Fermi National Accelerator Laboratory

FN-551

Longitudinal and Transverse Instabilities Around a γ_T Jump *

S. Y. Lee
Brookhaven National Laboratory
Upton, New York 11973

K.-Y. Ng
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, Illinois 60510

October 1990

* Submitted to the proceedings of the Fermilab III Instabilities Workshop, St. Charles, Illinois, June 25-29, 1990.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

LONGITUDINAL AND TRANSVERSE INSTABILITIES AROUND A γ_T JUMP

S.Y. Lee*

Brookhaven National Laboratory, Upton, NY 11973

and

K.Y. Ng

Fermi National Accelerator Laboratory,† *P.O. Box 500, Batavia, IL 60510*

1. Introduction

When a particle beam is in the transition region, the phase-slip parameter

$$\eta = \frac{1}{\gamma_T^2} - \frac{1}{\gamma^2} \tag{1}$$

goes through zero. This reduces the spread in revolution frequency inside the beam by so much that collective instability growths can no longer be Landau damped. With the implementation of a γ_T jump, η dips down to zero only for a very short but finite time interval, typically less than 0.5 ms or 1 ms, which is necessary in order to minimize transverse emittance growth.[1] Besides this, during the jump the phase-slip factor is roughly given by

$$|\eta| \approx \left|\frac{1}{(\gamma_T - \Delta\gamma_T/2)^2} - \frac{1}{\gamma_T^2}\right| \approx \left|\frac{\Delta\gamma_T}{\gamma_T^3}\right| \ , \tag{2}$$

where $\Delta\gamma_T$ is the amount of jump, which is negative for the Main Injector. The duration of the jump is typically about 10 ms. Thus, the spread in revolution frequency may still be small enough and the time long enough for collective microwave instabilities to develop. The purpose of this paper is to examine the microwave thresholds in both the longitudinal and transverse modes.

*Work performed under the auspices of the U.S. Department of Energy.

†Operated by the Universities Research Association, Inc., under contract with the U.S. Department of Energy.

2. Longitudinal microwave threshold

Let us start with the longitudinal microwave threshold for a Gaussian bunch developed by Krinsky and Wang.[2] The threshold longitudinal impedance per harmonic is given by

$$\frac{Z_\|}{n} = \frac{2\pi|\eta|(E/e)}{I_p}\left(\frac{\sigma_E}{E}\right)^2 , \tag{3}$$

where E is the energy of the synchronous particle and σ_E the rms energy spread of the beam. For a Gaussian bunch, the peak current is

$$I_p = \frac{eN}{\sqrt{2\pi}\sigma_\tau} \tag{4}$$

where N is the number of particles per bunch each carrying charge $e = 1.602 \times 10^{-19}$ Coulomb, and σ_τ is the rms bunch length in time. We try to express $Z_\|/n$ in terms of quantities which are constants near transition, such as the rf voltage $V_{\rm rf}$, rf phase ϕ_0, and 95% bunch area

$$S = 6\pi\sigma_E\sigma_\tau . \tag{5}$$

Since we are interested in the region around γ_T jump, where η given by Eq. (2) is finite, the rf bucket should be well defined and therefore Eq. (5) holds. One may argue that the bunch area is not a constant of motion because it usually grows during transition. However, the purpose of a γ_T jump is to eliminate the two tails developed in the longitudinal phase space due to nonlinear effects, as well as any collective instabilities if the jump were absent. Therefore, when a γ_T jump is properly implemented, the bunch area should remain the same before, during, and after the jump.

