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Impedance of Separators at Low Frequencies

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IMPEDANCE OF SEPARATORS AT LOW FREQUENCIES

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I. INTRODUCTION

Separators are used in $e^+ - e^-$ collider to separate the electron and positron bunches from collision during injection. When the injection stage is finished, the electrostatic potentials on the separators are turned off so that the bunches can collide. However, on passing through the separators, a bunch still sees a discontinuity and experiences an impedance. In a usual collider, besides radiation, the energy lost to the separators is rather significant. Therefore, an estimate of the impedance is necessary. Sakazaki[1] recently computed the transverse impedance of a pair of separators using an equivalent lumped circuit. His result, however, is definitely incorrect, because the real part of his transverse impedance is not positive semi-definite for positive frequencies. His most serious error was the mistaking of the potential on the separator plate as the wake function.

In this note, we try to compute both the longitudinal and transverse impedances of a pair of separators using also the equivalent lumped circuit of Sakazaki. As a result, our results are applicable only at low frequencies. As is discussed below, the model violate causality.

II. THE MODEL

The cross section of a pair of separators inside the vacuum chamber and its equivalent lumped circuit is shown in Fig. 1. The resistance, inductance and capacitance between a separator plate and the vacuum chamber are denoted by R, L, and C_1 respectively, while the capacitance between the two separator plates is $C_p/2$. In the lumped circuit the capacitance $C = C_1 + C_p$ representing the capacitance from the separator plate to the beam and to the vacuum chamber. The current source I_p is the image current flowing into the separator through the R-L-C circuit, while I_R is the current flowing through the resistance R which is dissipative.

The vacuum chamber and the separator plates are considered perfect conductors. Therefore, there is no electric field on their surface. In evaluating the wake functions, we compute the electric field E_z at a distance $d/2$ from the beam where the separator plates are situated. Thus, the only contribution comes from the upstream and down gaps. As a result, the potential $V_p(t)$ on the separator plate is what we need to compute. Because the separator plate is considered perfectly conducting, $V_p(t)$ is unique everywhere on the plate at the time t. This potential is represented by the point P in the equivalent circuit.

Assume a beam current I_0 of duration t_b passing through. The charge is therefore $Q = I_p t_b$. Later we let $t_b \to 0$ so that we have a point charge. We compute firstly the monopole mode. If the head of the bunch arrives at the upstream gap at time $t = 0$, the current flowing into the plate is

$$I_p(t) = -I_0[\theta(t) - \theta(t - t_b) - \theta(t - t_p) + \theta(t - t_p - t_p)] \; , \tag{2.1}$$

where t_p is the length of the separator plate in time. The voltage at P is

$$L\frac{dI_R}{dt} + RI_R = \frac{Q_b - Q_p}{C} , \tag{2.2}$$

where

$$Q_b = \int^t I_p dt \quad \text{and} \quad Q_p = \int^t I_R dt .$$

Differentiating, we get

$$L\frac{d^2 I_R}{dt^2} + R\frac{dI_R}{dt} + \frac{I_R}{C} = \frac{I_p}{C} . \tag{2.3}$$

This equation can be solved easily using Laplace transform. With the input current $I_p(t)$, the response is

$$I_R(t) = -I_0[G(t) - G(t - t_b) - G(t - t_p) + G(t - t_p - t_b)] , \tag{2.4}$$

where

$$G(t) = \theta(t)\left(1 + \frac{\omega_-}{2ib}e^{\omega_+ t} - \frac{\omega_+}{2ib}e^{\omega_- t}\right) . \tag{2.5}$$

In the above, $\omega_\pm$ are the two roots of $s^2 L + sR + 1/C = 0$ and are represented by

$$\omega_\pm = -a \pm ib , \tag{2.6}$$

with

$$a = \frac{R}{2L} \quad \text{and} \quad b = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} . \tag{2.7}$$

Note that $a^2 + b^2 = 1/LC$ and b can sometimes be imaginary depending on whether the R-L-C circuit is underdamped, overdamped, or just-damped.

