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CDF Luminosity Calibration *

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Abstract

This note deals with the calibration of the CDF luminosity at the two principal running energies of $\sqrt{s}$=546 and 1800 GeV.

We find that for the lower energy running the luminosity scale (or equivalently- the effective cross section seen by the BBC's) is best determined using UA4's measurement of the cross section to which their trigger counters are sensitive. The smallest uncertainty in the luminosity scale at higher energy is obtained by also making use of the accelerator calculation of the luminosity.

We study the reliability of the calculated luminosity by comparing it with measured BBC rates at each of the two energies. The accelerator calculation gives the most reliable way of determining the relative instantaneous luminosity in two runs, one at 546 GeV and the other at 1800 GeV.

We obtain the following results: $\sigma^{eff}_{BBC}(546)=36.0\pm1.8$ mb and $\frac{\sigma^{eff}_{BBC}(1800)}{\sigma^{eff}_{BBC}(546)}=1.30\pm.06$. As a consequence CDF cross sections at $\sqrt{s}$=1800 GeV, normalized using LUMBBC, should be multiplied by $\frac{46.8}{44.0}$ and assigned an uncertainty from luminosity of $\pm6.8\%$.

1 Introduction

The CDF luminosity monitor consists of scintillation counters which subtend the rapidity range $3.3\leq \eta \leq 5.9$ on either side of the interaction region [1]. The coincidence rate in the Beam-Beam Counters (BBC's) is corrected, on the basis of instantaneous rate, for multiple interactions and then integrated over the run to obtain $\int R_{BBC}\mathrm{dt}=N_{BBC}$.

An effective cross section, σ^{eff}_{BBC}, for the BBC coincidence rate is defined, which relates the rate to the integrated luminosity:

$$\sigma^{eff}_{BBC}\cdot(\int \mathcal{L}\cdot dt) = N_{BBC} \tag{1.1}$$

When defined in this way, σ^{eff}_{BBC} is a detector dependent quantity since it may include inefficiencies, sensitivity to photon conversions and other secondary effects. The integrated BBC rate is not corrected for (non beam-beam) background but, as we will see, this background is negligibly small.

The module 'LUMBBC', which is normally used to get integrated luminosity for CDF runs from the scaler information , uses a fixed value of σ^{eff}_{BBC}= 44.0 mb at all c-of-m energies. The same scaler information was provided to the accelerator control room during data taking, via a ratemeter next to our ACNET console. The instantaneous luminosity calculated from BBC rates was logged on the accelerator databases with the name 'B0LUMP'.

An ACNET process - *T106* [10]- computed the luminosity from flying wire scans and this luminosity as well as a number of relevant parameters were logged in the databases.

It is the purpose of this note to find the best value for σ_{BBC}^{eff} . We make frequent use of 'B0LUMP' (also referred to as $\mathcal{L}_{BBC}$) in conjunction with the accelerator based calculation of the instantaneous luminosity from Tevatron parameters ('flying wire data').

Our approach has been to:

- study how well variations in the luminosity with respect to machine related parameters are accounted for by the calculation.
- see whether, on average, the relevant machine parameters are similar enough at the two c-of-m energies that we can trust the relative luminosity calculation between runs at different energies.
- use the calculation to measure the effective cross section increase (i.e. the change in normalized trigger rates in going from $\sqrt{s}$=546 to 1800 GeV).

Because the absolute scale of the accelerator luminosity calculation has an 11% uncertainty we need an additional constraint at one energy. So we examine the UA4 total cross section measurement [4] in which $\sigma_{BBC}^{eff}(546)$ for a similar counter arrangement is a primary quantity from which other inelastic cross sections are derived.

1.1 Earlier versions

Previously the effective cross section seen by the Beam Beam Counters was calculated [1] using the UA4 and UA5 published values for the non-diffractive and diffractive inelastic cross sections. The CDF minimum bias Monte Carlo (MBR) was used for acceptance calculation to get σ_{BBC}^{eff} at SPS energy. At $\sqrt{s}$=1.8 TeV, reasonable extrapolations of the SPS cross-sections were input to the Monte Carlo. This method was estimated to have a reliability of ±15%.

An alternate determination of the 1.8 TeV effective cross section [2] used the accelerator luminosity calculation as will be described herein.

In this note we use an improved method to get σ_{BBC}^{eff} from the UA4 measurements, which has very little Monte Carlo dependence. Another important difference concerns the accelerator calculation of the luminosity which is now better understood. In this note we use a "full" calculation whereas in ref. [2] the *T106* calculation was adjusted by an average correction at each energy. We are also able to look for possible systematic errors in the calculation by comparing it to BBC rates for different beam conditions. These studies were made possible by a re-analysis of the *T106* input data by Norman Gelfand [7].

1.2 Organization

This note is divided into three main parts.

- In section 2. we discuss the UA4 experiment. A byproduct of that experiment was a determination of the fraction of the inelastic cross section which was seen by a two arm coincidence of their trigger counters. The angular coverage of these counters is very similar to that of the CDF Beam Beam counters and only a small correction is needed to relate the two. We also apply a correction for our counter efficiency.

- In section **3.** we discuss the accelerator calculation of the luminosity and the measurements (flying wire scans) from which it is derived. In conjunction with BBC rates, this is effectively a measurement of the BBC effective cross section:

$$\sigma^{eff}_{BBC} = \frac{R_{BBC}}{\mathcal{L}_{accel}} \tag{1.2}$$

  Every flying wire scan can be viewed as another measurement of this quantity. We focus on the relative calibration at the two energies obtained by this method.

- In the last section we collect results from sections **2.** and **3.** to obtain best values for the luminosity calibration at the two energies.

2 UA4 and the BBC Effective Cross-Section

Determining the scale of our integrated luminosity is equivalent to finding the BBC effective cross-section. The BBC rates have been summed during the live time of the run and the cross section for any given process, σ_i, is calculated from the number of events of class i observed in a run:

$$\sigma_i = N_i \cdot \frac{\sigma^{eff}_{BBC}}{N_{BBC}} \tag{2.1}$$

$$\mathcal{L}^{-1} \equiv \frac{\sigma^{eff}_{BBC}}{N_{BBC}}$$

In the rest of this discussion N_{events}(BBC) has been corrected (upward) for multiple interations (typically 8% at L $=10^{30}$). The rate, as we will see in Appendix B, is also essentially background free.

The motivation for using a UA4 based determination of the effective BBC cross-section at $\sqrt{s}$=546 GeV is simply that the absolute scale of the UA4 luminosity is better understood than our own .

In what follows we argue that the UA4 trigger counter effective cross section is at least as well determined as σ_{tot}. To be consistent with UA4's presentation of their analysis we derive this cross section from σ_{tot} . In fact UA4's total cross section is actually a derived quantity and $\sigma_{trigger}$ is more directly measured.

We've tried to evaluate $\sigma^{eff}_{BBC}(546)$ in a way that minimizes the sensitivity to the Monte Carlo input. Inevitably some choice of model enters into the derivation.

- first it enters into the UA4 derivation of σ_{tot} in the extrapolation to the unobserved part of the cross section.They argue that the data themselves guide the extrapolation and that its magnitude is small (Table 1). We discuss this further in the next section.

- secondly, a Monte Carlo is used to calculate the *small* difference between CDF and UA4 coverages in the double arm triggers. This is discussed in section 2.3.

Table 1: UA4 trigger Contribution (from [4])

Trigger Contribution	η-range	Fraction of N_{inel} %
Double-arm($=f_{DA}$)	3.0 - 5.6	82.7 $\pm$ 0.8
Single Arm	2.5 - 5.6	16.3 $\pm$0.6
Central Detector	$\leq$1.7	0.08 $\pm$0.04
Small-angle extrapolation	$\geq$ 5.6	0.9$\pm$0.2
Large angle correction	1.7 - 2.5	0.04 $\pm$ .02

2.1 The total cross section measurement

UA4's total cross section was obtained by what is now a classical method for colliding beam experiments [8]. The luminosity independent method uses the fact that by measuring the total interaction rate:

$$R_{tot} = \mathcal{L} \cdot \sigma_{tot} \tag{2.2}$$

$$R_{tot} = R_{elastic} + R_{inelastic}$$

simultaneously with the extrapolation of the differential elastic scattering rate to the optical point

$$(1+\varrho^2) \cdot \sigma_{tot}^2 = \mathcal{L}^{-1} 16\pi(hc)^2 \frac{dR_{elast}}{dt} \Big|_{t=0} \tag{2.3}$$

one can eliminate $\mathcal{L}$.

$$(1+\varrho^2) \cdot \sigma_{tot} = \frac{16\pi(hc)^2}{R_{elastic} + R_{inelastic}} \cdot \frac{dR_{elast}}{dt} \Big|_{t=0} \tag{2.4}$$

where ϱ is the ratio of the real to the imaginary part of the forward elastic scattering amplitude.

In fact, the UA4 inelastic analysis [4] uses measured trigger rates from a double arm (DA) coincidence with both sides of the interaction region (see Fig.1) and from a single arm (SA). An extrapolation of the rate to include events beyond the acceptance of these triggers is made using the distribution of tracks in the above events. Equation 2.2 could be rewritten as:

$$R_{elast} + R_{DA} + R_{SA} + R_{extrap} = \mathcal{L} \cdot \sigma_{tot} \tag{2.5}$$

where the inelastic rate has been divided into measured and unmeasured parts.

In the derivation below we take UA4 errors at their face value but it is interesting to see how extrapolation and other uncertainties influence the derived value of σ_{DA}. In terms of measured quantities we can write:

$$\sigma_{DA} = \frac{\sigma_{DA}}{\sigma_{tot}} \cdot \sigma_{tot} = \frac{R_{DA}}{(R_{elast} + R_{DA} + R_{SA} + R_{extrap})^2} \cdot \frac{16\pi(hc)^2}{(1+\varrho^2)} \cdot \frac{dR_{el}}{dt}|_{t=0} \tag{2.6}$$

$$\frac{\delta\sigma_{DA}}{\sigma_{DA}} = -2 \cdot \frac{\delta R_{extrap}}{R_{tot}} \tag{2.7}$$

For example, if the extrapolated rate were increased by 1% (this is the upper limit given in the UA4 paper) then σ_{DA} would decrease by 2%. UA4 found agreement between σ_{tot} measurements using equations 2.2 and 2.4. This effectively constrains R_{extrap}, since a change in R_{extrap} would effect the value of σ_{tot} obtained by these two equations in opposite directions.

Double arm triggers included in their analysis were verified to have tracks pointing to the interaction vertex. It is still interesting to see what would happen if there were inefficiencies or background to the double arm rate. Differentiating the above w.r.t. R_{DA} we have:

$$\frac{\delta\sigma_{DA}}{\sigma_{DA}} = \frac{\delta R_{DA}}{R_{DA}} \cdot (1 - 2 \cdot \frac{R_{DA}}{R_{tot}}) \tag{2.8}$$

Since, as we will see below, $\frac{\sigma_{DA}}{\sigma_{tot}} = \frac{38.9}{60.0}$ the fractional error in σ_{DA} is less than 1/3 of the fractional uncertainty in R_{DA}.