After some algebraic manipulation, we obtain the result

$$\frac{Z_\|}{n} = \frac{(2\pi)^{\frac{7}{4}} f_0^{\frac{3}{2}} h^{\frac{1}{4}}}{Ne}\left(\frac{S/e}{6\pi}\right)^{\frac{3}{2}}\left(\frac{|\eta|}{E/e}\right)^{\frac{3}{4}}(V_{\rm rf}\cos\phi_0)^{\frac{1}{4}} , \tag{6}$$

where $h = 588$ is the rf harmonic number of the Main Injector and $f_0 = 90.2$ kHz is the revolution frequency. Substituting Eq. (2) into Eq. (6), we obtain the threshold impedance

$$\frac{Z_\|}{n} = 10.0\ [\Omega]\left(\frac{N}{6\times 10^{10}}\right)^{-1}\left(\frac{S}{0.4\ [\text{eV-s}]}\right)^{\frac{3}{2}}\left(\frac{V_{\rm rf}\cos\phi_0}{2.78\ [\text{MV}]\cos 37.6^\circ}\right)^{\frac{1}{4}}(|\Delta\gamma_T|)^{\frac{3}{4}} , \tag{7}$$

where $\gamma_T = 20.4$ has been used. The result is plotted in Fig. 1 versus bunch area for different rf voltage $V_{\rm rf}$ with the rate of acceleration $\dot\gamma_T = 163$ sec^{-1} kept constant.

3. Transverse microwave threshold

The transverse impedance threshold for transverse microwave instability for a Gaussian bunch developed by Ruth and Wang[3] is given by

$$Z_\perp = \frac{4\sqrt{2}(E/e)}{I_p\bar{\beta}}\left(\frac{\sigma_E}{E}\right)|(n-\nu)\eta - \xi\nu| \ , \tag{8}$$

where $\bar{\beta}$ is the average beta function, ν the betatron tune, and ξ the chromaticity defined as the ratio of the fractional tune spread to fractional momentum spread. The harmonic number n is the position of the broad-band transverse impedance driving the instability. In terms of the 95% bunch area S, we have

$$Z_\perp = \frac{4}{3}\frac{\nu(S/e)}{NeR}|(n-\nu)\eta - \xi\nu| \ , \tag{9}$$

where $R = 528.3$ m is the average radius of the Main Injector. With a tune of $\nu = 22.42$, an evaluation gives

$$Z_\perp = 2.3\left[\frac{\mathrm{M}\Omega}{\mathrm{m}}\right]\left(\frac{N}{6\times 10^{10}}\right)^{-1}\left(\frac{S}{0.4\ [\mathrm{eV\text{-}s}]}\right)^{\frac{3}{2}}|(n-\nu)\eta - \xi\nu| \ . \tag{10}$$

This result is plotted in Fig. 2 as a function of bunch area.

The harmonic n of the driving broad-band is usually taken as the ratio of R to $b \approx 5$ cm, the half-width of the beam pipe; or $n \approx 10400$. Then, during the γ_T jump,

$$(n-\nu)\eta = \mp 1.23|\Delta\gamma_T| \ , \tag{11}$$

which can be comparable to $\xi\nu$ depending on the pulsing of the correction sextupoles. In other words, unless special care is taken in the control of the chromaticity, the transverse impedance threshold can dip down to a dangerously tiny value.