Now, we take the limit $t_b \to 0$ and obtain

$$I_R(t) = -Q[G'(t) - G'(t - t_p)] ,$$

where the differentiation of $G(t)$ can be written explicitly as

$$G'(t) = \theta(t)\left(\frac{\omega_+\omega_-}{2ib}\right)\left(e^{\omega_+ t} - e^{\omega_- t}\right) . \tag{2.8}$$

The voltage on the separator plate is

$$V_p(t) = \left(L\frac{d}{dt} + R\right) I_R(t) , \tag{2.9}$$

or

$$V_p(t) = -Q[(S(t) - S(t - t_p)] , \tag{2.10}$$

where

$$S(t) = \left(L\frac{d}{dt} + R\right) G'(t) . \tag{2.11}$$

III. THE MONOPOLE WAKE FUNCTION

A pair of separators is a nonuniform structure in the vacuum chamber. For a particle lagging behind the source particle by t_0, the longitudinal electric wake field seen will be different at different time. Therefore, an average the electric field E_z across the whole structure is required. Since we choose to compute E_z along the surface of the separator plate, the only contribution comes from the upstream and downstream gaps. As a result, the wake function or wake field can be defined as

$$W_0(t_0) = \frac{1}{t_p}\int_0^{t_p} E_z(t+t_0)dt \ . \tag{3.1}$$

Or,

$$W_0(t_0) = \frac{1}{t_p}\left[\left.\frac{V_p(t)}{Q}\right|_{t=t_0} - \left.\frac{V_p(t)}{Q}\right|_{t=t_0+t_p}\right] \ . \tag{3.2}$$

Note that the two terms in Eq. (3.2) are of opposite signs. This is because, going across the upstream gap, we climb up to the potential of the plate, and, going across the downstream gap, we climb down to ground again. Substituting Eq. (2.11) in Eq. (3.2), we get

$$W_0(t_0) = -\frac{1}{t_p}[2S(t_0) - S(t_0-t_p) - S(t_0+t_p)] \ . \tag{3.3}$$

As is given by Eqs. (2.9) and (2.11), the only limitation on the function $S(t)$ is that it vanishes when $t < 0$. The last term of Eq. (3.3) clearly indicates that the wake function is nonzero when $-t_p < t < 0$, a result that violates causality. In fact, this is the situation of our model. As an example, suppose the source particle is at time $t_0 < t_p$ after passing through the upstream gap but still has not reached the downstream gap. Some charges will reach the plate and a potential will build up. Because the plate is supposed to be perfectly conducting, this potential is detectable theoretically at the downstream gap. In other words, there is a wake field E_z at the downstream gap ahead of the source particle. Therefore, our model violates causality up to a time interval of t_p and the wake function should start from $t_0 = -t_p$ and go to infinity:

$$W_0(t_0) = \begin{cases} -t_p^{-1}[-S(t_p+t_0)] & -t_p < t_0 < 0 \ , \\ -t_p^{-1}[2S(t_0) - S(t_0+t_p)] & 0 < t_0 < t_p \ , \\ -t_p^{-1}[2S(t_0) - S(t_0-t_p) - S(t_0+t_p)] & t_p < t_0 \ . \end{cases} \tag{3.4}$$

The longitudinal impedance $Z_{0\|}$ or simply $Z_\|$ can then be computed by

$$Z_\|(\omega) = -\int_{-t_p}^{\infty} W(t_0)e^{i\omega t_0}dt_0 \ . \tag{3.5}$$

The result is

$$Z_{\|}(\omega) = 4\sin^2\frac{\omega t_p}{2}\frac{R - i\omega L}{1-\omega^2 LC - i\omega RC} , \tag{3.6}$$

with a real part

$$\mathcal{R}e\, Z_{\|}(\omega) = \frac{4R sin^2 \omega t_p/2}{(1-\omega^2 LC)^2 + (\omega RC)^2} , \tag{3.7}$$

which will never go negative. This real part can also be checked directly by computing the power loss. For a beam current $I_0 e^{-i\omega t}$, the current flowing into the plate is

$$I_p(t) = -I_0\left[e^{-i\omega t} - e^{-i\omega(t+t_0)}\right] , \tag{3.8}$$

where the first term is for the upstream gap and the second term the downstream gap. Solving Eq. (2.3), the current flowing through the resistance R is

$$I_R(\omega) = -I_0\frac{1-e^{-i\omega t_p}}{1-\omega^2 LC - i\omega RC} , \tag{3.9}$$

resulting in a power loss of

$$P(\omega) = \frac{1}{2}R|I_0|^2\frac{4 sin^2 \omega t_p/2}{(1-\omega^2 LC)^2+(\omega RC)^2} . \tag{3.10}$$

Equating this to $\mathcal{R}e\, Z_{\|}(\omega)|I_0|^2/2$, we get immediately Eq. (3.7).

