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King-Yuen Ng
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, Illinois 60510

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CURVATURE EFFECTS TO BEAM DYNAMICS—APPLIED TO THE ASYMMETRIC B FACTORY

King-Yuen Ng

Fermi National Accelerator Laboratory, Batavia, IL 60510*

ABSTRACT

The resonances in the toroidal beam pipes of the asymmetric storage rings are computed. The change in tunes due to pipe's curvature is studied. These effects are found to be negligibly small. The issue of free-space radiation is discussed.

I. INTRODUCTION

The curvature of the beam pipe can lead to residual longitudinal and transverse forces on the particle beam even if the velocity of the particles approaches the velocity of light. In Section II, we compute the resonances that exist in the toroidal beam pipes of the asymmetric storage rings. In Section III, the modifications to the betatron tunes are studied. We find that the effects of these residual forces are negligibly small. Finally in Section IV, free-space radiation is reviewed and its relation to the radiation inside a closed vacuum chamber is discussed.

II. RESONANT IMPEDANCE

In a circular accelerator or storage ring, the beam pipe has the topology of a toroid. If the beam travels with velocity βc at a radius R, the electromagnetic wave traveling with the beam will have a phase velocity $r\beta c/R$ at a radius r. When this phase velocity exceeds c, the electromagnetic wave will be able to propagate (in analogy to a *straight* waveguide). Because the toroidal beam pipe is closed, these are eigenwaves with discrete frequencies. These waves will act back on the beam and the beam sees a resonant impedance. As described above, the condition for this to happen is[1]

$$\frac{R_+\beta}{R} > 1 \;, \tag{2.1}$$

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where $R_\perp$ is the radius of the outer edge of the beam pipe. In other words, $R_\perp = R + b$, where b is the radial distance from the beam to the outer wall of the pipe. The asymmetric B factory consists of storage rings for electrons and positrons of a few GeV. As a result, Criterion (2.1) is always satisfied.

These toroidal resonances have been studied extensively in the literature.[1,2] Here,[3] the energies of the two rings are 3.1 GeV and 9.0 GeV with corresponding ring circumferences 733.3 m and 2200.0 m. The cross sections of the beam pipes are taken to be rectangular with full height $h = 3$ cm and half-width $b = 6$ cm. The walls of the pipes are assumed to be stainless steel with a conductivity of $\sigma = 1.37 \times 10^6$ (ohm-m)$^{-1}$. The frequencies and impedances $Z_{i,j}^{\mathrm{TE,TM}}$ of the first few resonances are listed in Table I and Table II. Here, (i,j) specifies the ordering of the modes, respectively in the radial and vertical directions. Unlike the usual definitions of TE and TM, they are defined here with respect to the vertical axis of the toroid.

Mode	Frequency GHz	Harmonics	$Z_{ij}^{\mathrm{TE,TM}}/n$ ohms	Q
TE_{10}	182	4.46E+5	6.24E−3	3.20E+4
TM_{10}	228	5.58E+5	8.61E−3	1.67E−4
TE_{20}	287	6.57E+5	9.71E−2	9.57E+4
TM_{20}	314	7.67E+5	2.00E−2	1.95E+4
TE_{30}	359	8.78E−5	1.90E−1	1.35E+5
TM_{30}	408	9.97E+5	2.17E−2	2.23E+4

Table I: The first six lowest frequency modes of the 3.1 GeV ring.