4. Conclusion

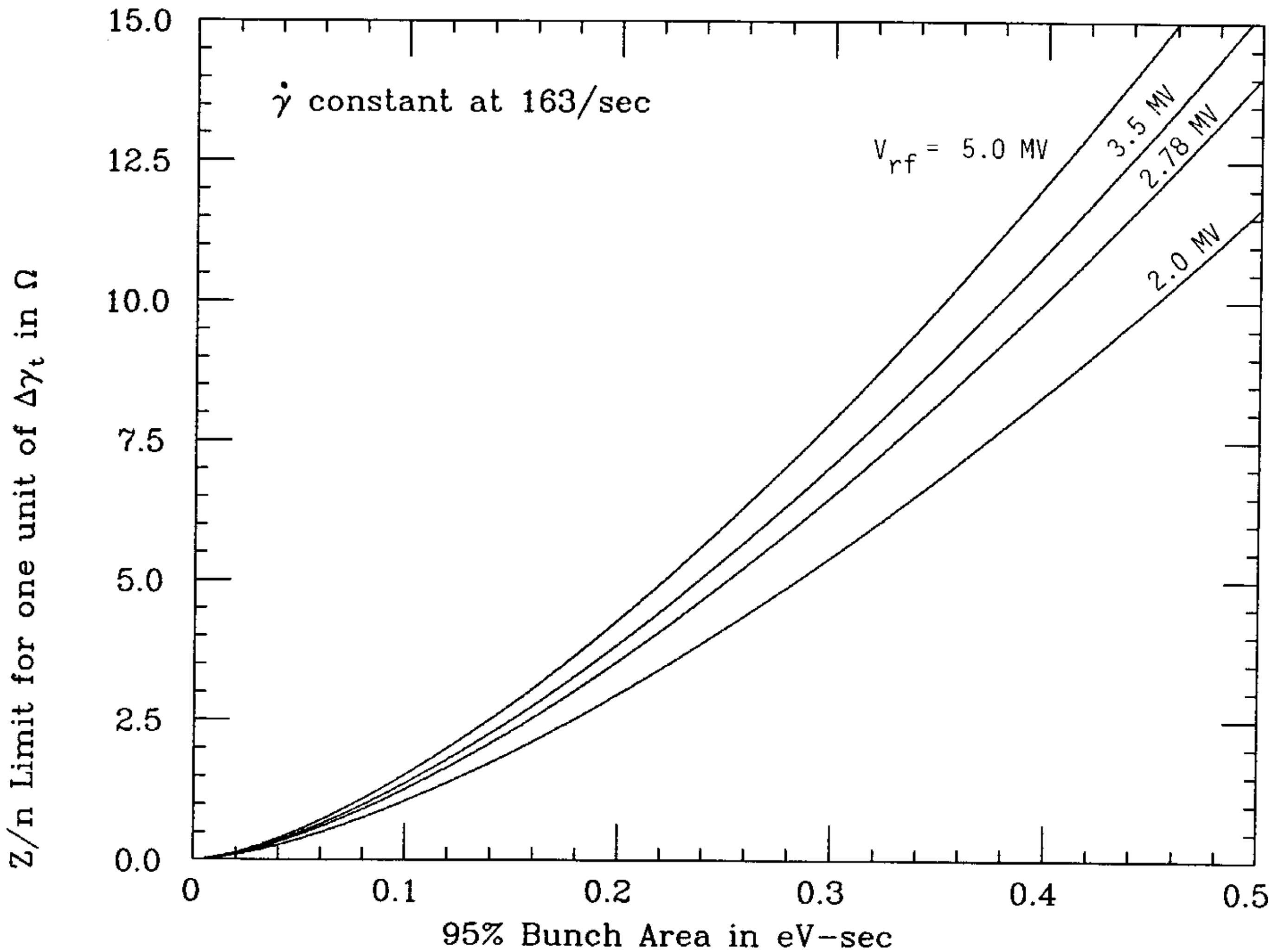
We see that the longitudinal microwave threshold before and after a γ_T jump of one unit is only $Z_\parallel/n = 10\ \Omega$ in the present proposed operation.[4] It is in fact the lowest in the whole cycle of the Main Injector as is shown in Fig. 3 for a possible cycle (without γ_T jump). Although the total time for the γ_T jump is of the order of

10 ms, away from which η increases rapidly, nevertheless, microwave growth of the bunch area can still be important.

The transverse microwave threshold can become more stringent if the natural chromaticity is mostly corrected. Thus, the impedance budget is still an important issue even when a γ_T jump is implemented.