So, in conclusion, the quantity σ_{DA} which is of interest to us for luminosity calibration is given with conservative errors in the derivation below.

2.2 UA4 results

UA4 has published [4] the following measurements

- 1) $(1+\varrho^2)\cdot\sigma_{tot}$=63.3±1.5 ±0.6 mb.

  (using the luminosity independent method).

- 2) σ_{el}/σ_{tot}=0.215±0.005

- 3) $\sqrt{(1. + \varrho^2)} \cdot \sigma_{tot}$= 61.7± 3.0 mb. (using the machine luminosity)

- 4) ϱ=0.24±0.04

  From the measured value of (ϱ) and its errors (combining 1,4) they obtain

- σ_{tot}=59.5±1.4±1.2 mb.

  which includes uncertainties due to the error on ϱ.

  Similarly (combining 3,4) they obtain:

- σ_{tot}=60.0 from the machine luminosity.

To obtain the effective cross section seen by UA4's trigger counters we use 2) to derive $\sigma_{inelastic}$ and multiply by the fraction of inelastic events seen by the double arm trigger (1st entry of Table 1). We denote this by:

$$\sigma_{DA} = (1 - \frac{\sigma_{el}}{\sigma_{tot}}) \cdot \sigma_{tot} \cdot (f_{DA}) \tag{2.9}$$
$$= 38.9 \pm 1.8 mb$$

2.3 Comparison with CDF Min-bias Monte Carlo

At this point we need to use a model to calculate the effect of difference in acceptances between the UA4 double arm trigger and the CDF Level-0 trigger (i.e. BBC coincidence rate) for minimum bias events. The difference is, in fact, small so we don't expect the choice of Monte Carlo to be critical. The CDF minimum bias Monte Carlo [3] reproduces most of the features (track multiplicity, rapidity distributions) observed in UA4 and UA5 which are relevant to calculating trigger acceptance. In this sense there is no discrepancy between the Monte Carlo and UA4 data, on which it is tuned.

An apparent discrepancy arises when UA4's published cross- sections are used as input to the Monte Carlo to predict Double Arm and Single Arm rates. The predicted entries in Table 1 for different triggers are sensitive to input cross sections (particularly the relative amount of single diffraction dissociation cross section, σ_{SD}). This is the main reason for the large uncertainty quoted in CDF-552 [1] for $\sigma^{eff}_{BBC}(1800)$. From Table 2 it is clear that the choice of 14 mb. for $\sigma_{SD}(540)$, which is the default in the MBR generator, doesn't reproduce the trigger fractions of UA4 (Table 1). An input value of 8.2 mb. [6] gives better agreement but is incompatible with the UA4 published value [5] of σ_{SD}= 10.4$\pm$0.7 mb (the values quoted of 9.4$\pm$ 0.7 for $\frac{M_X^2}{s} \leq .05$ and 10.0 for $\frac{M_X^2}{s} \leq .10$ should be corrected to 10.4 to be consistent with the $\frac{M_X^2}{s}$ range defined in MBR).

There is another sense in which the CDF Minimum Bias simulation doesn't reproduce the data. The BBC multiplicities are higher than expected from mean multiplicity densities ($\frac{dN}{d\eta}$) and this difference is attributed to conversions, albedo and other secondary effects. The double arm fraction in Table 1 comes from an analysis (UA4's) in which tracks hitting the trigger counters were required to point back to the interaction vertex. This should eliminate spurious hits from albedo, etc. Nevertheless, because of the discrepancy between Tables 1 and 2 it is natural to ask how much the acceptance could be altered if conversions and nuclear interactions are not properly accounted for. To check this we ran a simulation in which all charged *or neutral* particles in the acceptance of the UA4 trigger counters registered as charged tracks. In this case the predicted fraction, f_{DA}, increased from 66.6% to 72% (even this cannot account for the UA4 measurement of 82.7%).

We conclude that the discrepancy between the UA4 results in Table 1 and the MBR simulation summarized in Table 2 arises primarily from a disagreement in the fraction of the inelastic cross section with a single diffractive (i.e. single arm topology). We note, however, that the relative acceptance correction we will apply is essentially insensitive to this fraction.

2.4 Relative Acceptance Correction

We now relate the UA4 measured rate to σ_{BBC}^{eff}. We anticipate the following (small) corrections to σ_{DA}:

- The trigger counters cover slightly different η ranges in the 2 experiments (3.3 to 5.9 vs 3.0 to 5.6 in UA4).
- The BBC's suffered radiation damage at the beginning of the '88-89 run and have less than 100% efficiency.

The results of a calculation of these effects (using the MBR Monte Carlo) are presented in Table 2. To illustrate the insensitivity of the small net correction to details of the Monte Carlo, two versions of MBR were compared. In the first case the default inelastic cross-sections were used. In the second case the single diffractive part was decreased by a factor of 2.

Table 2: Monte Carlo calculation of Acc(546) (%of inelastic events)

Cut	Standard MBR generator	With reduced σ_{diff}
BBC η-range	64.9	74.0
UA4 Double Arm η-range	66.6	75.6
BBC Full Simulation	68.1	77.5
BBC Simulation with counter inefficiencies(not used)	64.0	72.7
Resulting Correction ($=A_{\eta}^{BBC}/A_{\eta}^{UA4}$)	64.9/66.6=**.975**	74.0/75.6=**.979**

Table 2 shows that we can determine σ_{BBC}^{eff} from UA4 rates with little sensitivity to Monte Carlo input. Even with a drastic change in Monte Carlo input all acceptances seem to track each other.

In contrast to this there is good reason to believe that (because the Monte Carlo calculation neglects the effects of secondary interactions and other sources of extra tracks) the BBC efficiency is actually higher than given in Table 2. The actual efficiency correction which we've used (2.2%) is based on a direct check of BBC inefficiency as described in Appendix A. At present the uncertainty on the BBC overall efficiency is $\pm 2.2\%$. We now determine σ_{BBC}^{eff} from the calculated quantities in Table 2.

$$\sigma_{BBC}^{eff}(546) = (\frac{Acc_{\eta}^{BBC}}{Acc_{\eta}^{UA4}}) \cdot (\varepsilon^{BBC}) \cdot \sigma_{DA}$$

$$= (.975) \cdot (.978) \cdot (38.9mb.)$$

$$= 37.1 \pm (2.5\% \pm 2.2\% \pm 4.5\% = 5.6\%)mb \leftarrow (UA4 - derived)$$

Taking the weighted average with $\sigma_{BBC}^{eff}(546)$ calculated from the accelerator Luminosity as discussed in the following sections we get

$$\sigma_{BBC}^{eff}(546) = 36.0 \pm 1.81 mb \leftarrow (\textit{combined})$$

The situation at $\sqrt{s}$=546 GeV is summarized in Table 3.

Table 3: Derivations of BBC Effective Cross Sections, SPS energy.

1)	From Tevatron Luminosity calculation	0.746·44 mb =	32.8±3.6 mb
2)	From acceptance · SPS cross sections (corrected for efficiency)[1]	0.978·34.8 mb =	34.1±3.3 mb
3)	From UA4 σ_{DA} (corrected for acceptance and eff., derived in Section 2)	0.978·37.9 mb =	37.1±2.1 mb
	Weighted average of 1) and 3)		= 36.0±1.81 mb

3 Luminosity Measurement from Accelerator Parameters

The luminosity in bunched beam colliders can be calculated in terms of the product of the number of particles in the two colliding bunches divided by transverse beam size, summed over the number of bunch crossings at the interaction region. If the bunch length is negligible and the bunches are equal in intensity and size, the total luminosity can be expressed as

$$\mathcal{L} = B \frac{N_p N_{\overline{p}}}{4\pi\ \sigma_x \sigma_y} f \qquad (3.1)$$

where:

$N_p(N_{\overline{p}})$ = number of protons (antiprotons) per bunch,

σ_x (σ_y) = horizontal (vertical) size of the bunch at the interaction point taken as the r.m.s. average of the corresponding σ_p and $\sigma_{\overline{p}}$ (assuming gaussian distributions in both the vertical and horizontal dimensions),

B = number of bunches,

f = revolution frequency of the beams.

In fact, beam parameters can vary from bunch to bunch. The luminosity at a given interaction region must then be calculated as the sum of the luminosities for collinding bunches. Furthermore, the bunches have longitudinal distributions that can be approximated by gaussians, with σ_l typically of 30-40 cm. The overlap of the p and $\overline{\text{p}}$ longitudinal distributions will still be a gaussian with

$$\sigma_z = \sigma_{overlap}(z) = \frac{\sigma_{l,p}(z)\sigma_{l,\overline{p}}(z)}{\sqrt{\sigma_{l,p}^2(z) + \sigma_{l,\overline{p}}^2(z)}} \qquad (3.2)$$

The luminosity for two bunch crossing will then be expressed by

$$\mathcal{L} = \sum \mathcal{L}_{i,j} = N_p N_{\bar{p}} \int \frac{1}{\sqrt{2\pi}\sigma_z} \frac{e^{-\frac{z^2}{2\sigma_z^2}}}{4\pi\sigma_x(z)\sigma_y(z)} dz \tag{3.3}$$

In general, $\sigma_{l,p}(z)$,$\sigma_{l,\bar{p}}(z)$ differ from bunch to bunch and evolve with time in the store. The transverse dimensions of the bunches vary inside the integral and are functions of the lattice parameters at the interaction region. Moreover, the horizontal size is also affected by the momentum dispersion. The transverse sizes can be written as:

$$\sigma_x^2(z) = \frac{1}{6\pi\gamma}(\beta_x(z)\epsilon_x) + (\eta(z)\frac{dp}{p})^2 \tag{3.4}$$

$$\sigma_y^2(z) = \frac{1}{6\pi\gamma}\beta_y(z)\epsilon_y \tag{3.5}$$

where:

$\frac{dp}{p}$ is the momentum dispersion,

$\epsilon_{x,y}$ are the (95%) normalized emittances in $m \cdot rad$,

η is the momentum dispersion in meters,

$\sigma_{x,y}$ are the transverse bunch sizes in m, and

$\beta_{x,y}$ are the lattice functions in meters.

3.1 Accelerator Measurements

For all the runs considered in this analysis the accelerator run in the mini-β lattice configuration, at both energies, with $\beta_x \simeq \beta_y \simeq .5$ m.

During the run, wires are flown periodically, usually every two hours, through the beams at two locations around the ring (C48 and A17) to measure the beam's transverse size. A resistive wall current monitor (Sampled Bunch Display) [14] measured the bunch intensities and logitudinal profiles.

