 cases might be different. A hadron strikes the HAD compartment and all knowledge of fo is lost; a jet strikes the segmented calorimeter and Fo is explicitly tagged in the EM segment.

REFERENCES

1. R. Wigmans, NUC, Inst. Meth. A 259 (1987)

2. Review of Particle Properties, Phys. Lett. B 239 (1990)

3. D. Green, Dijet Mass Resolution and Compensating Calorimetry, Fermilab TM-1740 May, 1991

4. A. Para et al., Jet Energy Resolution of the Dec., 1990 SDC Detector, Fermilab TM-1720

5. D.E. Groom, Proceedings of the Workshop on Calorimetry for the SSC, Ed. R. Donaldson and M.G.D. Gilchriese, World Scientific (1990)

FIGURE CAPTIONS

Figure 1. Gedanken experiment for uniform, homogeneous calorimeter

a. Single hadron incident on a calorimeter.
b. An ensemble of hadrons of total energy E incident on a calorimeter.

Figure 2. Gedanken experiment for a segmented calorimeter with 2 compartments, an electromagnetic (EM) and an hadronic (HAD). The response of these compartments to neutral pions and hadrons is different, and labelled by ε_E, ε'_E and ε'_H.

a. Incident neutral pions and hadrons
b. An incident jet, J, consisting of a neutral fraction Fo of neutral pions. The fractional jet energy carried by the fragment is defined by zi. In

the hadron compartment, the fractional neutral energy in the hadron shower is defined to be fo.

Figure 3. Results of a numerical calculation for the fastest 20 jet fragments.

a. Fractional jet energy as a function of fragment number, where fragment 1 is the fastest.
b. Error in the determination of the fragment energy. The dashed line corresponds to $dk = 0.5\sqrt{k} \oplus 0.02$. The solid line is the error due to $\varepsilon'_E/\varepsilon'_H = 1.3$ alone.

Figure 4. Results of a Monte Carlo simulation for 10 TeV dijets. Histogram of the ratio of reconstructed to generated mass.

a. For a compensating calorimeter.
b. For a calorimeter with $\varepsilon'_E/\varepsilon'_H = 1.0/1.3$ which is calibrated at all energies.
c. For a calorimeter as in b. except that it is only calibrated at asymptotic energies.

Figure 5. Fractional mass resolution for dijets as a function of mass. The points/curves correspond to Monte Carlo results for ,•, compensating calorimeter ,o, energy calibrated, noncompensating ($\varepsilon'_E/\varepsilon'_H = 1.3$) calorimeter, and ,$\nabla$, asymptotically calibrated, noncompensating calorimeter.