One may raise the question that since the real part of the impedance can be computed through dissipative loss, can one obtain the whole impedance by Hilbert transform? The answer is no. Because causality has been violated in our model, Hilbert transform no longer holds. This can also be illustrated directly by the mathematical expression of $Z_{\|}(\omega)$. The factor $\sin^2\omega t_p/2$ is not analytic in either the whole upper ω-plane or the lower ω-plane.

IV. THE DIPOLE MODE

For the transverse impedance, we look at the dipole or differential mode. The beam current I_0 is displaced by a transverse distance Δ from the axis of the vacuum chamber toward one of the separator plates. The differential image current flowing into one of the plate is

$$I_p(t) = -\frac{I_0\Delta}{d}\lim_{t_b\to 0}[\theta(t)-\theta(t-t_b)-\theta(t-t_p)+\theta(t-t_p-t_b)] , \tag{4.1}$$

where d is the distance between the separator plates. Since this current differs from the one in the monopole mode, Eq. (2.4), by the factor Δ/d only, we expect the current $I_R(t)$ through the resistance R also differs by the same factor. The longitudinal wake

$W_1(t_0)$ (for the dipole mode) is the average electric field seen by a test dipole which lags the source dipole by the time interval t_0. This means that we need to divide the average electric field E_z by $Q\Delta$ and $d/2$. The result is exactly the same as $W_0(t_0)$ for the monopole mode except for an extra factor of $2/d^2$. The longitudinal impedance for the dipole mode has exactly the same definition[2] as Eq. (3.5):

$$Z_{1\|}(\omega) = t_p \int_{-\infty}^{\infty} W_{1\|}(t_0) e^{-i\omega t_0} dt_0 \ , \tag{4.2}$$

which gives

$$Z_{1\|}(\omega) = \frac{8}{d^2} \sin^2 \frac{\omega t_p}{2} \frac{R - i\omega L}{1 - \omega^2 LC - i\omega RC} \ . \tag{4.3}$$

The transverse impedance $Z_{1\perp}$ or simply $Z_\perp$ for the dipole mode is related to the longitudinal impedance for the dipole mode by[2]

$$Z_{1\perp}(\omega) = \frac{c}{\omega} Z_{1\|}(\omega) \ , \tag{4.4}$$

where c is the velocity of light. Therefore, the final result is

$$Z_\perp(\omega) = \frac{8c}{\omega d^2} \sin^2 \frac{\omega t_p}{2} \frac{R - i\omega L}{1 - \omega^2 LC - i\omega RC} \ . \tag{4.5}$$

Note that the real part is again non-negative when the frequency is positive. We can verify it also by computing the power dissipation. For a source current $I_0 e^{-i\omega t}$ displaced by Δ, similar to the monopole case, the differential (or dipole-mode) currents passing through the resistances R on either side are

$$I_{p\pm}(\omega) = \mp \frac{I_0}{d} \frac{1 - e^{-i\omega t_p}}{1 - \omega^2 LC - i\omega RC} \ , \tag{4.6}$$

leading to a power dissipation of

$$P(\omega) = \frac{1}{2} R(|I_{p+}|^2 + |I_{p-}|^2) \ . \tag{4.7}$$

On the other hand, the dipole beam current sees an opposing electric field or an impedance whose real part must be equal to $\mathcal{Re}\, Z(\omega) = 2P(\omega)/|I_0|^2$. This impedance can be related to the transverse force driving the dipole current further apart by applying Faraday's law, and can therefore be expressed in terms of $Z_\perp$:[3]

$$\mathcal{Re}\, Z = \frac{\omega \Delta^2\, \mathcal{Re}\, Z_\perp}{c} \ . \tag{4.8}$$

Combining Eqs. (4.6) to (4.8), one can easily verify that the dissipative part of Eq. (4.5) is indeed correct.

V. LOSS FACTORS

The loss factors for the monopole and dipole modes can be computed using the formulas:

$$k_{||}(\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega Z_{||}(\omega)e^{-(\omega\sigma)^2} \ , \tag{5.1}$$

$$k_{\perp}(\sigma) = \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega Z_{\perp}(\omega)e^{-(\omega\sigma)^2} \ , \tag{5.2}$$

where σ is the rms length of a gaussian bunch measured in time. These integrals are rather hard to perform in closed forms. At zero bunch length, however, $k_{||}$ has the simple form

$$k_{||}(0) = \frac{1}{C}\left\{1 - e^{-at_p}\left[\frac{a}{b}\sin bt_p + \cos bt_p\right]\right\} \ . \tag{5.3}$$

It is worth mentioning that $k_{\perp}(0)$ here need not be zero because our wake function does not vanish at $t_0 = 0$.

