Fermi National Accelerator Laboratory

FN-483

Effects of Copper Coating the RHIC Beam Pipe*

King-Yuen Ng
SSC Central Design Group
c/o Lawrence Berkeley Laboratory 90/4040
Berkeley, California 94720
and
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois

April 1988

*Submitted to the Proceedings of the Workshop on RHIC Performance at Brookhaven, Upton, New York, March 21-26,1988

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

FN-483

EFFECTS OF COPPER COATING THE RHIC BEAM PIPE

King-Yuen Ng

*SSC Central Design Group**

c/o Lawrence Berkeley Laboratory 90/4040, Berkeley, CA 94720

and

*Fermi National Accelerator Laboratory**

Batavia, IL 60510

(April 1988)

Submitted to the Proceedings of the Workshop on RHIC Performance

at Brookhaven, Upton, NY, March 21-26 1988

*Operated by the Universities Research Association, Inc., under contracts with the U.S. Department of Energy.

I. INTRODUCTION

There has been a suggestion that the beam pipe of the RHIC main ring[1] should be coated with a layer of copper. Since the conductivity of copper at cryogenic temperature is $\sim$ 1000 higher than that of stainless steel, the obvious advantages of the copper coating are the lowering of the transverse coupled-bunch growths and parasitic heating. However, the coating can bring in mechanical problems to the beam pipe. During a quench, the Lorentz force generated by the eddy current in the highly conductive copper layer can lead to bigger deformation of the beam pipe. The partially vaporized helium outside, if not allowed to escape fast enough, can easily push the beam pipe towards stress failure. Also if the copper coating is not even, this Lorentz force generated during a quench will produce a torque, which if strong enough can cause failure to the position-fixing keys attached to the beam pipe.

A transition jump may be necessary during the acceleration cycle. In order that the fast changing quadrupole field initiating the jump will not be blocked, it is suggested that the beam pipe in the quadrupole regions should not be coated. Undoubtedly, the coupling impedances will be affected by these gaps and so are the instability growths and parasitic heating.

All these problems will be studied in detail in the following sections. For the RHIC main ring, we take the mean radius as $R = 610.18$ m, the beam pipe internal radius $b = 3.645$ cm with a wall thickness of $t = 0.165$ cm. At cryogenic temperatures, the conductivity of bulk copper is taken as $\sigma_c = 2.3 \times 10^9$ $(\Omega\text{-m})^{-1}$ corresponding to a residual resistance ratio (RRR) of 200. The conductivity of stainless steel is taken as $\sigma_s = 2.0 \times 10^6$ $(\Omega\text{-m})^{-1}$.

II. TRANSVERSE COUPLED-BUNCH GROWTHS

For $M = 56$ equal *point* bunches, there are M modes of transverse coupled-bunch motion. The growth rate of the μth mode driven by the transverse coupled impedance $Z_\perp$ is[2]

$$\frac{1}{\tau_\mu} = -\frac{Z^2}{A}\frac{MI_bc}{4\pi\nu E/e}\sum_{k=-\infty}^{\infty} \mathcal{R}e\, Z_\perp[(kM-\mu+\nu)\omega_0] \ , \tag{2.1}$$

where Ze, A, and E are respectively the charge, atomic number, and average total energy per amu of an ion in the bunch ($Z = 1$, $A = 1$ for proton and $Z = 79$, $A = 197$ for gold), ZI_b is the average bunch current, c is the velocity of light, ν is the betatron tune, and $\omega_0/2\pi$ is the mean revolution frequency. Besides a resonance, the resistive wall can contribute to Eq. (2.1) because the impedance goes to infinity at zero frequency. At very low frequencies, the image wall currents fill the whole wall thickness t, the transverse resistive wall impedance of the whole ring is given by

$$Z_\perp(\omega) = \frac{2Rc}{\sigma_s t\omega b^3} \; . \tag{2.2}$$

The ring has a designed betatron tune of $\nu = 28.82$. Therefore the coupled-bunch mode that has the fastest growth is $\mu = 29$, corresponding to the spectral line with $k = 0$ or at $\omega = -0.18\omega_0$ where $Z_\perp = 25.9$ MΩ/m for a stainless steel pipe wall. The next lines are $M = 56$ units of ω_0 away and give negligible contribution. The designed average bunch current is

$$ZI_b = \begin{cases} 1.25 \text{ mA} & \text{for p with } N = 1.0\times 10^{11} \\ 1.09 \text{ mA} & \text{for Au with } N = 1.1\times 10^{9} \; , \end{cases} \tag{2.3}$$

and the mean total energy per amu is

$$E = \begin{cases} 29.4 \text{ GeV} & \text{p} \\ 11.6 \text{ GeV} & \text{Au} \; . \end{cases} \tag{2.4}$$