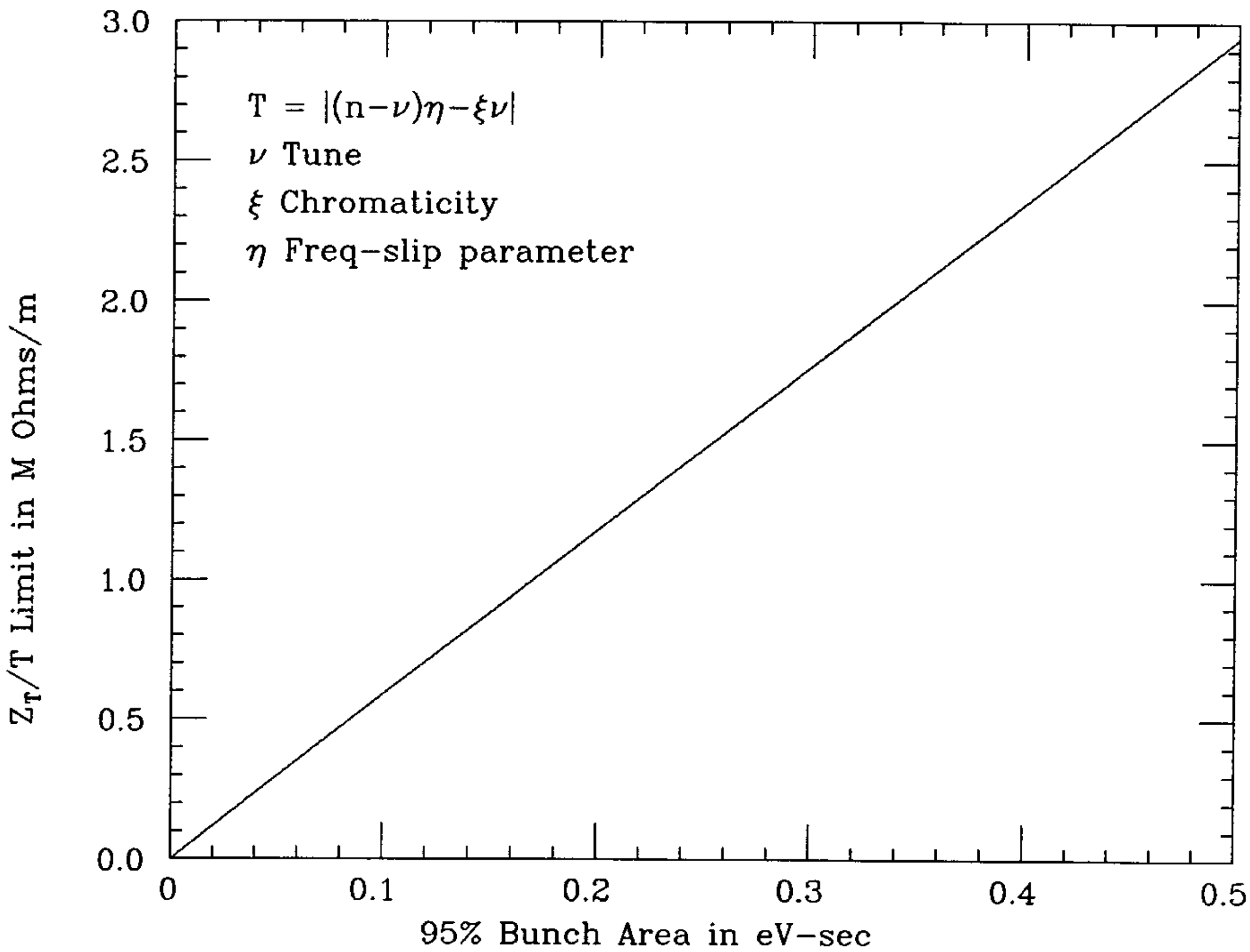
REFERENCES

1. K.Y. Ng, *Emittance Growth During* γ_T *Jump*, Fermilab Report FN-552, 1990.
2. S. Krinsky and J.M. Wang, Particle Accelerators **17**, 109 (1985).
3. R. Ruth and J.M. Wang, IEEE Trans. Nucl. Sci. **NS-28**, 2405 (1981).
4. K.Y. Ng, *Main Injector rf Cycle and Z/n Limit*, Fermilab MI-0029, 1990.



$\dot{\gamma}$ constant at 163/sec
V_{rf} = 5.0 MV
3.5 MV
2.78 MV
2.0 MV
15.0
12.5
10.0
7.5
5.0
2.5
0.0
0
0.1
0.2
0.3
0.4
0.5
95% Bunch Area in eV-sec
Z/n Limit for one unit of $\Delta\gamma_t$ in Ω

Figure 1



3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
0.1
0.2
0.3
0.4
0.5
$T = |(n-\nu)\eta - \xi\nu|$
ν Tune
ξ Chromaticity
η Freq-slip parameter
Z_T/T Limit in M Ohms/m
95% Bunch Area in eV-sec