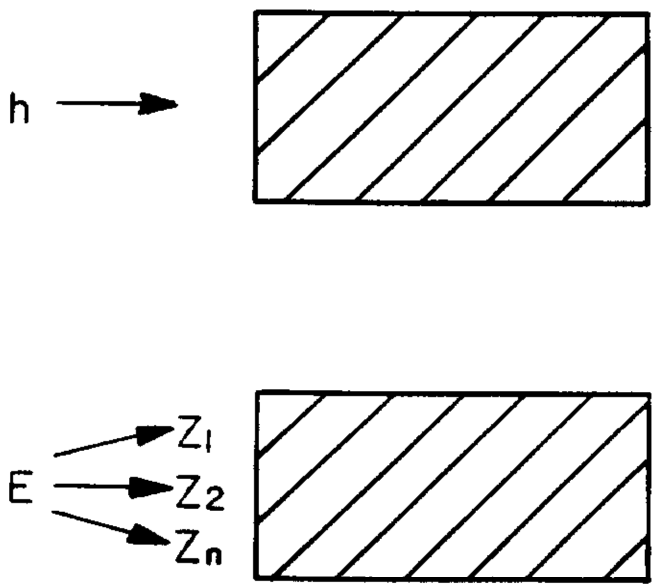


Fig. 1

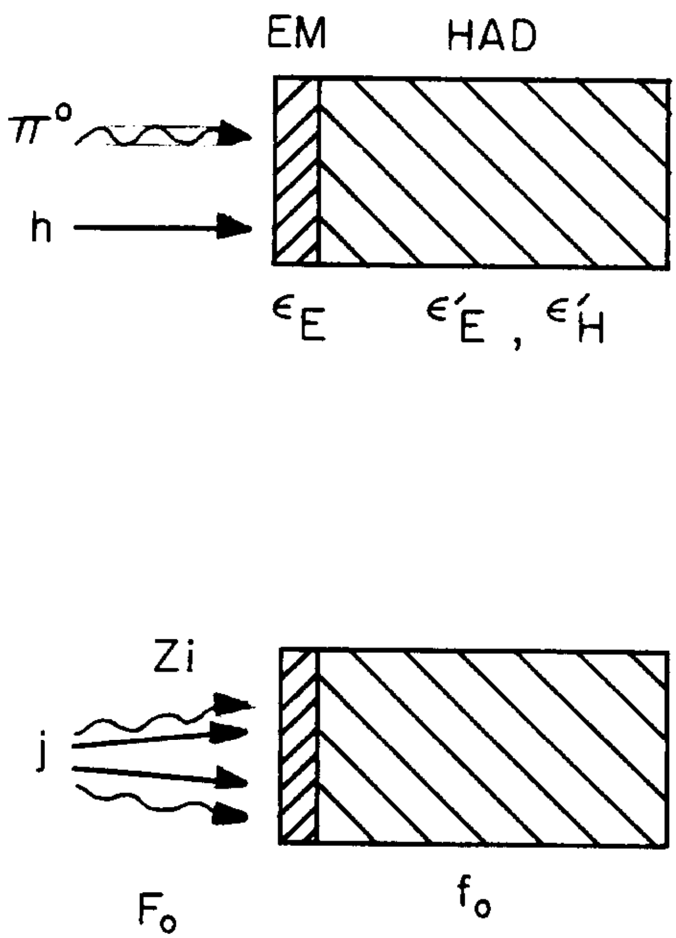