The quantities necessary for the actual luminosity calculation are then:

N_p,$N_{\bar{p}}$ measured by the Sampled Bunch Display (SBD)

$\sigma_{l,p}(z)$,$\sigma_{l,\bar{p}}(z)$ measured by SBD

$\sigma_{x,y}^{p,\bar{p}}$ from flying wires

$dp/p_{p,\bar{p}}$ derived from $\sigma_{l,p}(z)$, $\sigma_{l,\bar{p}}(z)$and the measured RF voltage

These parameters were measured for each of the twelve bunches and stored in the accelerator database, either directly or in terms of derived quantities. The transverse size at the interaction point in B0 can be extrapolated from the measurement of the transverse size at the flying wires scan location, C48, where the dispersion is small (using eqns. 3.4,3.5). The momentum spread could also be inferred from the horizontal size measured at A17 where the dispersion is large. The momentum spread (or longitudinal emittance) derived in this way from wire scan data ($\epsilon_l(w)$) could then be checked against the derivation from the SBD profile (yielding $\epsilon_l(S)$) for consistency. The accelerator luminosity was calculated on line by a program, T106, and stored in the same database, together with B0LUMP. Because of known problems and approximations in the original T106 calculation, the data were re-analyzed and the luminosity recalculated. Some of the raw measurements were not directly available in the database, but had to be extracted by inverting the results of calculations. This implies the ability to reconstruct the state of the programs used at any given time. The effort of reconstructing the data from the database has been done by Norman Gelfand [7] for all the 1989 collider runs, at both energies, $\sqrt{s} = 1800$ and 546 GeV.

Due to this indirect route, a number of data points look questionable; we have required internal consistency in the measurements to eliminate bad data points.

The ratio $\mathcal{L}_{BBC}/\mathcal{L}_{accel}$, (where $\mathcal{L}_{BBC}$ and $\mathcal{L}_{accel}$ are defined by equations 1.1 and 3.3 respectively), for all the data at high energy is shown in Fig. 2. Even for the raw sample, there is a very good correlation between the two measurements of the luminosity. To understand the tails we have compared, where possible, the derivation of a given quantity from independent measurements.

The longitudinal emittance can be obtained both from the bunch length measured from the SBD, and from $\frac{dp}{p}$ inferred from flying wire measurements. A comparison of these two quantities for both p and $\overline{p}$ at $\sqrt{s} = 1800$ GeV shows good correlation for the majority of the points, with clusters of obviously bad data. Assuming that the difference between these two quantities comes from measurement uncertainty , rather than fundamental problems, we have fit the larger clusters in Fig.3 to a straight line and then required that the data fall in a band of width 0.2 $eV \cdot sec$ around this line.

The vertical and horizontal beam sizes are expected to be similar. Again, the data in Figs.4 a&b fall in a tightly correlated band, with satellites. A straight line fit to the main band gives a slope of 1.11. A cut of $\pm$1mm was applied. Most points eliminated by this and the cuts above are correlated, as can be seen in Figs.4 b& d, where the effect of the longitudinal (transverse) emittance cut on the transverse (longitudinal) one is shown. A cut on the $\overline{p}$ transverse emittances was not necessary. The above cuts (summarized in Table 4) eliminate most of the tails in the luminosity ratio, while the mean value is changed by $\approx 1\%$. The distribution has been fitted to a gaussian, as indicated in Fig. 5.

The data sample for $\sqrt{s} = 546$ GeV consists of three low-β stores, for which we have 19 flying wire scans. Figure 6 shows the vertical versus horizontal emittances for both p and $\overline{p}$. The small cluster of points above the main band in Fig 6b were eliminated. It is interesting to note that the same points deviate from a straight line in the scatter plot of two measurements of the longitudinal emittance, from flying wires and SBD data. These points correspond to very early times in the stores. Figures 7&8 show the distribution of $\mathcal{L}_{BBC}/\mathcal{L}_{accel}$ with and without cuts. Again there is no systematic shift between the mean of the uncut distribution and that of the gaussian fit after the cut (see Table 5). At both energies, $\mathcal{L}_{BBC}$ has been calculated as $\frac{R_{BBC}}{\sigma_{BBC}}$ with $\sigma_{BBC} = 44$mb. One can write

$$\sigma_{BBC}^{eff} = \frac{R_{BBC}}{\mathcal{L}_{accel}} = \frac{\mathcal{L}_{BBC}}{\mathcal{L}_{accel}} 44mb$$

and

$$\frac{\sigma_{BBC}(1800)}{\sigma_{BBC}(546)} = \frac{\frac{\mathcal{L}_{BBC}(1800)}{\mathcal{L}_{accel}(1800)}}{\frac{\mathcal{L}_{BBC}(546)}{\mathcal{L}_{accel}(546)}}. \tag{3.6}$$

Note that the latter directly relates σ_{BBC}^{eff} at the two energies, independently of individual inelastic cross sections and relative beam-beam counter efficiencies, provided that energy dependent corrections to $\mathcal{L}_{accel}$ are properly accounted for.

Table 4: Cleanup Cuts

Cut	No. of Scans	
	$\sqrt{s}$ =1800 GeV	$\sqrt{s}$ =546 GeV
Raw	1353	20
$\epsilon_l(S) - 1.2 \cdot \epsilon_l(w) - .1 < 0.2(\mathrm{eV} \cdot \mathrm{sec})$	686	–
$\epsilon_y - (1.11 \cdot \epsilon_x + .19) < 1.$	509	–
$(\cdot\pi \cdot mm \cdot mrad)$		–
$\epsilon_y/\epsilon_x < 1.18$	–	15

Table 5: Ratio $\mathcal{L}_{BBC}/\mathcal{L}_{accel}$-before 1% correction to $\mathcal{L}_{BBC}$

	$\sqrt{s}$=546 GeV		$\sqrt{s}$=1800 GeV	
	mean	σ	mean	σ
Raw	.753±.002 (poor fit)	.007±.002	.994±.001	.044±.001
Cleanup	.753±.002	.007±.002	1.004±.001	.029±.001
Correc. for L dep.	–		.981±.001	

3.2 Corrections

This analysis uses data as stored in the accelarator database; corrections to $\mathcal{L}_{BBC}$ are discussed in App. II.1. There we find that a common 1% correction must be applied to all points independent of cms energy.

One can observe a dependence of $\frac{\mathcal{L}_{BBC}}{\mathcal{L}_{accel}}$ on luminosity (Fig.9). Dynamic beam-beam interaction effects, which change the focusing properties of the lattice at the 12 crossing points around the ring and have not been taken into account in the calculation, would predict a linear dependence of the ratio with $\mathcal{L}$. This effect is estimated to have magnitude consistent with that of Fig.9, but a calculation with the current lattice has not yet been performed [12]. The data can be fitted with both a linear and a quadratic polynomial. The slope of the linear fit is 0.06 per $10^{30} cm^2 sec^{-1}$.

Since the two sets of data have different average instantaneous luminosity, with the 1800 GeV data covering a much larger range, we extrapolate these data to the mean luminosity for the 546 GeV data. No correction is applied to the 546 data. This corresponds to a correction to $\frac{\mathcal{L}_{BBC}}{\mathcal{L}_{accel}}(1800)$ of -3$\pm$2%. We take the difference in the linear and quadratic extrapolations as estimate of the uncertainty on this correction.

3.3 Relative Uncertainties.

We now look for possible systematic errors to the ratios in eqn. 3.6. These are listed in Table 7 and the method used to estimate them is discussed below.

We are mostly concerned with uncertainties in the luminosity calculation ($\mathcal{L}_{accel}$) at one energy relative to another, not in the absolute luminosity. We therefore concentrate on variations of the ratio relative to the parameters of the calculation.

- We have checked that the ratio is independent of time over the duration of the 1800 Gev run, since the 546 Gev stores were concentrated in a very short time period.

- We estimate the sensitivity of the ratio to the cleanup cuts, by comparing the value obtained from cut and uncut samples.

- The transverse emittance measurements are independent of the beam energy; the accelerator was run with the same lattice at 546 and 1800 GeV, so, in principle, all calculations dependent on lattice parameters should also be energy independent. However, the actual beam parameters have different values at the two energies: the emittances are smaller at the lower energy and consequently the effect of the momentum dispersion is larger.

  To get some estimate of possible systematic uncertainties we compare the ratio of $\mathcal{L}_{BBC}$ to $\mathcal{L}_{accel}$ as function of the transverse and longitudinal beam widths in Fig.10, where the points are the 1800 GeV data and the asterisks the 546 GeV. Although the beam parameters are not identical in the two cases, there is no indication of systematic dependence on these parameters (the ratios need not be identical, since they depend on the 44.0 mb default value for σ_{BBC}^{eff} used to get $\mathcal{L}_{BBC}$ from BBC rates). The longitudinal bunch sizes ("sigzsp") partially overlap for stores at two energies: we have calculated $\frac{\mathcal{L}_{BBC}}{\mathcal{L}_{accel}}$ for data in the overlap region and compared to the ratio for the complete set. We find a 1.5% change. The bunch size is inversely related to the instantaneous luminosity, therefore the size and luminosity dependence seen before are understood as manifestation of the same effect, and we don't make this additional correction (we do include this as an additional uncertainty, though).

- We have also checked that the luminosity profiles calculated with accelerator parameters are consistent with the vertex distribution of CDF events. There is a remarkably good agreement, both in shape and width of the distribution, between the BBC vertex distribution and the bunch profile, which tracks the profile evolution during the course of the store. Figure 11 a& b show vertex distributions from the first and last file of run 20296 (store 2193), and the beam profile calculated with the parameters from the first and last flying wires measurement for that store in the database (Table 6).

Table 6: Comparison of Luminosity Profile with CDF vertex distribution (546-GeV)

	BBC vertex Fit		Luminosity Profile	
Store 2193	mean (cm)	σ (cm)	mean (cm)	σ (cm)
Beginning of store	3.6±.85	32.5± .71	1.4± .27	32.33± .21
End of store	5.6±1.0	42.6±.9	3.3±.4	40.5± .3

- There is a 5% uncertainty in the determination of the dispersion. Unlike the vertical betatron amplitude, for example, whose errors cancel in the ratio of calculations at different energies (see eqn. 3.3,3.4) the dispersion uncertainty as well as that of $\frac{dp}{p}$ need not cancel. We have estimated the effect on the ratio by changing $\frac{dp}{p}$ by 10% in the calculation of $\mathcal{L}_{accel}$. This results in a decrease of $\mathcal{L}_{accel}$ of 3(6)% at 1800(546)GeV respectively, and a 2.3% change in the ratio. The error given in Table 7 from this source is likely an overestimate of possible uncertainty in these two measurements.

Table 7: Uncertainties in Relative Luminosity

σ_z dependence	± 1.5%
$\mathcal{L}$ dependence	±2%
Cleanup cuts	± 1%
dp/p scale	±3%
Longitudinal bunch profile	±1.5%
net uncertainty	±4.3%

3.4 Errors on Absolute Luminosity

The errors in the luminosity calculation come from both errors in beam parameter measurements and uncertainties in the lattice function. β values have been checked against calculations at various locations in the Tevatron [11]: an uncertainty of 5% on the absolute luminosity was estimated. The bunch intensities were measured with the SBD, whose intrinsic precision was measured to be .5%; however, an uncertainty of 5% on the absolute calibration was estimated by comparing the SBD readout to a current monitor, T:IBEAM. This correlated uncertainty in N_p and $N_{\bar{p}}$ results in a 10% error which dominates the overall 11% uncertainty in the luminosity scale . Both uncertainties (β and N_p ,$N_{\bar{p}}$) should be energy independent and cancel out in the ratio of $\mathcal{L}_{accel}$ at 1800 and 546 GeV.