Figure 2

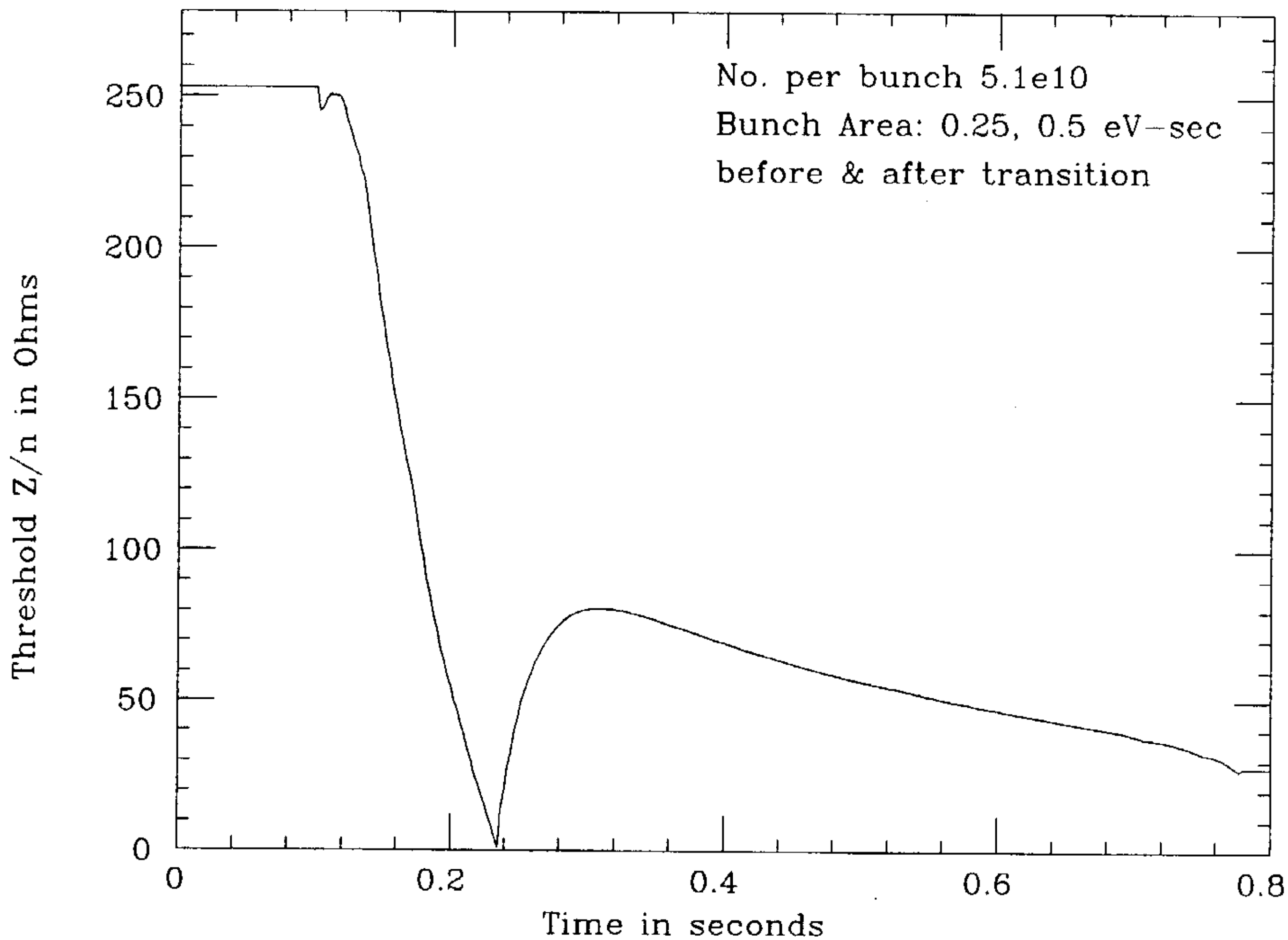

No. per bunch 5.1e10
Bunch Area: 0.25, 0.5 eV−sec
before & after transition
Threshold Z/n in Ohms
0
50
100
150
200
250
0
0.2
0.4
0.6
0.8
Time in seconds

Figure 3