Fig. 2

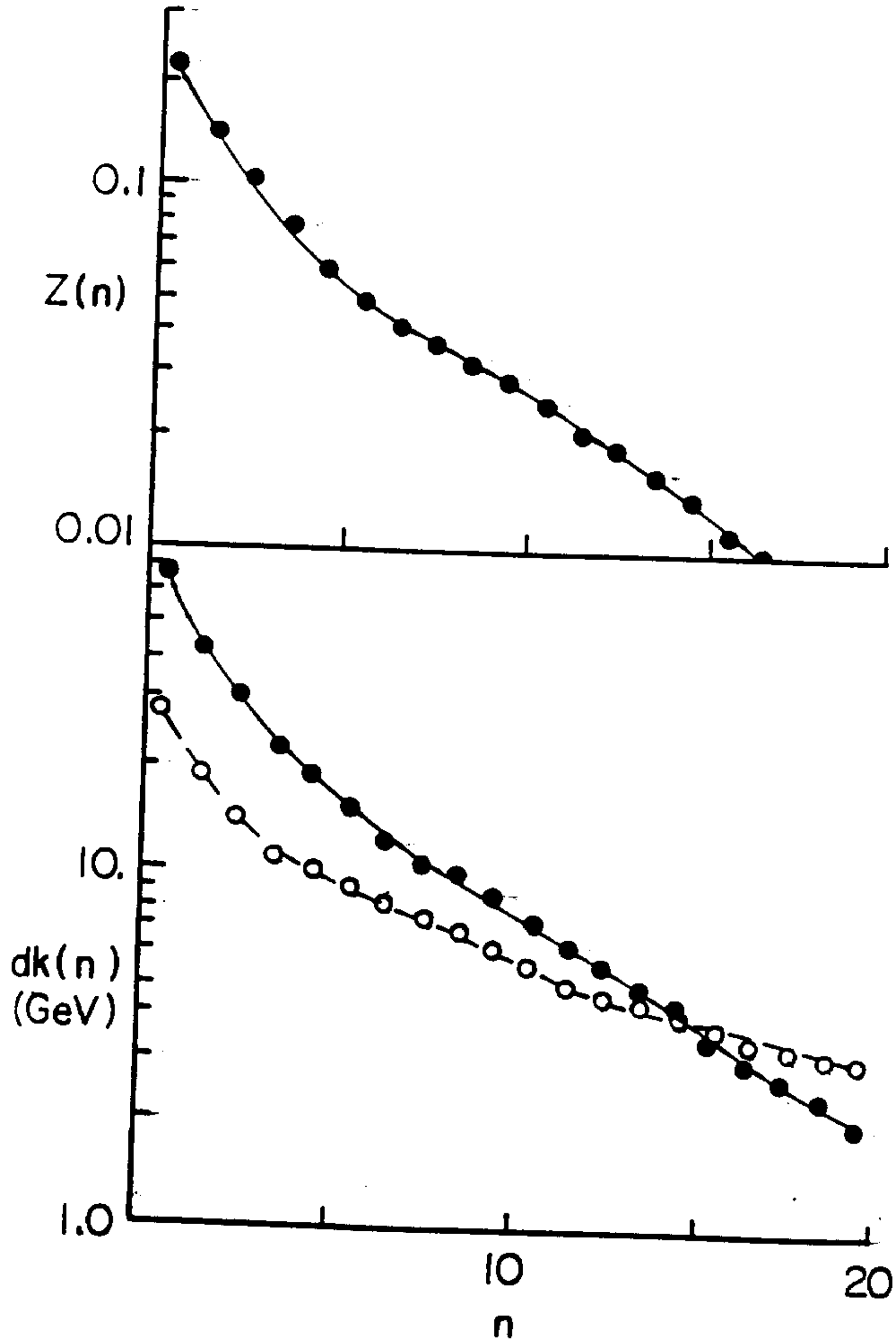
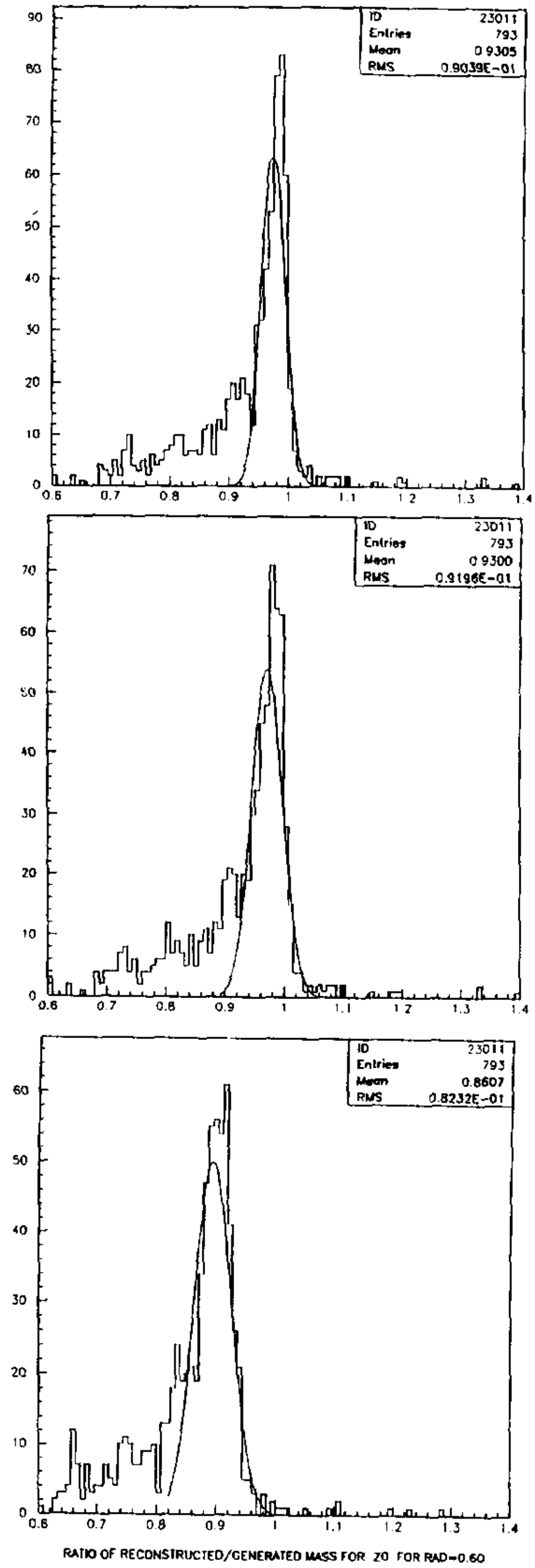