4 Results.

At $\sqrt{s} = 546GeV$, we can use $\mathcal{L}_{accel}$ and the rate in the beam-beam counters to calculate σ_{BBC}^{eff}. We obtain σ_{BBC}^{eff}=(32.8±3.6)mb, that we average with the result from UA4 to get σ_{BBC}^{eff}=(36.0±1.81)mb.

Finally, from equation 3.6 we calculate

$$\sigma_{BBC}^{eff}(1800) = (46.8 \pm 2.35 \pm 2.16)mb,$$

where the first error represents the contribution from $\sigma_{BBC}(546)$ and the second error the contribution from the luminosity ratios. Since CDF cross sections have been calculated using $\sigma_{BBC}^{eff}(1800)$=44 mb, they should be corrected upward (as in eqn. 2.1) accordingly. This correction is well within the quoted error of CDF-552 [1], but we have been able to reduce the luminosity uncertainty by a factor of 2. Table 8 summarizes the history of our calculation which relates the luminosity at different energies. Note that for comparisons of CDF cross sections at $\sqrt{s}$=546 and 1800 GeV the luminosity error is ±4.6% since the overall luminosity scale is irrelevant.

Table 8: History of luminosity scaling factor

Stage of calculation	$\frac{\sigma_{BBC}^{eff}(1800)}{\sigma_{BBC}^{eff}(546)}$
CDF#1031	.976/.742 = 1.32
2/90 Collab Mtg.	1.03/.81 = 1.27
(Triangular weighting fun.)	
THIS NOTE	.971 /.746 = 1.30

Acknowledgements.

We thank Norman Gelfand, for countless helpful discussions about the accelerator business. We also thank Rol Johnson and Dave Finley. We thank Tom Chapin for running the Monte Carlo calculations used in Table 2.

A BBC Inefficiency Correction

There are a number of indications that the beam-beam counters were inefficient for minimum ionizing particles during the '88-'89 run [9]. For the purposes of this note, the BBC efficiency is pertinent for 2 reasons.

- 1) To calibrate the luminosity at $\sqrt{s}$=546 GeV we need to go from a cross section· acceptance to "effective" cross section(which includes detector dependent corrections).
- 2) It will simplify our job if we know that the inefficiency and therefore the effective cross section are constant throughout the run (it is constant as well as we can tell).

A.1 Correction for inefficiency at $\sqrt{s}$=546 GeV

We have two complementary ways of approaching the inefficiency- neither of which is complete by itself.

- We can take the individual counter inefficiencies measured at the end of the run [9] and calculate the expected reduction in Level-0 trigger rate using Monte Carlo simulation. This is likely to underestimate the efficiency since the CDF simulation we use doesn't account for all secondaries.
- On the other hand, we can use data from Level-0 query runs in '88-'89 to compare the BBC trigger efficiency with that of the '87 data (before radiation damage occured). This has the shortcoming that only $\sqrt{s}$=1800 GeV Level-0 query data exist.

  Using the Level-0 data the efficiency is found to be [9]:

$$L0_{eff-corr} = 0.993 \pm .014 \; (\sqrt{s} = 1800)$$

We then use the relative efficiency between $\sqrt{s}$=546 and 1800 GeV as calculated with the simulation. We expect the efficiency to increase with energy because of the log-s rise of $\frac{dN_{ch}}{d\eta}$ (we found about a 30% incease for $\mid \eta \mid \leq 3.0$ -[15]).

The calculation in Table 2 used successively 25,50,75,100 % efficiency in the inner to outer rings of beam counters. This is an approximation to the measured efficiencies for minimum ionizing particles.

The table does show that the efficiency correction (94%) at $\sqrt{s}$ =546 GeV is not very sensitve to the fraction of single diffractive cross-section used as input to the Monte Carlo.

The same calculation at $\sqrt{s}$=1800 GeV yields 95.4%, so we have:

$$L0_{eff-corr} = \frac{.94}{.954} \cdot (0.993 \pm .014) = 0.978 \pm .022 \leftarrow (\sqrt{s} = 546)$$

This is the correction applied in Table 3.

Note that we haven't used the Monte Carlo program to calculate the overall efficiency. We rely only on the fact that the efficiency change from one c-of-m energy to another must be smaller than in this simulation- which doesn't take into account secondary effects that increase the probability of detecting an inelastic event.

Alternatively, we can estimate the BBC inefficiency from the data. The L0 trigger required one beam-beam counter on each side of the interaction to fire. From the distribution of the number of counters that fired on each side, we can extrapolate to the number of zero's and of missed triggers. From a $\sqrt{s}$=1800 GeV run, we get a 1.5% rate of zero counters firing on each side, with a global inefficiency of 3%. Similarly, at $\sqrt{s}$=546 GeV, we get a 2.2% probability of missing a side, and a 4.4% global inefficiency. We use the two values to evaluate the relative BBC inefficiency at the two energies. The result is consistent with that obtained by the Monte Carlo.

B Corrections to BBC Rates

In this note we are using BBC rates that have been corrected for multiple interactions but not corrected for accidentals. We are using these rates for a measurement of the effective BBC cross section. Here we consider two corrections to the rates and therefore the quantity LumBBC which appears in the plots.

B.1 Ratemeter Scale

The quantity 'B0LUMP' stored in Accelerator data bases is obtained by a calculation which duplicates the CDF module 'LUMBBC'[13]. BBC coincidence signals are fed into rate meters in the CDF control room whose outputs are digitized in ACNET ADC's. The ACNET program that converts from ADC channel to Luminosity also corrected for the multiple interaction probability.

Here we check the relative scale of what was originally calculated by our online program (read off the data tapes) with respect to what is in the databases. We look at typical runs at 546 and 1800 GeV. The ratemeter scale was changed by a nominal factor of 10 when we ran at 546 GeV.

For each run we correlated data from CDF tapes with 'RIM' events using the time stamp in Logical Record I.D. time of day. We plotted 'RIM" recorded values of 'B0LUMP' and superposed instantaneous luminosity from 'LUMBBC'. We found that the above sequence has a gain of 1.01 at both energies.

We do not correct the plots for this factor. However tables which use results derived from the plots do have this correction. In Tables 8 (but not 5) LUMBBC has been multiplied by a correction factor of 0.99.

B.2 Background correction to BBC Coincidence Rate:

The most straightforward way to get the background to the BBC rate =(E$\geq$ 1)$\odot$ (W$\geq$ 1) is to examine rates with missing bunches.

Table 9 shows the situation during run 20339 which was a low beta 1800 GeV store with one missing p and one missing $\overline{p}$ bunch. The conditions at the time that numbers in Table 9 were measured corresponded to a luminosity of $3 \cdot 10^{29}$ (but the overall L was lower since 2 bunches were empty).

The background in this store should be typical or slightly worse than in the average 1800 Gev store: the luminosity in this store (per crossing) is about a factor of 2 lower than average.

We can consider two types of background to the coincidence rate - i.e. the fraction of the rate which cannot be due to beam-beam interactions. These are:

- The coincidence rate due to single beam interactions with residual gas, etc. From Table 9 we have

Table 9: BBC rates used to derive background (Missing bunches Run 20339 - low-β, 1800 GeV)

	Bunch Intensities			Counting rate		
Bunch	Np(10^9)	N$\bar{p}$(10^9)	$\mathcal{L}_{BBC}(10^{29})$	E.W	E	W(khz)
A	53.0	6.2	.54	2.38	3.5	2.8
B	46.2	6.7	.51		(like A)	
C	53.1	6.6	.51		(" ")	
D	0.13	7.9	.00	.002	.008	.130
E	47.0	0.17	.0014	.013	.858	.015
F	51.0	7.9	.65		(like A)	

$$SingleBeamBackground = \frac{R_{E \cdot W}(missing - p(\bar{p}))}{R_{E \cdot W}(p\bar{p})} = 0.1 \ to \ 0.5\%$$

– The accidental coincidence rate due to singles rates from each beam is calculated from the bunch "D" and "E" rates in Table 9.

$$AccidentalBackground = \frac{R_E(p_{only}) \cdot R_W(\bar{p}_{only})}{R_{crossing}} = 2.4hz$$

This also amounts to a 0.1% background. This background rate is lower than the background rate we had in the '87 run [15] of 4% at $\mathcal{L} = 4 \cdot 10^{28}$. The difference is presumably due to both higher luminosity and cleaner conditions (the low beta tune was essentially the same in both runs).

B.3 Background in the 546 Run

We examined the rates corresponding to those of Table 9 for Run 20277. This was also a store with missing p and $\bar{p}$ bunches but the luminosity was extremely low during this run ($\mathcal{L} = 5 \cdot 10^{27}$). The background was found to be 2% and we consider this to be an upper limit to the background contribution to the rates at $\sqrt{s} = 546$ GeV.

No Background correction applied: We make no background correction to rates for the analysis in this CDF note. The rates given herein are small and taken to be upper limits.

References

[1] T. Liss , CDF Note # 552 (Luminosity Monitoring and Beam-Beam counter performance)

[2] S. White, CDF Note # 1031 (Reducing Luminosity Uncertainties)

[3] S. Belforte and K. Goulianos, CDF Note # 256 (A Complete Minimum Bias Event Generator)

[4] UA4 σ_{tot} Phys.Lett. 147 B('84) p.392, and UA4 ϱ-value ,Phys.Lett. 198 B ('87) p.583

[5] Phys.Lett. 147 B('87) p.227

[6] This cross section is arrived at arbitrarily by discarding half of single diffractive events generated by the MBR Monte Carlo and holding the inelastic cross section constant.