Thus the worst growth rates are

$$\frac{1}{\tau_{29}} = \begin{cases} 51.6 \text{ sec}^{-1} & \text{p} \\ 45.8 \text{ sec}^{-1} & \text{Au} \; , \end{cases} \tag{2.5}$$

or growth times

$$\tau_{29} = \begin{cases} 19.3 \text{ ms} & \text{p} \\ 21.9 \text{ ms} & \text{Au} \; . \end{cases} \tag{2.6}$$

For the SSC, any transverse coupled-bunch growth times $\tau > 8.5$ ms can be stabilized by the designed transverse damper. Using this as a criterion, the present RHIC design is quite safe against transverse coupled-bunch instabilities and no copper coating is required. However, with a copper layer of $t_c = 1$ mil (25.4 micron) coated inside the beam pipe everywhere, the transverse impedance is reduced $\sigma_c t_c/\sigma_s t = 17.7$ times, and the growth times will be increased by the same ratio. Here we have assumed the the wall currents will reside mostly in the copper layer only because the latter is so much more conductive than stainless steel.

III. PARASITIC HEATING

The power lost to wall resistivity can be obtained by integrating the longitudinal coupled wall impedance over the bunch power spectrum to get[3]

$$P = \Gamma(3/4)M(NZI_bR)^2 \left(\frac{Z_0}{2\sigma}\right)^{1/2} \left(\frac{1}{b\sigma_z^{3/2}}\right) , \tag{3.1}$$

where $\Gamma(3/4) = 1.23$ is the gamma function, $Z_0 = 377\ \Omega$, σ_z is the rms bunch length, and σ is the conductivity of the pipe wall. Here we have neglected all contributions due to higher multipoles and assumed that the wall currents flow in a skin depth of the pipe wall only. The latter is true except at very low frequencies.

For RHIC, most of the parasitic heating that will endanger the cryogenic is due to wall resistivity because other discontinuities such as the rf cavities are at room temperature. Therefore coating the pipe wall with a copper layer of reasonable thickness everywhere will lower the parasitic heating by a factor of $\sqrt{\sigma_c/\sigma_s} \sim 34$ times.

IV. STRESS FAILURE DURING A QUENCH

During a quench, the helium outside the beam pipe will be partially vaporized and the external helium pressure will rise. If the beam pipe of average radius $\bar{b}$ and thickness t is perfectly cylindrical, it will be buckled when the radially symmetric helium pressure p reaches the critical buckling pressure

$$p_c = \frac{Yt^3}{4(1-\mu^2)\bar{b}^3} , \tag{4.1}$$

where Y is the modulus of tensile elasticity and μ the Poisson's ratio for lateral contraction. The buckled pipe can reach the elastic limit easily when the helium pressure exceeds p_c slightly. Note that p_c is very sensitive to the radius to thickness ratio $m = \bar{b}/t$.

For the analysis in this section and the next, we shall approximate the proportional limit, below which Hooke's law holds, by the yield limit. For many materials, the yield limit comes just slightly beyond the proportional limit.

During a quench, the sudden drop in vertical magnetic dipole flux density B induces eddy current in the pipe wall which flows mainly in the highly conductive copper layer. Thus a horizontal outward Lorentz pressure

$$P_L(\varphi) = P_m \sin\varphi , \tag{4.2}$$

is generated, where φ is the polar angle. This pressure has a maximum at the equator, $\varphi = \pi/2$, given by

$$P_m = \sigma_c t_c b |B\dot{B}| \ . \tag{4.3}$$

Here, the conductivity of copper σ_c, the dipole magnetic flux density B, and its time derivative $\dot{B}$ are all time dependent during the quench, but the maximum is implied in Eq. (4.3). Note that this Lorentz pressure is directly proportional to the thickness of the copper layer. Equation (4.3) is in mks units. If B is in tesla, $\dot{B}$ in tesla/sec, b and t_c in m, and σ_c in $(\Omega\text{-m})^{-1}$, then the pressure P_m will be in newton/m^2. To convert to psi or atmospheres, the factors 1 newton/m^2 $= 1.45 \times 10^{-4}$ psi and 1 atm $= 14.68$ psi should be used.