VI. DISCUSSION

We have derived the longitudinal and transverse impedances of a pair of separators. The model used has been a lumped equivalent circuit in which the transmission of current takes zero time and the separator plates have been regarded as perfect conductor. As a result, the impedances are good at low frequencies only probably when $\omega t_p \leq 1$. Also, causality has been violated up to a time t_p. Because of this, the impedances do not satisfy Hilbert transform. The computation of loss factors may also be affected by these assumptions. To improve the model, we need to consider the finite rate of transmission across the separators. This brings in many propagating modes at high frequencies and the problem will become very complicated.

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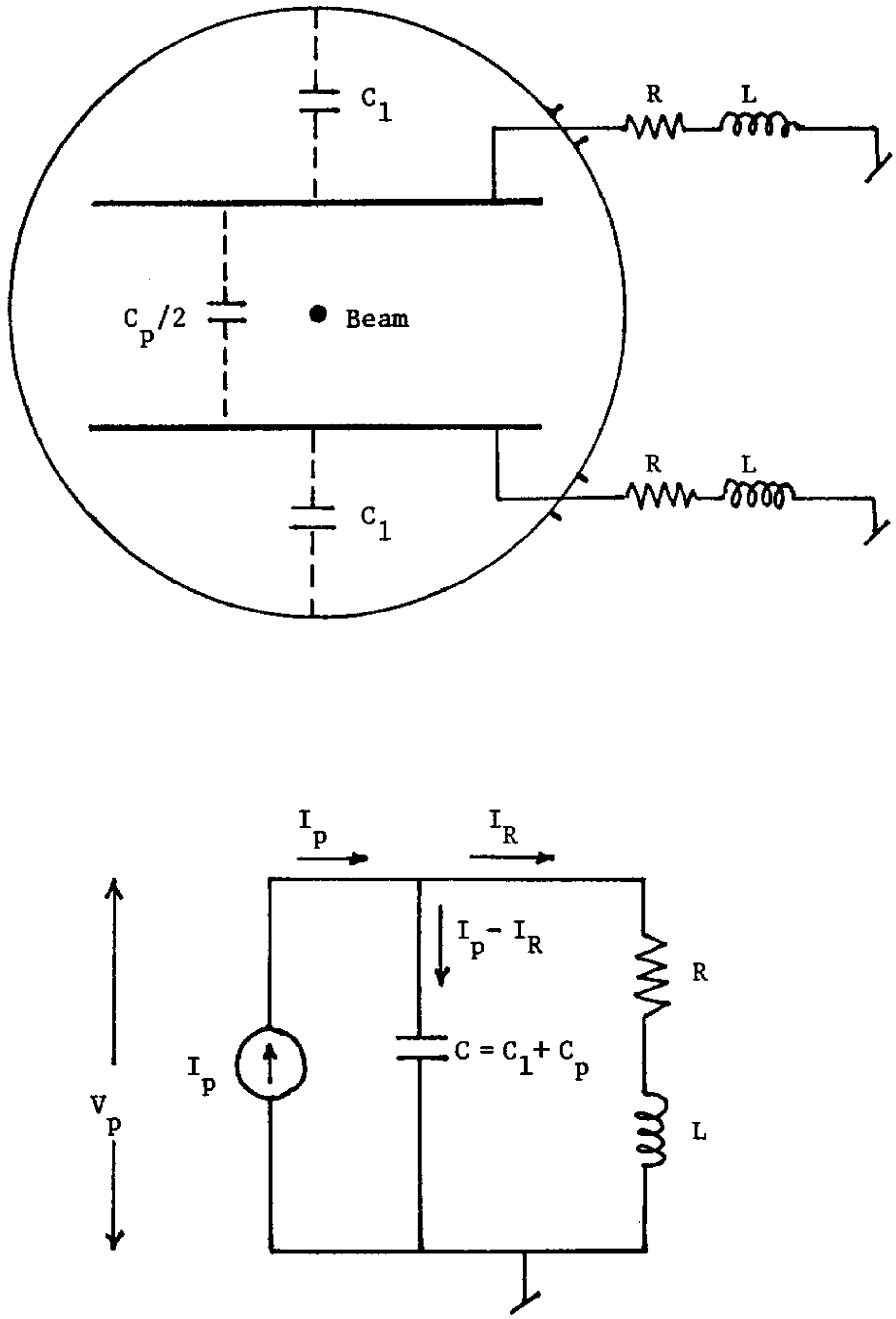

Figure 1. A cross section of a pair of separators in the vacuum chamber and its equivalent circuit.