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Further Parametric Studies of the Accelerator System for Heavy Ion Fusion - Addendum

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We consider the case of filling N rings from one injector at kinetic energy T_i. For the time being we shall assume the injector to be an rf linac. The possibility of using other types of accelerator as injector will be discussed later. The beam in each ring is divided into k/N segments, compressed by a bunching factor b, and extracted at kinetic energy T_f to form k/N beams. Thus, the total number of beams striking the target simultaneously is k. If the injection energy T_i equals the final energy T_f the rings are d.c. and serve only as accumulator/compressor rings. If $T_i < T_f$, the rings are synchrotrons and serve as accumulator/accelerator/compressor rings.

The targeting requirements are given by

P = total power on target

W = total energy on target

E = specific energy deposition in target

r = target radius

The principal design considerations are the following.

A. First, we have to deliver the required specific energy deposition. This gives a condition on the range λ of the ions in the target material

$$\lambda = \frac{1}{\pi r^2}\,\frac{W}{E}\,. \tag{I}$$

The kinetic energy T_f for a given ion type is, then, given by the range/energy curve.[1] The heaviest ion gives the highest T_f, hence the least demand on the current.

B. Then, we have the two conditions for delivering the required total energy and power. In terms of the number of ions n in the ring, the tune shift $\Delta\nu$ at injection is given by

$$\Delta\nu = q^2 \frac{r_o}{2\pi} \frac{n}{\varepsilon/\pi} \frac{1}{\eta_i \gamma_i} \tag{0}$$

where

$$\left\{ \begin{array}{l} r_o = \frac{e^2}{mc^2} \ (= 1.44\text{x}10^{-18}\,\text{m for } mc^2 = 1 \text{ GeV}) \\ q = \text{charge number of ion} \\ \varepsilon = \text{normalized emittance} \\ \gamma,\ \beta,\ \eta \equiv \beta\gamma = \frac{p}{mc} \text{ are conventional relativistic parameters} \\ \text{subscript i denotes injection value} \end{array} \right.$$

But the required number of ions per ring is

$$n = \frac{1}{N} \frac{W}{T_f} \qquad \text{(N = number of rings).}$$

Hence

$$\frac{N}{q^2} = \frac{r_o}{2\pi} \frac{W/T_f}{(\varepsilon/\pi)\Delta\nu} \frac{1}{\eta_i \gamma_i} . \tag{II}$$

In terms of the final d.c. particle current i_f we have

$$n = \frac{2\pi R}{c\beta_f} i_f$$

where the ring radius R is given by

$$R = \frac{mc^2}{qe} \frac{\eta_f}{B_f} = \frac{1}{qr_o} \frac{e}{B_f} \eta_f \tag{1}$$

($\frac{mc^2}{e} = \frac{100}{3}$ kGm for $mc^2 = 1$ GeV) with B_f = average final bending field in ring. Thus

$$\Delta\nu = q \frac{e\gamma_f}{cB_f} \frac{i_f}{\varepsilon/\pi} \frac{1}{\eta_i \gamma_i} .$$

But the required final particle current per beam before bunching is

$$i_f = \frac{1}{kb} \frac{P}{T_f}$$

where

$$\begin{cases} k = \text{total number of beams on target} \\ b = \text{bunching factor} = \dfrac{\text{peak current of bunched beam}}{\text{current of d.c. beam}} . \end{cases}$$

Hence

$$\frac{kb}{q} = \frac{e\gamma_f}{cB_f} \frac{P/T_f}{(\varepsilon/\pi)\Delta\nu} \frac{1}{\eta_i \gamma_i} . \qquad \text{(III)}$$

We have assumed here that the tune-shift limitation needs not be applied during the fast final bunching as was shown to be true by the BNL experiment.[2]

We might add here the electric current of the d.c. beam in the ring at injection

$$I_i = qei_f \frac{\beta_i}{\beta_f} = \frac{qe}{kb} \frac{P}{T_f} \frac{\beta_i}{\beta_f} \qquad (2)$$

and the voltage of the injector linac

$$V_{linac} = \frac{T_i}{qe} \qquad (3)$$

Clearly we would like the left-hand-side quantities in conditions (II) and (III) to be small. This implies that we should have

1. r_o small, hence heavy ions.
2. T_f large, hence heavy ions.
3. γ_f small. But within the range of interest $\gamma_f \cong 1$ anyway.
4. B_f large. Conventional magnets give $B_f \cong 10$ kG. Superconducting magnets give $B_f \cong 20$ kG and consume no power.
5. $\eta_i \gamma_i$ large. This is largest when injecting at the final energy ($T_i = T_f$), but must be compromised with the cost of the injector.
6. $\Delta\nu$ large. If the rings are used as synchrotrons ($T_i < T_f$) rf acceleration requires some bunching, hence $\Delta\nu$ must be $<\frac{1}{4}$. We shall take $\Delta\nu \cong \frac{1}{8}$. If the rings are used only as accumulator/compressors we can take $\Delta\nu \cong \frac{1}{4}$. (It may be possible to waive the tune-shift limitation also during the rather fast process of multiturn injection. If, further, space charge neutralization is applied, we can ignore the space charge effect altogether. But these possibilities have yet to be demonstrated.)
7. ε/π large. This is limited either by the reactor vessel and beam port dimensions or by the strength of the final focusing quadrupoles

$$B_Q \ell_Q \cong \frac{mc^2}{qe} \frac{\varepsilon/\pi}{r} \qquad (4)$$

where B_Q and ℓ_Q are respectively the pole-tip field and the length of the quadrupole.