[7] N. Gelfand, FN # 938 (Computation of Tevatron Luminosity using measured machine parameters) Apr. 1990

[8] U. Amaldi, in "Laws of Hadronic Matter" A.Zichichi ed. ('75) p.673

[9] N. Giokaris, CDF Note # 1213 and 'BBC Inefficiency from the Data and Bench Measurements', CDF note # 1214

[10] R.Johnson, CDF Note # 876

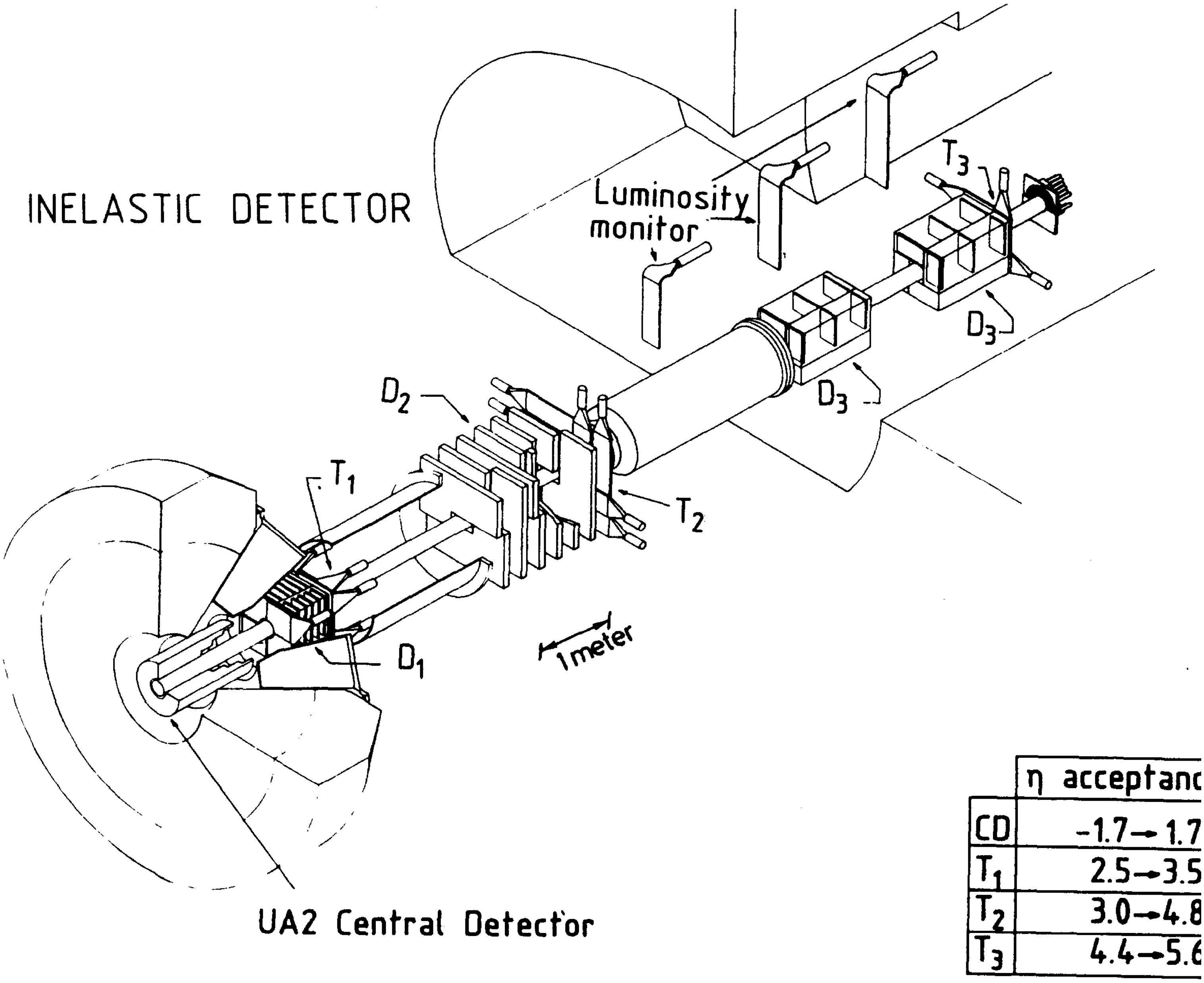
[11] D. Finley, Technical Memo #137

[12] Dave Johnson TM-1554 (Linear Beam-Beam Tune Shift Calculations for the Tevatron Collider) June 1989

[13] J.E.Elias, private communication.

[14] C. D. Moore, *et al*, Proceedings of the 1989 IEEE Particle Accelerator Conference, Chicago (Single Bunch Intensity Monitoring System Using an Improved Wall Current Monitor)

[15] Abe et al. PRL 61 ('88) p.1819



	η acceptanc
CD	-1.7→ 1.7
T_1	2.5→3.5
T_2	3.0→4.8
T_3	4.4→5.6

Fig. 1 - UA4 apparatus and acceptances. Note that counters T2 and T3 together cover approximately the same angles as the CDF Beam-Beam counters.

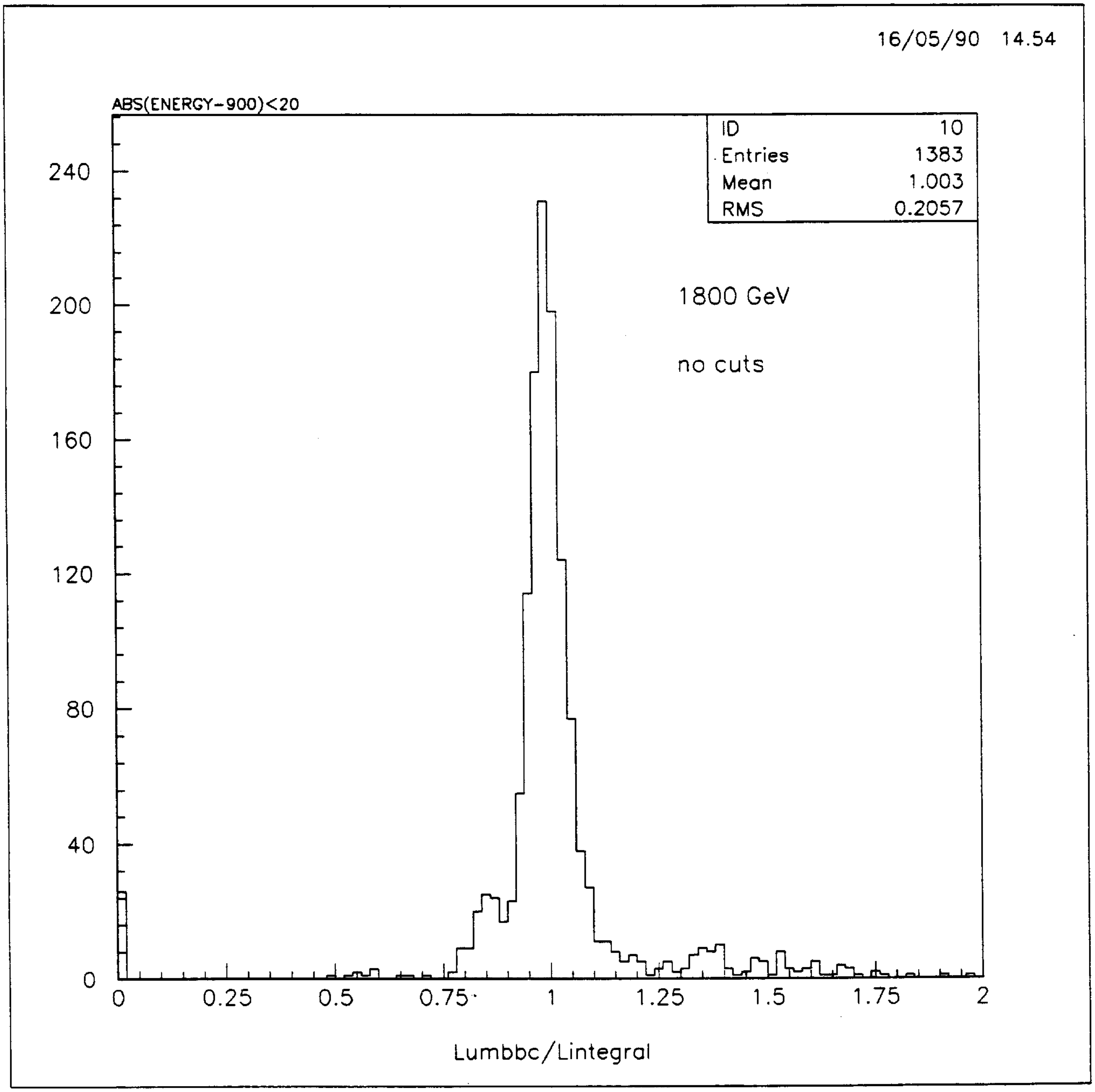


Fig. 2 - Ratio of $\mathcal{L}_{BBC}$ to $\mathcal{L}_{accel}$ for all flying wire scans at $\sqrt{s}$=1800GeV

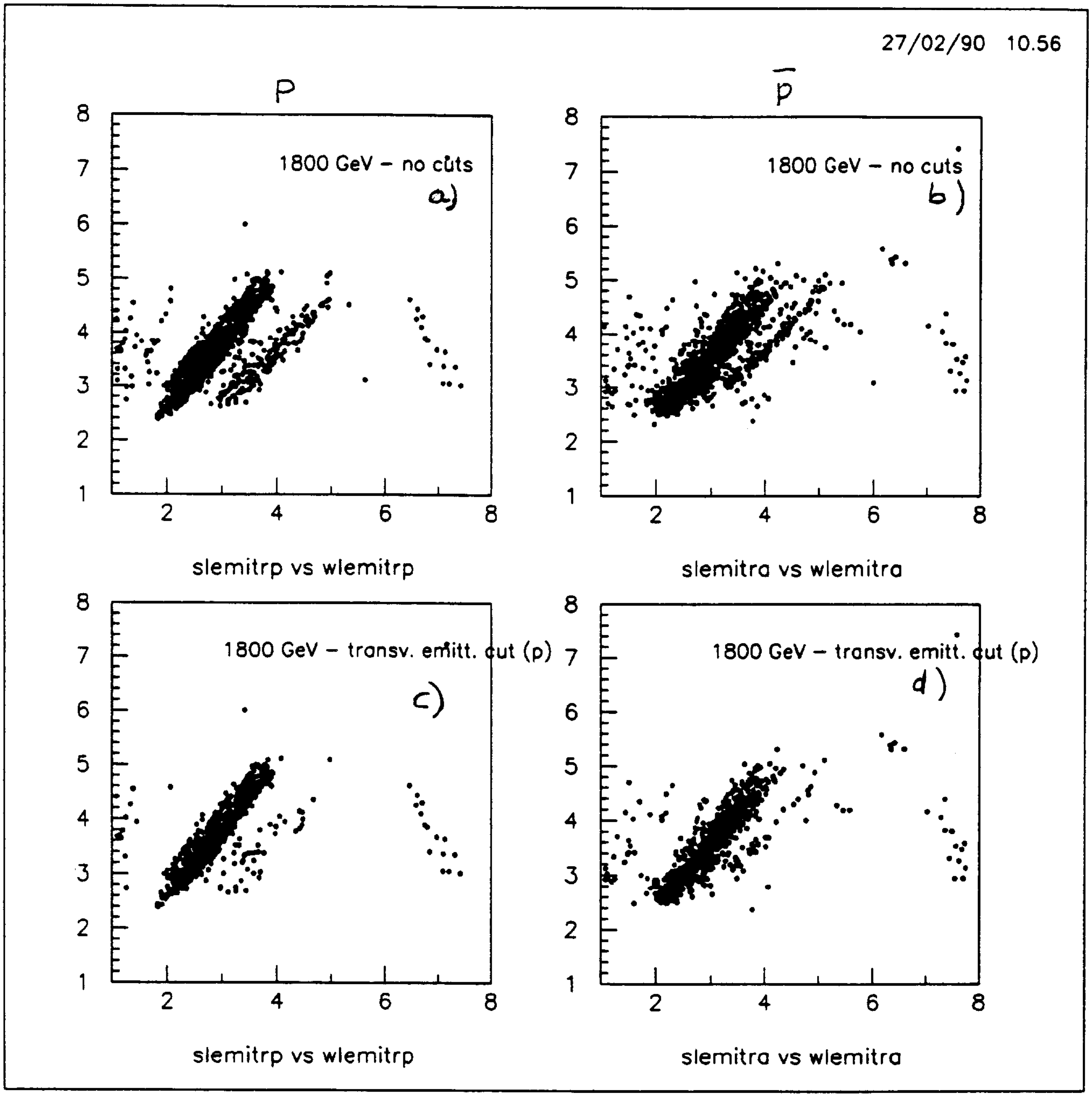


Fig. 3 - Comparison of longitudinal emittance (in eV-sec) measurement from flying wire and SBD data, for p and $\bar{p}$ for 1800 GeV stores. No cuts are applied to the data in Fig. a) and b), showing deviation from a 45^0 line, due to bad measurements. A cut is applied to these variables to select the main band. The same points are shown in Fig. c) and d) after a cut on the tranvserse emittance.