With such a Lorentz pressure, the beam pipe will be flattened. Then the helium pressure required to further deform the pipe to the elastic limit will be less than the critical buckling pressure p_c. For a given equatorial Lorentz pressure P_m, the yield helium pressure p is given by[4]

$$\sigma_{yp} = \begin{cases} p\dfrac{\bar{b}}{t} + \dfrac{3P_m}{2}\dfrac{\bar{b}^2}{t^2}\dfrac{p_c}{p_c - p} & \text{when } P_m < 2p \\[2ex] (P_m - p)\dfrac{\bar{b}}{t} + \dfrac{3P_m}{2}\dfrac{\bar{b}^2}{t^2}\dfrac{p_c}{p_c - p} & \text{when } P_m > 2p \ , \end{cases} \tag{4.4}$$

where σ_{yp} is the tensile yield stress of the pipe material. For the first relation of Eq. (4.4), the pipe yields at the equatorial points $\varphi = \pi/2$, and for the second relation, the pipe yields at the polar points $\varphi = 0$ and π.

Taking for stainless steel at 20 K, the modulus of tensile elasticity $Y = 30 \times 10^6$ psi, the Poisson's ratio $\mu = 0.278$, and the yield-point stress $\sigma_{yp} = 115 \pm 25$ kpsi, the limiting helium and Lorentz pressures are computed and are plotted in Fig. 1 with the range of uncertainty indicated by dotted curves. Here, we take the average pipe radius as $\bar{b} = 3.7275$ cm and the pipe thickness as $t = 1.65$ mm. As a safety factor, the working yield stress is usually taken as about one half of the actual one. The limiting pressures corresponding to the working yield stress have also been included in the figure.

The maximum equatorial Lorentz pressure P_m can be estimated using a model to compute or performing an experiment to measure the maximum of $|B\dot{B}|$ during a quench. For the SSC, with a pipe radius of $b = 1.66$ cm and copper thickness of 2 mils, the estimated P_m is about 90 psi. Here at RHIC, the product tb is roughly the same. However the dipole field is a factor of two smaller. We may expect P_m to be ~ 22.5 psi, a factor of four smaller. From Fig. 1, with the working yield stress, the maximum allowable helium pressure is ~ 30 atm, which should be able to maintain without much difficulty.

We see from Fig. 1 that most of the time failure will occur at the equatorial points corresponding to the first criterion of Eq. (4.3), which can be rewritten as

$$P_m = \frac{2}{3m^2}\left(1 - \frac{p}{p_c}\right)(\sigma_{yp} - mp) \ , \tag{4.5}$$

where $m = \bar{b}/t$. Together with the definition of the critical buckling pressure p_c in Eq. (4.1), it is easy to observe that the limiting pressure curve is every sensitive to m. If, for example, the pipe radius is increased by a factor of two or the pipe thickness is decreased by a factor of two so that m is doubled, the working limiting curve will starts off from a new critical pressure of ~ 6 atm, a factor of 8 smaller and ends at Lorentz pressure $P_m \sim 18$ psi, a factor of 4 smaller.

V. BEAM-PIPE TORQUE LIMITATION

If, for example, the copper thickness varies with the polar angle φ as

$$t(\varphi) = t_c(1 + \epsilon \sin 2\varphi) \ , \tag{5.1}$$

the torque per unit length per radius squared twisting the beam pipe about its axis during a quench is given by[5]

$$\frac{\tau}{\ell\bar{b}^2} = \frac{1}{2}\,\pi\epsilon P_m \ . \tag{5.2}$$

This torque is resisted by the keys attached to the beam pipe. If the unevenness is big enough, these keys can suffer a stress failure. The detail depends on the actual design and material of the keys. If a tensile stress failure occurs, the limiting torque per unit length and radius squared is given roughly by[4]

$$\frac{\tau}{\ell\bar{b}^2} = \frac{w^2 g}{3\ell b h}\sigma_{yp} \ , \tag{5.3}$$