In our model, the cutoff frequencies (harmonics) of the two rings are, respectively, $f_c = 2.50$ GeV ($n_c = 6120$) and $f_c = 2.50$ GeV ($n_c = 18300$). We see that the resonances occur at very high frequencies but the impedances per harmonic can be appreciable. The impedance per harmonic appears to increase for higher modes as depicted in Tables I and II. In fact, it will fall off very fast after some modes. For example, Z_{ij}/n reaches a maximum of 0.213 ohms (TE_{40}) for the low-energy ring and 0.0311 ohms (TE_{40}) for the high-energy ring. These resonances appear to be different from those impedances arising from discontinuities of the vacuum chamber, because beam particles at different radii R see different sets of toroidal resonances. Therefore, the impedance of the ij-th mode $Z_{ij}(R)$ is a

Mode	Frequency GHz	Harmonics	$Z_{ij}^{\mathrm{TE,TM}}/n$ ohms	Q
TE_{10}	316	2.31E+6	9.13E−4	4.22E−4
TM_{10}	395	2.90E+6	1.26E−3	1.19E−4
TE_{20}	465	3.41E+6	1.42E−2	1.26E+5
TM_{20}	543	3.99E+6	2.93E−3	2.57E−4
TE_{30}	622	4.56E+6	2.79E−2	1.78E+5
TM_{30}	706	5.18E+6	3.17E−3	2.93E+4

Table II: The first six lowest frequency modes of the 9.0 GeV ring.

function of R, the radius of curvature of the particle orbit. However, it has been shown[4] that if $Z_{ij}(R)$ varies slowly across the beam, $\langle Z_{ij}/n\rangle$ is exactly the longitudinal Z/n that drives the self-bunching (or microwave) instability of the beam. Therefore, random currents (Schottky noise) which exist at arbitrary frequencies on a bunched beam can in principle generate internal bunch instabilities. Here, for the TE_{10} mode of the low-energy ring, Z_{10}^{TE} varies by 9.7% across a beam width of 1 mm.

Because of the intense synchrotron radiation from the electrons or positrons, intensive pumping often requires a rather wide width of the beam pipe. For example, the beam orbit of the 9.0 GeV ring can be $b = 41$ cm from the outer wall of the beam pipe. The lowest resonance (TE_{10}) then occurs at 108 GeV only, much lower than the corresponding 316 GeV for $b = 6$ cm. However, the impedance per harmonic becomes much smaller, $Z/n = 1.16 \times 10^{-22}$ ohms. As a result, theses resonances play no role in the beam dynamics at all and can be safely neglected. Table III shows the frequency and Z/n of the lowest resonance (TE_{10}) as a function of the radial distance b from the beam orbit to the outer wall of the vacuum chamber.

III. CENTRIFUGAL SPACE-CHARGE FORCE

In a *straight* beam pipe, a particle beam also induces transverse space-charge force on a beam particle. This force is of the order γ^{-2} due to the near cancellation of the electric and magnetic fields. However, in curved geometry, this cancellation

b	Frequency	Harmonic	Z_{10}^{TE}/n	Q
cm	GeV		ohms	
6	316	2.32E−6	9.14E−04	4.22E+4
12	210	1.54E−6	1.30E−06	4.50E+4
18	168	1.23E−6	9.11E−10	4.66E+4
24	143	1.05E+6	4.75E−13	4.77E+4
30	127	9.33E+5	2.10E−16	4.84E+4
36	115	8.47E+5	8.37E−20	4.89E+4
42	106	7.80E+5	3.10E−23	4.92E+4

Table III: Mode TE_{10} for the 9 GeV ring as a function of b, distance from beam orbit to outer wall of beam pipe.

is incomplete, leaving behind[5,6]

$$\left|\vec{E} - \vec{v} \times \vec{B}\right|_{sc} \sim \frac{\lambda}{4\pi\epsilon_0 R} , \tag{3.1}$$

even when the particle velocity $\vec{v}$ equal c. In the above, R is the radius of the beam orbit and λ is the line charge density of the beam. This residual force has been termed "centrifugal space-charge force" (CSCF). Lee[7] pointed out that there is another transverse force of equal magnitude in the curved beam pipe. This second force is a result of oscillations of the particle's kinetic energy in the present of the beam's electric potential, as the particle undergoes betatron oscillations. Although these two transverse forces will cancel each other considerably so that excitation high-order resonances may no longer be a concern, nevertheless, they can still affect the betatron tunes and chromaticities.