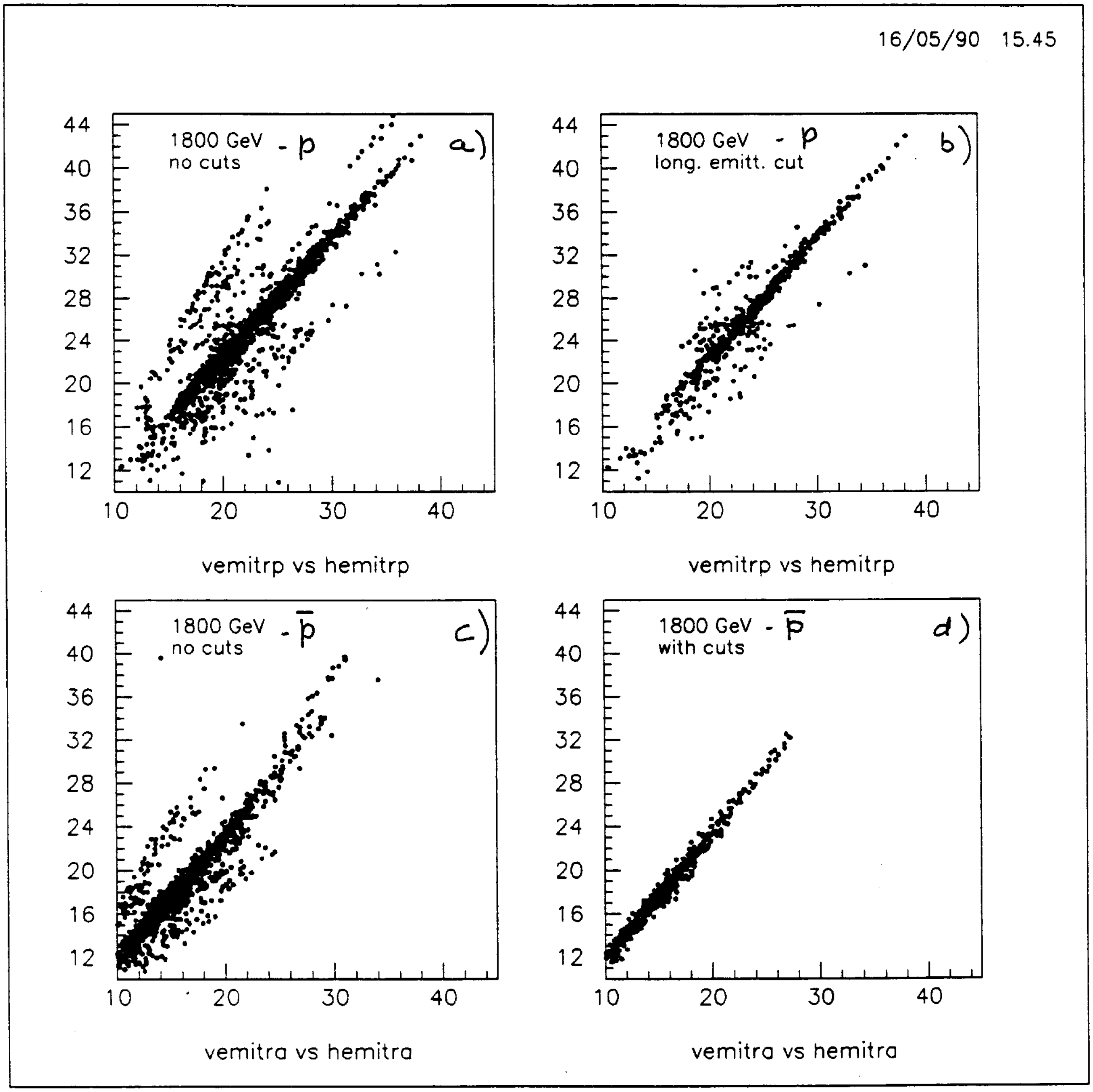


Fig. 4 - Vertical vs horizontal emittance (in units of $\pi \cdot mm \cdot mrad$) for protons and antiprotons without cuts, a) and c), and with cut on the longitudinal emittance, b) and d)

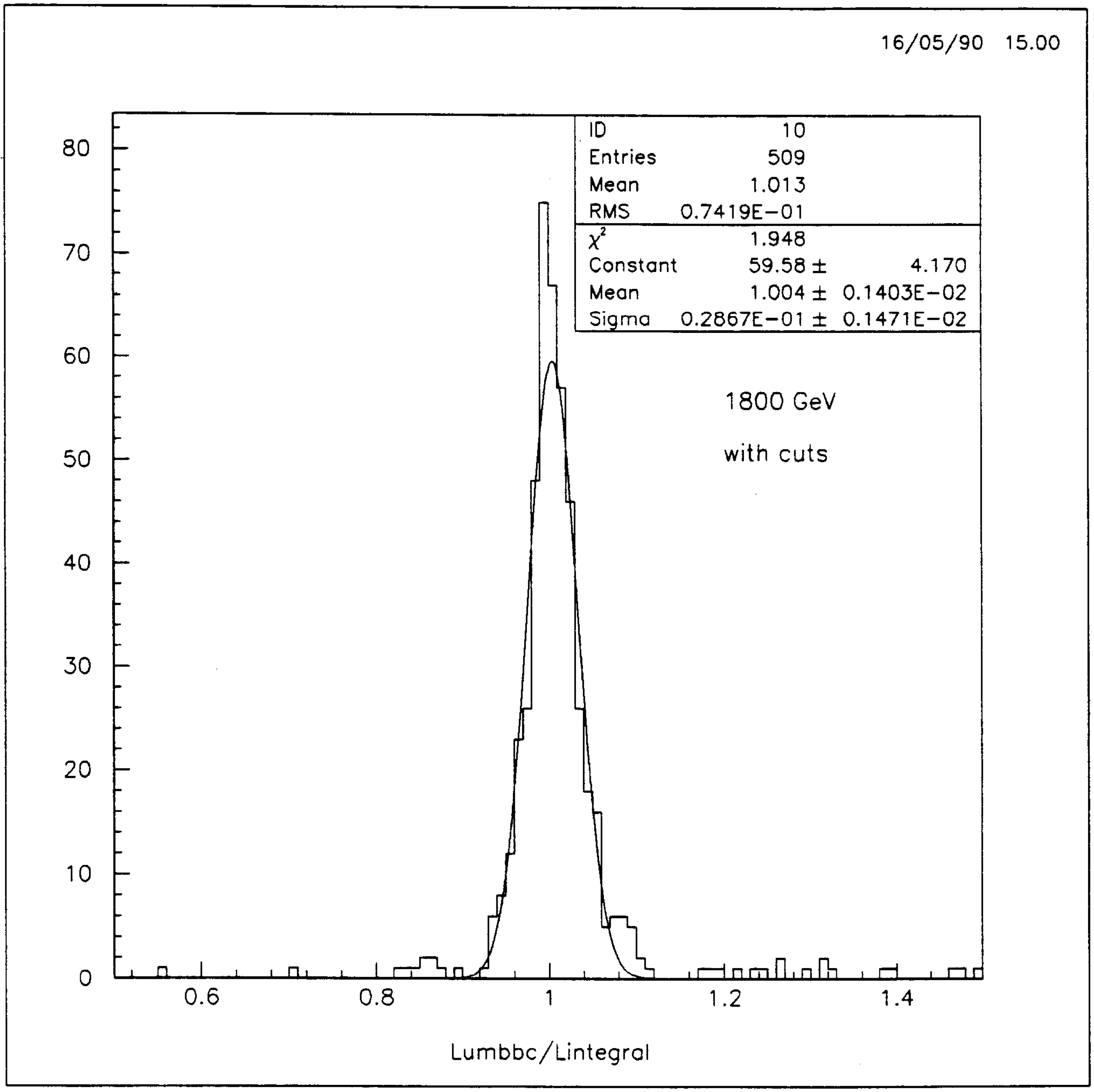


Fig. 5 - Ratio of $\mathcal{L}_{BBC}$ to $\mathcal{L}_{accel}$ for 1800 GeV stores, after cleanup cuts, with gaussian fit.