Fig. 3



ID 23011
Entries 793
Mean 0.9305
RMS 0.9039E-01
ID 23011
Entries 793
Mean 0.9300
RMS 0.9196E-01
ID 23011
Entries 793
Mean 0.8607
RMS 0.8232E-01
RATIO OF RECONSTRUCTED/GENERATED MASS FOR Z0 FOR RAD=0.60

Fig. 4

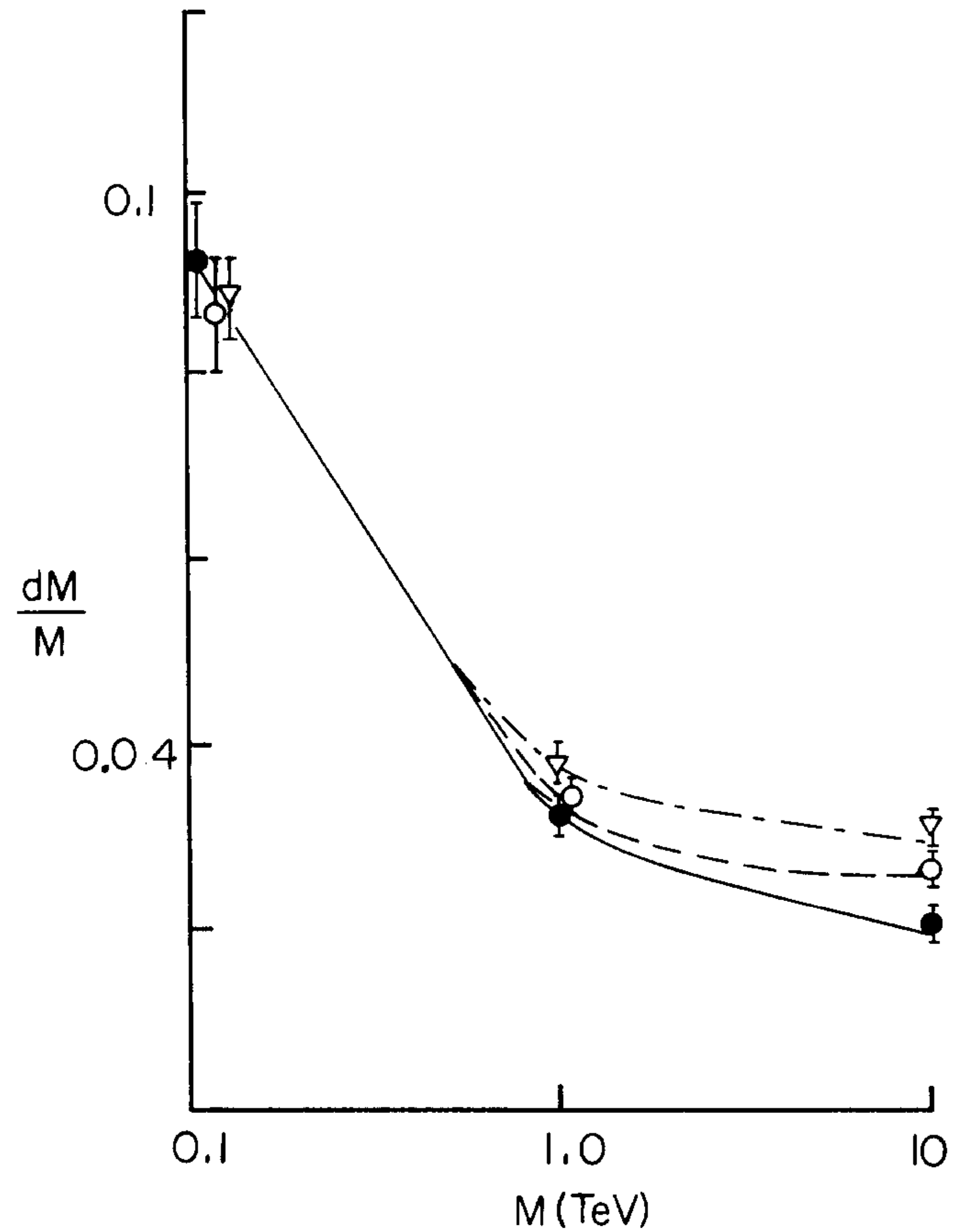


Fig. 5