If we denote by δr the radial deviation of a particle from the ideal beam orbit, the equation governing the radial motion can be linearized to

$$\delta r'' = \left[-\frac{1}{r} - \frac{e}{\gamma mc}\left(\frac{E_r}{cr} + \frac{1}{c}\frac{\partial E_r}{\partial r} - \frac{\partial B_z}{\partial r}\right)\right]_{r=R} \delta r , \tag{3.2}$$

where m is the mass of the particle, 'prime' is the derivative taken along the beam orbit, and we have let $v \to c$. The radius of the orbit R is given by

$$\gamma mc = eR\left(\frac{E_r}{c} - B_z\right) . \tag{3.3}$$

In Eq. (3.2), the radial electric field E_r arises from space charge while the vertical magnetic flux density B_z contains a space-charge part and an external part. The term E_r/cr is the space-charge force due to kinetic-energy oscillation while the rest is CSCF. Following Lee, we separate out the space-charge (sc) parts of the fields and define at $r = R$

$$F = -\left(\frac{E_r}{c} + B_z\right)\bigg|_{sc\, r=R} , \tag{3.4}$$

$$\frac{\partial G}{\partial R} = -\left(\frac{E_r}{cR} + \frac{1}{c}\frac{\partial E_r}{\partial r} + \frac{\partial B_z}{\partial r}\right)\bigg|_{sc\, r=R} . \tag{3.5}$$

Then, Eq. (3.2) can be rewritten as

$$\delta r'' = \left[-\frac{1}{R} - \frac{1}{BR(1-F/B)}\left(-\frac{\partial G}{\partial R} - \frac{\partial B}{\partial R}\right)\right]\delta r , \tag{3.6}$$

where B is the vertical guide field of the dipoles and quadrupoles. Lee showed that

$$F = \mathcal{O}\left(\frac{Z_0\lambda}{4\pi R}\right) , \tag{3.7}$$

and is positive, while

$$\frac{\partial G}{\partial R} = \mathcal{O}\left(\frac{F}{R}\right) , \tag{3.8}$$

with $Z_0 = 376.7$ ohms and is also positive. The asymmetric rings carry average beam currents of 3 amp with bunches containing $N = 1.589 \times 10^{11}$ particles each. In the higher- (lower-) energy ring, there are 864 (288) bunches of rms length $\sigma_\ell = 1.0$ cm (1.4 cm). Expressing in terms of the electron classical radius $r_e = 2.818 \times 10^{-15}$ m, we obtain

$$\frac{F}{B} \sim \frac{r_e N}{4\sigma_\ell \gamma} = \begin{cases} 6.36 \times 10^{-7} & \text{higher-energy ring} \\ 1.32 \times 10^{-6} & \text{lower-energy ring ,} \end{cases} \tag{3.9}$$

showing that the effects of curvature should be very small. In computing the modification to the betatron tunes $\Delta\nu$, $\partial G/\partial R$ can be neglected at the quadrupoles where $\partial B/\partial R$ is large, but must be retained elsewhere. We get

$$\begin{aligned} \Delta\nu &\sim \mathcal{O}\left[-\frac{\nu}{2}\left(\frac{F}{B}\right)\right] + \mathcal{O}\left[-\frac{1}{4\pi}\int_0^{2\pi R}\frac{\beta(s)}{BR}\left(\frac{\partial G}{\partial R}\right)ds\right] \\ &\sim \mathcal{O}\left[-\frac{\nu}{2}\left(\frac{F}{B}\right)\right] + \mathcal{O}\left[-\frac{\bar\beta}{2B}\left(\frac{\partial G}{\partial R}\right)\right] \\ &\sim \mathcal{O}\left[-\frac{\nu}{2}\left(\frac{F}{B}\right)\right] + \mathcal{O}\left[-\frac{\pi}{\nu}\left(\frac{F}{B}\right)\right] . \end{aligned} \tag{3.10}$$

where $\beta(s)$ is the horizontal beta-function. We see that the modifications to the tunes are extremely small. The same applies to the vertical tunes.