where each key is assumed to have a width w, length g, height h and located at interval ℓ along the length of the beam pipe. Taking a key similar to that in the SSC, $w \sim 0.2$ in., $g \sim 3$ in., $h \sim 3$ mm, $\ell \sim 18$ in., and $\sigma_{yp} \sim 5000$ psi for the key material, we obtain $\tau/\ell\bar{b}^2 \sim 66$ psi. If as a safety factor, the working yield stress is again taken as one half the actual one, then with the equatorial Lorentz pressure of $P_m \sim 22.5$ psi per mil of copper layer thickness as estimated in Section VI, Eqs. (5.2) and (5.3) lead to

$$t_c\epsilon \approx 0.47 \text{ mil} \ . \tag{5.4}$$

VI. GAPS IN THE COPPER LAYER

In order to allow rapid changing quadrupole field to penetrate the beam pipe during a transition jump, the portion of the beam pipe inside the quadrupoles are *not coated* with copper.

During a quench, the eddy currents flowing in the copper layer are of the same magnitude regardless of whether there are any gaps in the coating. Therefore the allowable stress of the copper coated portion of the beam pipe will not be affected by the quadrupole gaps at all.

Parasitic heat generation is directly proportional to the longitudinal coupling impedance $Z_{\|}$, if higher-multipole contributions are neglected. The longitudinal wall impedance is just the sum of the impedance due to the copper coated part and that due to the stainless steel gaps. According to the Conceptual Design,[1] there are 246 quadrupoles per ring occupying a total length of $\ell_q = 360.42$ m. We assume that the beam pipe inside all these quadrupoles are not coated. We further assume that all other parts of the beam pipe $\ell_c = 3833.87 - 360.42 = 3473.45$ m are copper coated. From Eq. (3.1), the parasitic loss due to wall resistivity alone is

$$\frac{\ell_q/\sqrt{\sigma_s} + \ell_c/\sqrt{\sigma_c}}{(\ell_q + \ell_c)/\sqrt{\sigma_s}} = 0.12 \tag{6.1}$$

of the parasitic loss if there is no copper coating at all. This fraction becomes $\sqrt{\sigma_s/\sigma_c} = 0.029$ if 100% of the pipe is coated.

We next consider the effect of the quadrupole gaps to transverse coupled-bunch growths.[6] For the dipole mode, the wall currents flow in opposite directions on each side of the beam pipe. Therefore, on approaching a gap, the dipole currents flowing in the copper layer have the option of turning around in front of the gap or flowing through the gap. The choice of the two paths depends on the resistivity of the gap and the inductive reactance of the turn-around loop. The latter is linearly proportional to the frequency under consideration. This picture can be represented by an equivalent circuit with two resistors Z_g and two inductors L_g as depicted in Fig. 2. The resistor Z_g is just the wall resistivity of roughly a quarter of the circumference of the gap,

$$Z_g \simeq \begin{cases} \dfrac{2g}{\pi b t \sigma_s} & t \lesssim \delta_s \\[2ex] (1+j)\dfrac{2g}{\pi b}\sqrt{\dfrac{nZ_0}{2R\sigma_s}} & t \gtrsim \delta_s \ , \end{cases} \tag{6.2}$$

where g is the length of the quadrupole gap, $Z_0 = 377\ \Omega$, δ_s is the skin depth of stainless steel at the frequency $f = nf_0$ under consideration, and f_0 is the revolution frequency. Take the arc quadrupole as an example, where $g = 1.35$ m. We get

$$Z_g \simeq \begin{cases} 7.15 \times 10^{-3}\ \Omega & t \lesssim \delta_s \\ (1+j)\sqrt{n}9.27 \times 10^{-3}\ \Omega & t \lesssim \delta_s\ , \end{cases} \tag{6.3}$$

The reactance of the inductor L_g is related to the change in image contribution to the space-charge impedance at low frequencies, and is given by

$$Z_L = j\omega L_g \simeq= jn\frac{Z_0 g}{\pi R} = jn0.266\ \Omega\ . \tag{6.4}$$

The lowest frequency of interest is $(\nu - m)$ times the revolution frequency, where m is the integer nearest to the betatron tune ν, because the most dangerous transverse coupled-bunch mode is driven at this frequency. Taking $\nu = 28.82$ or $n = |\nu - m| = 0.18$, the wall current I_W sees Z_g and Z_L in parallel, or an effective impedance of

$$Z_W = 0.00699 + j0.00104\ \Omega\ . \tag{6.5}$$

We see that most of the wall currents will cross the gap instead of turning around. This is different from the situation of the SSC. Here the main ring radius of not as big. The lowest frequency considered $|\nu - m|\omega_0/2\pi = 14.1$ kHz is not as low as that in the SSC (where 0.36 KHz is considered). As a result, the inductive reactance of the turn-around path is big compared with the wall resistivity.