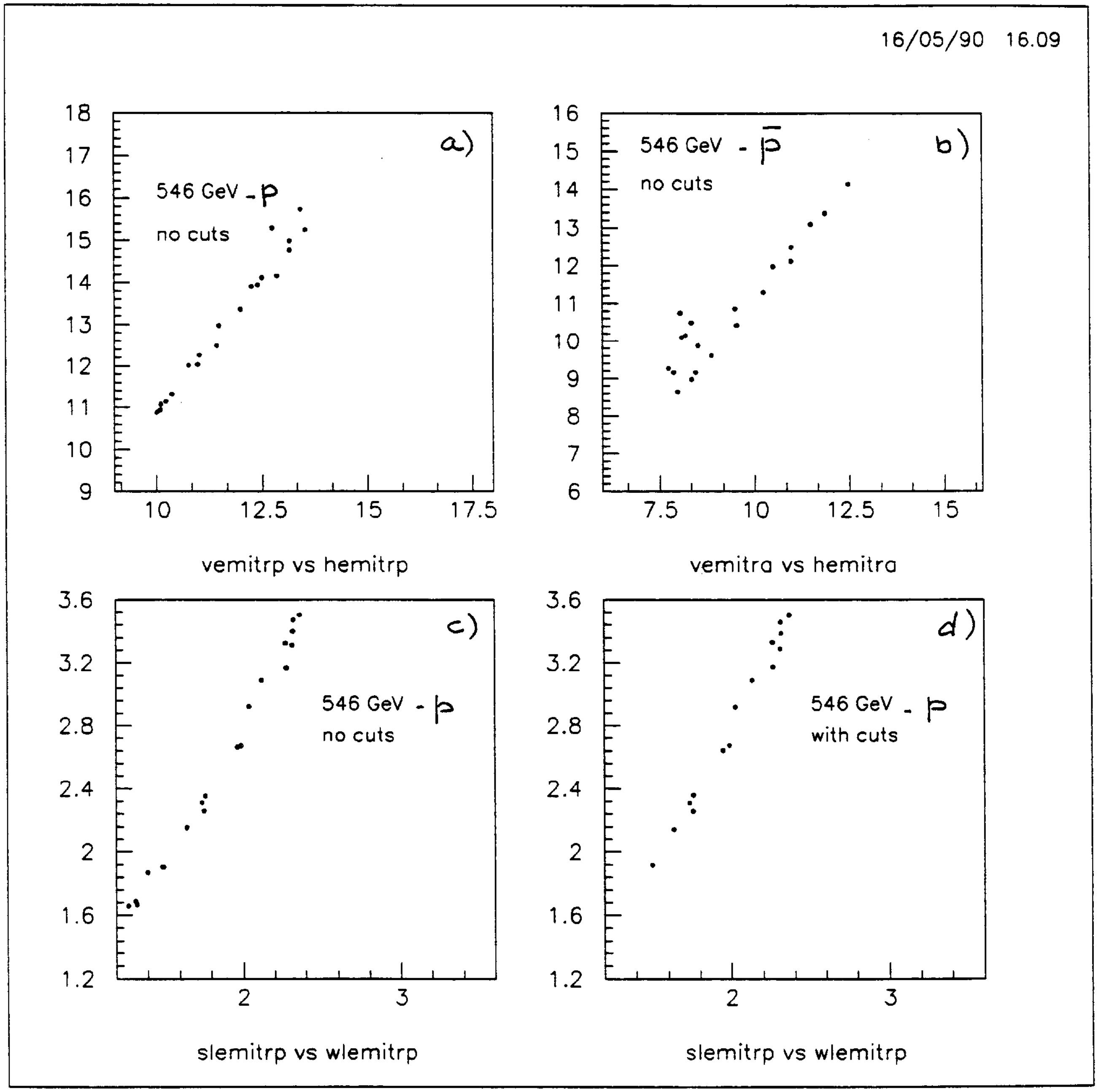


Fig. 6 - Vertical vs horizontal emittance (in units of $\pi \cdot mm \cdot mrad$) for proton and antiproton, a) and b), without cuts for $\sqrt{s}$=546 GeV stores, and SBD vs flying wires derived longitudinal emittance (in eV·sec) for protons, without cuts, c), and with cut on the proton transverse emittance, d).

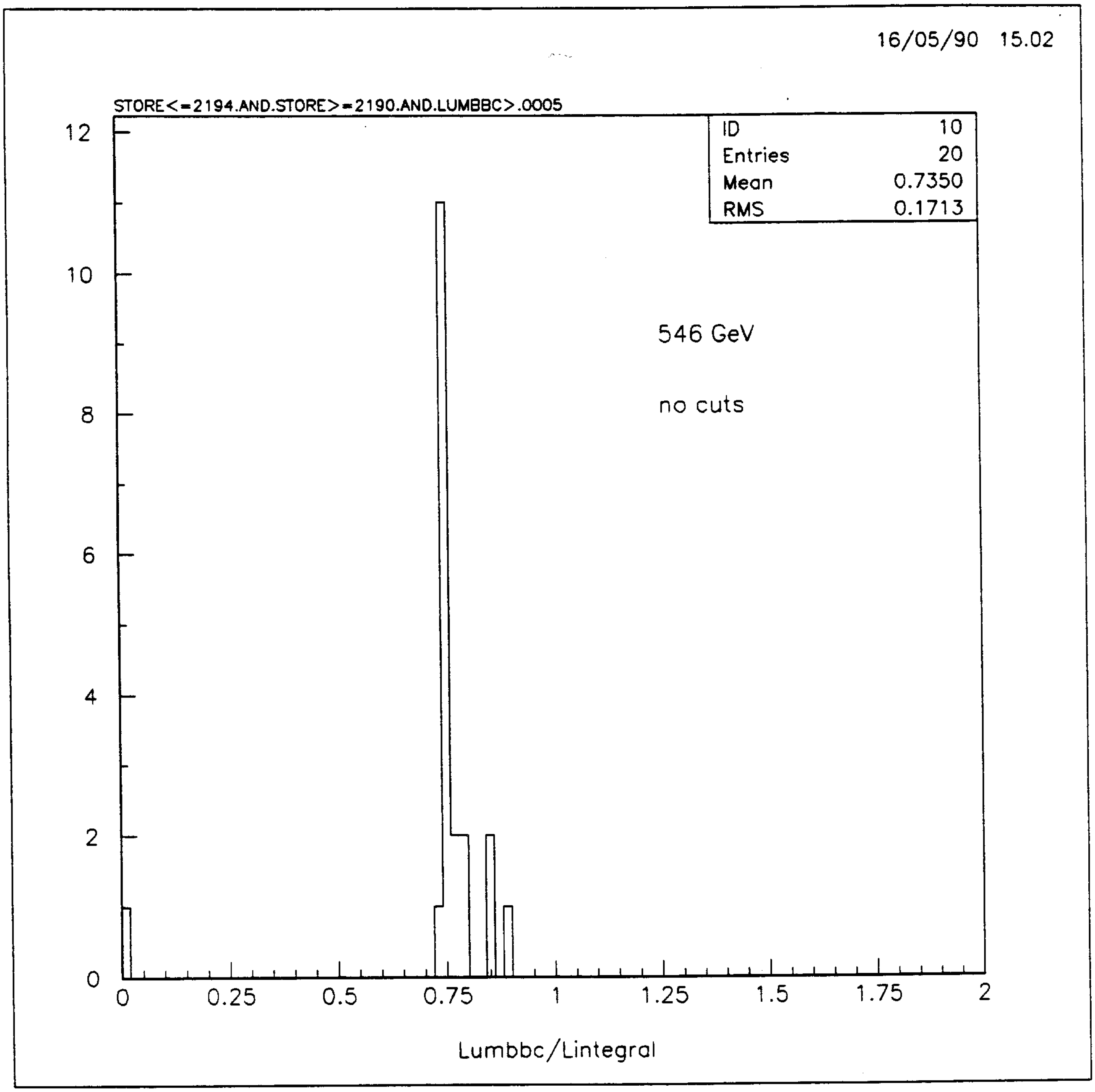


Fig. 7 - Ratio of $\mathcal{L}_{BBC}$ to $\mathcal{L}_{accel}$ for all 546 GeV points.

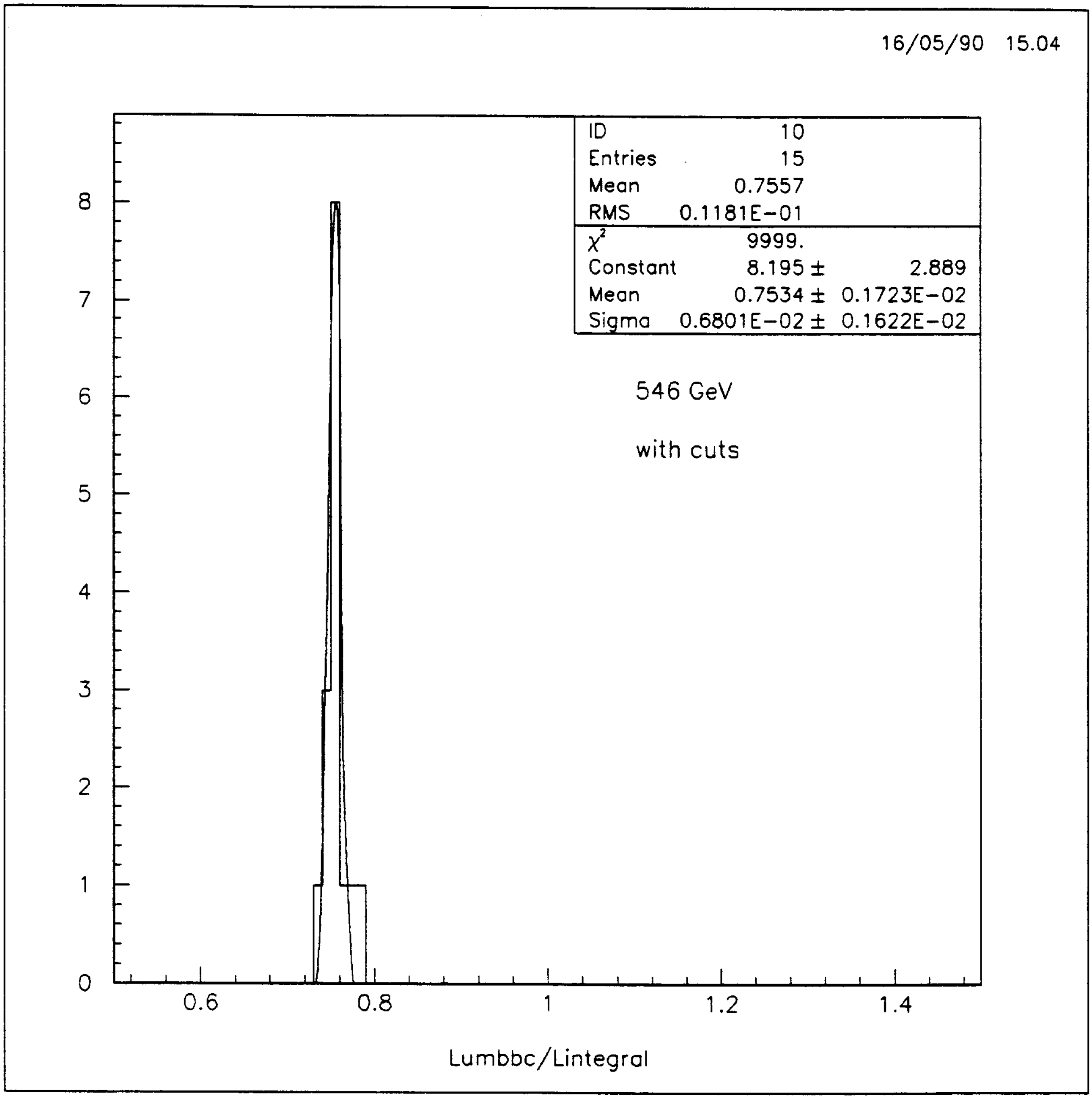


Fig. 8 - Ratio of $\mathcal{L}_{BBC}$ to $\mathcal{L}_{accel}$ for 546 GeV stores after cleanup cut, with gaussian fit.