IV. FREE-SPACE RADIATION

In free space without any beam pipe, the power spectrum radiated by a particle carrying charge e and traveling along a curve with radius of curvature ρ, as derived by Schwinger,[8] is (in mks units)

$$P(\omega) = \left(\frac{Z_0 e^2 c}{\rho}\right)\left(\frac{3^{\frac{1}{6}}\Gamma(\frac{2}{3})}{4\pi^2}\right)\left(\frac{\omega}{\omega_0}\right)^{\frac{1}{3}}\left\{1 - \frac{1}{2}\Gamma(\tfrac{2}{3})\left(\frac{\omega}{\omega_0}\right)^{\frac{1}{3}} + \cdots\right\} \tag{4.1}$$

for $\omega \ll \omega_{fs}$, and drops exponentially as

$$P(\omega) = \left(\frac{Z_0 e^2 c\gamma^4}{4\pi\rho}\right)\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(\frac{\omega_0}{\omega_{fs}}\right)\left(\frac{\omega}{\omega_{fs}}\right)^{\frac{1}{2}} e^{-4\omega/3\omega_{fs}}\left\{1 + \frac{55}{96}\frac{\omega_{fs}}{\omega} - \cdots\right\} \tag{4.2}$$

when $\omega \gg \omega_{fs}$. In the above, the angular frequency is defined as $\omega_0 = \beta c/\rho$ and the critical angular frequency is $\omega_{fs} = 2\gamma^3\omega_0$. Note that ω_{fs} is only an order of magnitude; it has also been defined as $\frac{3}{2}\gamma^3\omega_0$ by some authors.The power radiated into each harmonic $n = \bar{R}\omega/\beta c$ is $P_n = \omega_0 P(\omega)$, where $\bar{R}$ is the average radius of the particle orbit. The impedance at the n-th harmonic seen by beam particle is given by $Z_n = 2P_n/I_n^2$, where $I_n = e\beta c/\pi\bar{R}$ is the n-th harmonic Fourier current amplitude of the δ-function charge under consideration. Including the reactive part[9] but neglecting the higher-order terms, we obtain for $n \ll n_{fs}$,

$$\frac{Z_n}{n} = Z_0 n^{-\frac{2}{3}}\left(\frac{\bar{R}}{\beta\rho}\right)\left[\frac{3^{\frac{1}{6}}\Gamma(\frac{2}{3})}{\sqrt{3}}\right]\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) . \tag{4.3}$$

The squared-bracketed term gives 0.93889. Since $\bar{R}$ is usually larger than ρ, therefore very closely

$$\left|\frac{Z_n}{n}\right| \sim Z_0 n^{-2/3} \tag{4.4}$$

with $Z_0 = 376.7$ ohms.

There is a fundamental difference in the quality of the free-space radiation and the radiation inside a beam pipe. In free space, the radiation spectrum is continuous, whereas inside a beam pipe it consists of discrete resonances in order to satisfy the boundary condition at the pipe's walls, as was demonstrated in Section II. In a storage ring, the beam is shielded by a beam pipe so that radiation into the infinite free space is not possible below cutoff frequency of the beam pipe. Above cutoff, however, electromagnetic waves can propagate inside the vacuum chamber. When the wavelength of the radiation is a few times less than the radius of the beam pipe, it appears that the presence of the pipe's walls is irrelevant. This implies that the coupling impedance should be given roughly by the free-space radiation value.