The transverse impedance for the gap is given by[6]

$$Z_\perp = \frac{2c}{\omega b^2}\left[\frac{Z_W}{4}\right]\ . \tag{6.6}$$

where $\mathcal{R}e\, Z_W/4 \approx 0.430\ \Omega$ for a total of 246 gaps. The copper-coated part of length $\ell_c = 3473$ m gives a contribution of

$$Z_\perp = \frac{2c}{\omega b^2} Z_\parallel\ , \tag{6.7}$$

with $Z_\parallel = \ell_c/2\pi b\sigma_c t_c = 0.259\ \Omega$ for a copper layer of $t_c = 1$ mil. If there is no copper coating anywhere, $Z_\parallel = R/b\sigma_s t_s = 5.07\ \Omega$ instead. Therefore, with the copper coating and the gaps, the most dangerous transverse coupled-bunch growth is reduced by $5.07/(0.430 + 0.259) = 7.4$ times. The reduction is 17.7 times if there are no gaps in the coating.

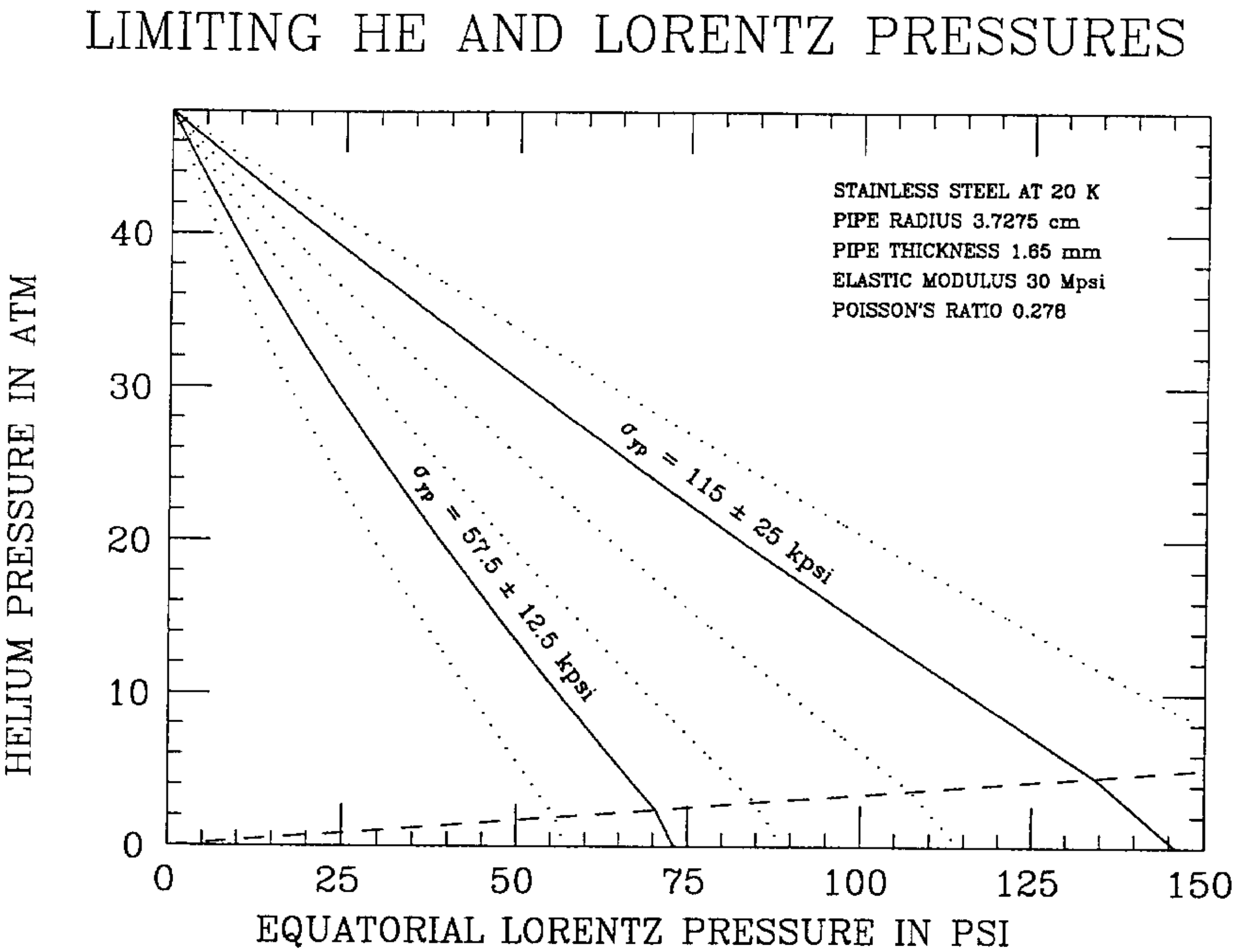
VII. CONCLUSIONS

From the above analysis, we conclude that

1. A copper coating of thickness 1 mil will reduce parasitic heating due to resistive wall by 34 times and transverse coupled-bunch growth by 18 times (or 7.4 times if the beam pipe inside the quadrupoles is not coated).
2. A copper layer will lead to the flattening of the beam pipe as a result of the eddy current during a quench, thus decreasing the external helium pressure necessary to cause stress failure in the beam pipe. For a 1 mil layer, we expect the maximum Lorentz pressure to be $\sim$ 22.5 psi and the beam pipe will be safe if the external helium pressure is kept under $\sim$ 30 atm. There should not be much difficulty in maintaining these limits.