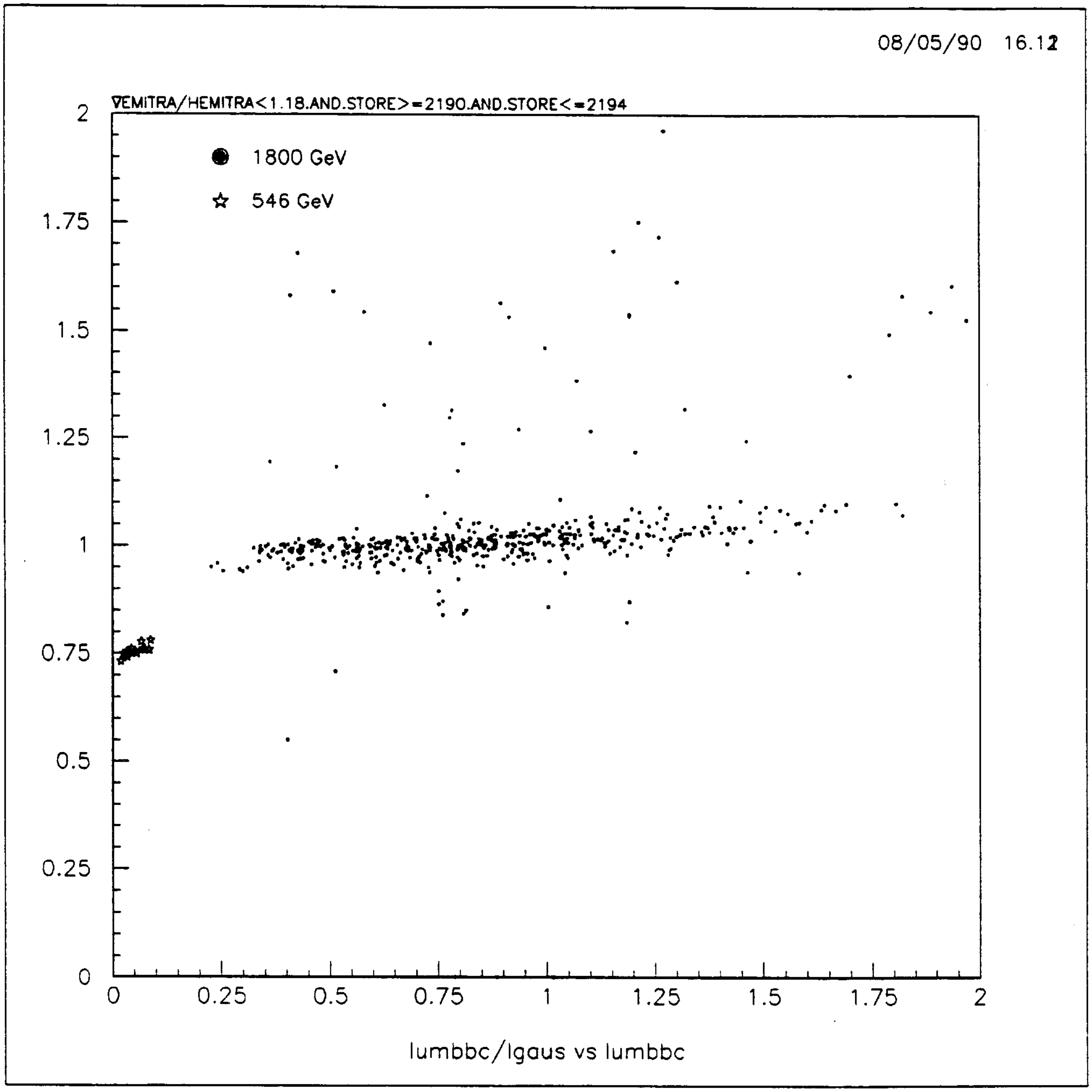


Fig. 9 - Luminosity dependence (scale is $10^{30} cm^{-2} sec^{-1}$) of $\frac{\mathcal{L}_{BBC}}{\mathcal{L}_{accel}}$. The increase of the ratio with instantaneus luminosity is qualitatively consistent with estimates of beam-beam effects.

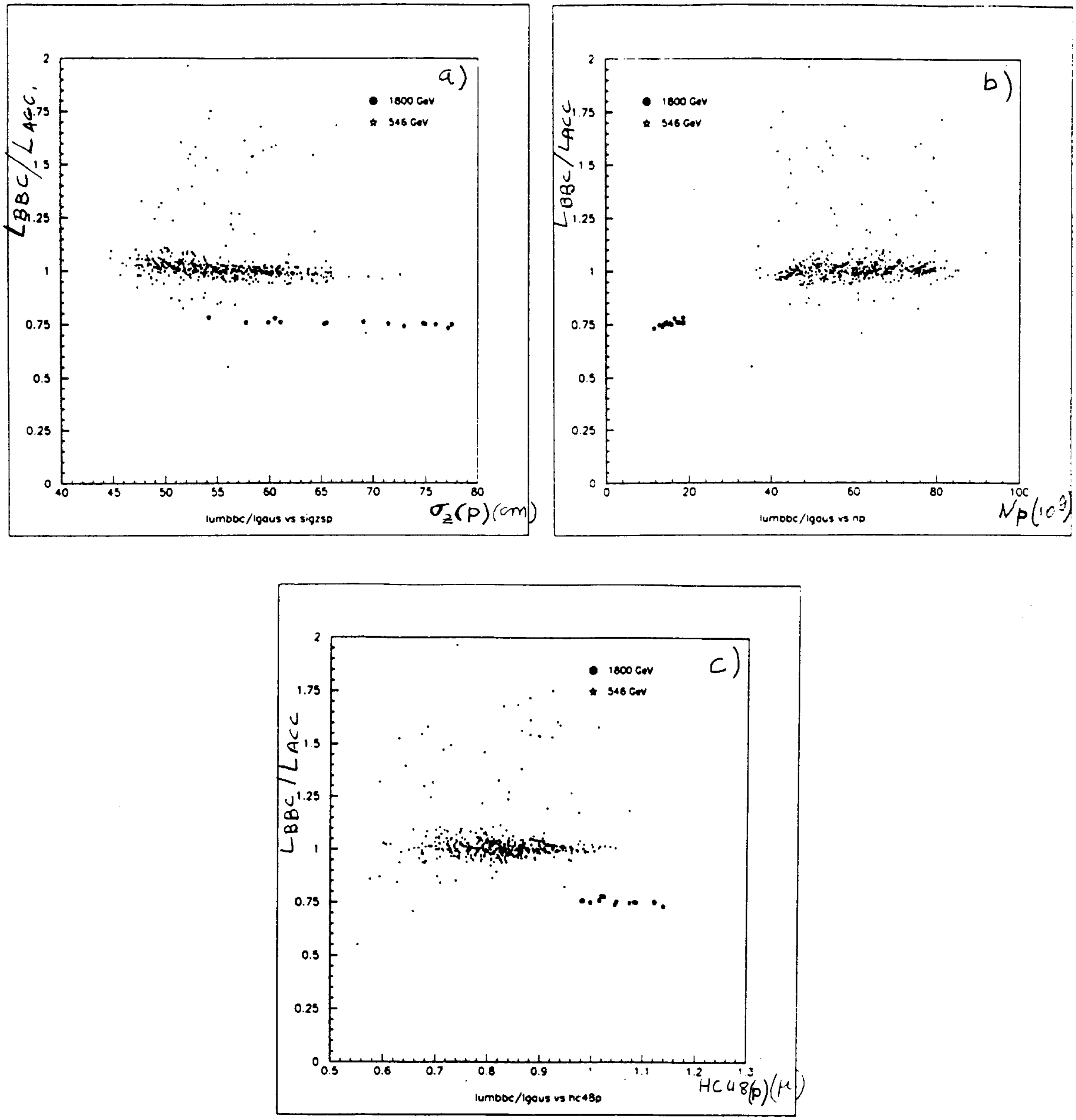


Fig. 10 - a) Dependence of $\frac{\mathcal{L}_{BBC}}{\mathcal{L}_{accel}}$ on the longitudinal bunch size of the proton. b) Dependence of $\frac{\mathcal{L}_{BBC}}{\mathcal{L}_{accel}}$ on the number of particles in the proton bunches. c) Dependence of $\frac{\mathcal{L}_{BBC}}{\mathcal{L}_{accel}}$ on the horizontal size of the proton bunches as measured by the flying wires at location C48.

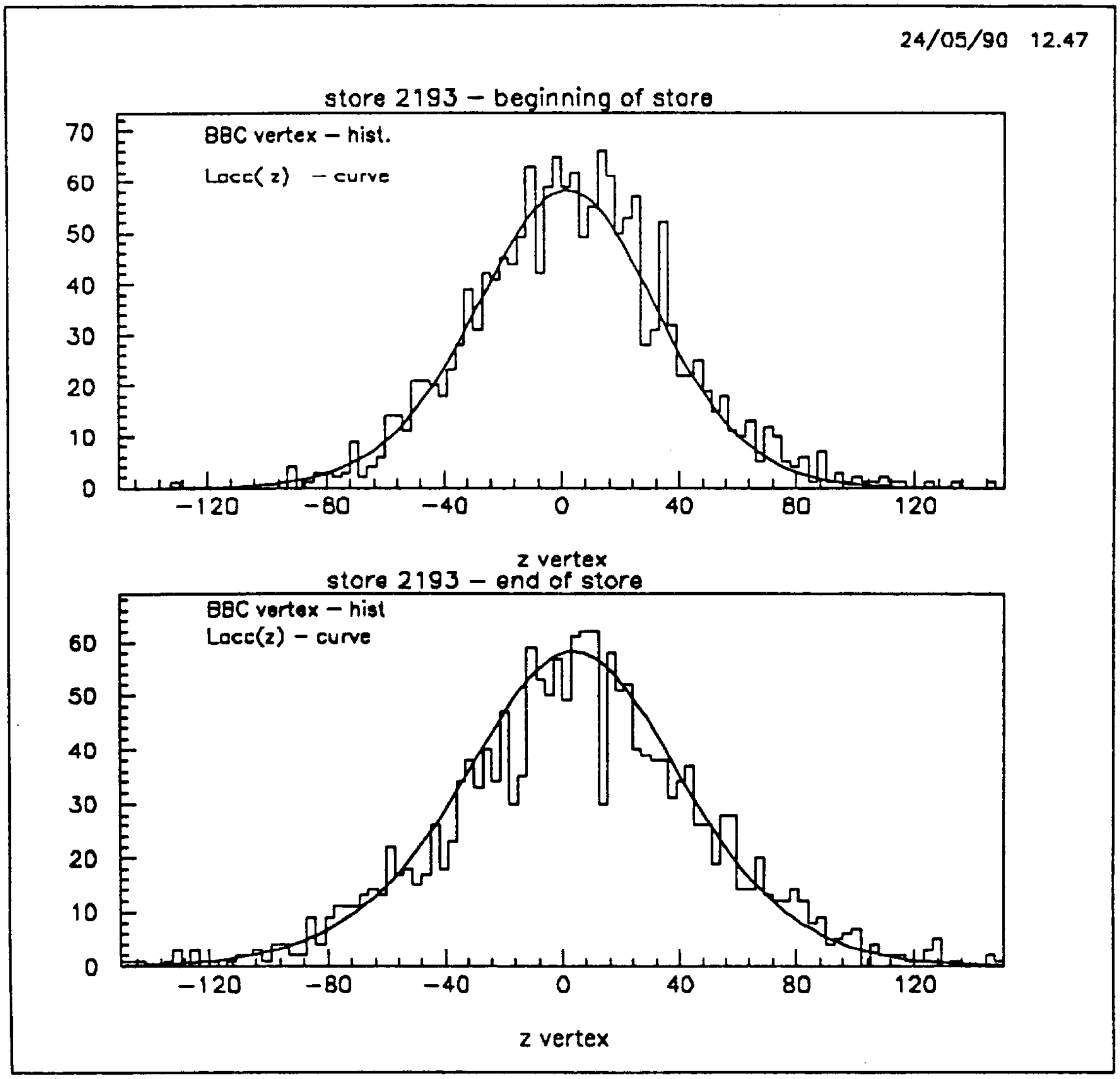


Fig. 11 - Vertex distribution as measured by BBC timing for a data sample at the beginning, a), and at the end of a store, b). The superimposed curves are the luminosity profiles calculated from accelerator parameters from flying wire scans at corresponding times.