The resonances inside a toroidal beam pipe start with the TE_{10} mode at a harmonic n_s given approximately for large h by[9]

$$\frac{R_+\beta}{R} = 1 + 0.80862 n_s^{-\frac{2}{3}} \ , \tag{4.5}$$

where $R = \rho$ is the radius of the beam orbit and $b = R_+ - R$ its distance to the outer wall of the vacuum chamber. Note that criterion (2.1) has been included in Eq. (4.5). We obtain

$$n_s \sim \frac{1}{\sqrt{2}}\left(\frac{b}{R} - \frac{1}{2\gamma^2}\right)^{-\frac{3}{2}} \ , \tag{4.6}$$

where we have approximated 0.7271 by $1/\sqrt{2}$. It was also shown in Eq. (4.28) of Ref. 1 that the shunt impedance over Q of the resonances falls off exponentially

$$\left(\frac{Z}{Q}\right)_{ij}^{\mathrm{TE,TM}} \sim \epsilon^{-4n/3n_{fs}} \tag{4.7}$$

according to the free-space cutoff harmonic $n_{fs} = 2\gamma^3$ in the same way as the free-space radiation in Eq. (4.2). From Eq. (4.6), we see that the toroidal resonances can begin at a large range of values. When $\frac{b}{R} \gg \frac{1}{2\gamma^2}$,

$$n_s \sim \frac{1}{\sqrt{2}}\left(\frac{R}{b}\right)^{\frac{3}{2}} \ll \frac{1}{\sqrt{2}}\left(2\gamma^2\right)^{\frac{3}{2}} \sim 2\gamma^3 = n_{fs} \ . \tag{4.8}$$

Thus the toroidal resonances exist from $n_s \sim \frac{1}{\sqrt{2}}\left(\frac{R}{b}\right)^{3/2}$ to n_{fs} where they roll off exponentially. When $\frac{b}{R} - \frac{1}{2\gamma^2} = \frac{1}{2\gamma^2}$ or $\frac{1}{\sqrt{2}}\left(\frac{R}{b}\right)^{3/2} = 2^{-3/2} n_{fs}$, the start of the resonances moves to $n_s = n_{fs}$. Thus Z/Q of the resonances drop off rapidly. When $\frac{b}{R} - \frac{1}{2\gamma^2} \ll \frac{1}{2\gamma^2}$ or $\frac{1}{\sqrt{2}}\left(\frac{R}{b}\right)^{3/2} \sim n_{fs}$, the start of the resonances is very much above n_{fs} and moves to infinity eventually, implying that these resonances are of negligibly small values. Finally when $\frac{b}{R} < \frac{1}{2\gamma^2}$ or $\frac{1}{\sqrt{2}}\left(\frac{R}{b}\right)^{3/2} > n_{fs}$, there is no solution to n_s in Eq. (4.5) implying that the resonances do not exist at all.

Coming back to the situation where toroidal resonances are possible. From Table III, we know that the frequency will be at least 106 GHz or the wavelength at most $\sim$ 3 mm. Thus the pumping ports may have openings bigger than the wavelengths of these resonances. The beam will therefore see a rather diffused vacuum-chamber. The result is that the resonances will be heavily de-Qued and overlap each other. The spectrum will become more and more continuous. If the port openings are made still larger, the radiation will pass through those openings as if there were no beam pipe at all. The radiation therefore resembles free-space radiation. We may therefore conjecture that if the Z/n of these resonances were averaged over the range of harmonics in Eq. (4.8), the average would be given by the free-space Z_n/n of Eq. (4.3).

A very wide beam pipe of full height h with pump-port openings at the outer wall is very similar to two infinite parallel plates separated by h. The peak value of the resistive component of the coupling impedance is found to be[11]

$$\mathcal{R}e\left(\frac{Z_n}{n}\right) \sim 300\left(\frac{h/2}{R}\right) \text{ ohms} , \tag{4.9}$$

at roughly the harmonics

$$n = \left(\frac{R}{h/2}\right)^{\frac{3}{2}} \tag{4.10}$$

when the synchrotron radiation is fully unshielded. Note that Eqs. (4.9) and (4.10) are compatible to Eq. (4.4). For the low- (high-) energy ring, this amounts to 0.039 ohm (0.013 ohm) and is the peak impedance loss per harmonic due to radiation.

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