3. A thicker copper layer will not improve the reduction in parasitic heating. It will, however, lead to further reduction in transverse coupled-bunch growth [see for example Eq. (6.7)]. But a thicker copper layer will carry more eddy current and lead to higher horizontal Lorentz pressure during a quench. As a result, the allowable helium pressure will be very much reduced.
4. If the thickness of the copper coating is uneven along the pipe circumference, the keys positioning the beam pipe will suffer a Lorentz torque in the event of a quench. Stress failure of the keys will ocuur when $t_c\epsilon \approx 0.47$ mil. This implies an unevenness limit of 47% for a 1-mil coating or 23.5% for a 2-mil coating.

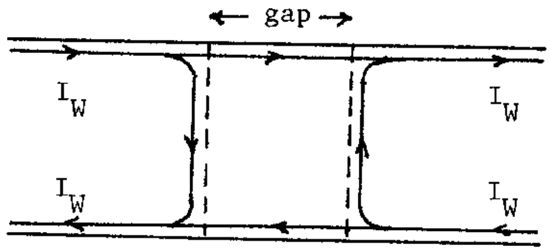
REFERENCES

1. *Conceptual Design of the Relativistic Heavy Ion Collider*, Brookhaven Report BNL 51932 (1986).
2. F. Sacherer, CERN Divisional Report No. CERN/SI-BR/72-5 (1972); F. Sacherer, IEEE Trans. Nucl. Sci. **NS-24**, 1393 (1977).
3. M. Furman, SSC Central Design Group Internal Report No. SSC-N-142 (1986).
4. K.Y. Ng and J. Peterson, SSC Central Design Group Report No. SSC-168 (1988).
5. A. Chao, SSC Central Design Group Internal Report No. SSC-N-434 (1987).
6. K.Y. Ng, SSC Central Design Group Internal Report No. SSC-N-494 (1988).

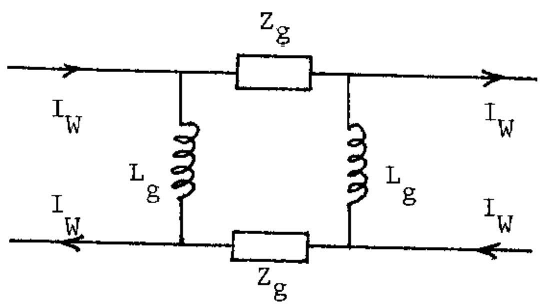


LIMITING HE AND LORENTZ PRESSURES
STAINLESS STEEL AT 20 K
PIPE RADIUS 3.7275 cm
PIPE THICKNESS 1.65 mm
ELASTIC MODULUS 30 Mpsi
POISSON'S RATIO 0.278
σ_{yp} = 115 ± 25 kpsi
σ_{yp} = 57.5 ± 12.5 kpsi
HELIUM PRESSURE IN ATM
EQUATORIAL LORENTZ PRESSURE IN PSI
0
10
20
30
40
0
25
50
75
100
125
150

Fig. 1



(a)



(b)

Fig. 2.