C. In addition to the conditions (II) and (III) the requirements on the bunching and accelerating rf systems will help in making a choice of the parameters q, N, k, b and T_i. The peak momentum spread (in mc units) $\Delta\eta_i$ of the beam from the injector linac is given by[3]

$$\frac{\Delta\eta_i}{\eta_i} \cong 2\mathrm{x}10^{-4} \; \frac{\sqrt{q}}{\left[T_i\,(\mathrm{GeV})\right]^{5/8}} \; . \tag{5}$$

This momentum spread of the injected beam in the ring (considered as a d.c. beam) is conserved through acceleration. That of the final bunched beam is, then, $\Delta\eta_b = b\Delta\eta_f = b\Delta\eta_i$. First, we must check that $\Delta\eta_b$ is allowed by chromatic aberration in the final transport elements. The condition is

$$\frac{\Delta\eta_b}{\eta_f} = \frac{b\Delta\eta_i}{\eta_f} < \frac{r}{a} \tag{6}$$

where a is the radius of the beam port. Second, to bunch the beam on the ring flattop the momentum width of the stationary bucket of the bunching rf must at least be equal to $\Delta\eta_b$. This gives

$$\Delta\eta_b = \left(\frac{2}{\pi h} \, \frac{qeV_b}{mc^2} \, \frac{\gamma_f}{\frac{1}{\gamma_f^{\,2}} - \frac{1}{\gamma_t^{\,2}}} \right)^{1/2}$$

where

$$\begin{cases} h = \frac{k}{N} = \text{harmonic number} \\ V_b = \text{peak rf voltage per turn} \\ \gamma_t = \text{transition } \gamma \text{ (generally >>1).} \end{cases}$$

Neglecting $\frac{1}{\gamma_t^{\,2}}$ compared to $\frac{1}{\gamma_f^{\,2}}$ we get

$$V_b \cong \frac{\pi h}{2} \frac{mc^2}{qe} \frac{(\Delta\eta_b)^2}{\gamma_f^3} . \tag{7}$$

We note that since $\Delta\eta_b \propto \sqrt{q}$, V_b is independent of q. The bunching rf runs at the fixed final frequency

$$F_f = h \frac{c\beta_f}{2\pi R} . \tag{8}$$

The accelerating rf is given by the desired energy gain per turn. Assuming a linear ramp we have

$$V_a \sin\phi_s = \frac{2\pi R}{c} \frac{\eta_f - \eta_i}{\Delta t} \frac{mc^2}{qe} \tag{9}$$

where

V_a = peak rf voltage per turn

ϕ_s = synchronous phase (determined by the necessary bucket area)

Δt = acceleration time (generally must be <0.1 sec to avoid excessive beam loss due to charge exchange interactions)

The accelerating rf must be frequency modulated between the injection frequency

$$F_i = h \frac{c\beta_i}{2\pi R} \tag{10}$$

and the final frequency F_f given by Eq. (8). Therefore it is much more difficult to produce the accelerating rf voltage V_a than the fixed-frequency bunching rf voltage V_b. But if V_b is not much larger than V_a it may be economically advantageous to combine the two rf systems.

There are, in addition, a large number of other accelerator parameters whose choice depends on compromises between performance, reliability, economy etc. based on the experience of the designers. We shall not enter into these discussions here.

We now apply the procedure outlined above to a few interesting examples.

D. Example 1 - Prototype power plant

$$\begin{cases} P = 100 \text{ TW} \\ W = 1 \text{ MJ} \\ E = 40 \text{ MJ/g} \\ r = 1 \text{ mm} \end{cases}$$

Condition (I) gives

$$\lambda = 0.796 \text{ g/cm}^2$$

and the range/energy curve gives for U_{238} (mc^2 = 222 GeV, $r_o = 6.5\text{x}10^{-21}$ m, $\frac{mc^2}{e}$ = 7400 kGm)

$$T_f = 26.2 \text{ GeV} = 4.2\text{x}10^{-9}\text{J} \qquad \begin{cases} \gamma_f = 1.118 \\ \eta_f = 0.500 \\ \beta_f = 0.447 \end{cases}$$

<u>Case A</u> $T_i = T_f$. Taking

$$\begin{cases} \Delta\nu = \frac{1}{4} \\ B_f = 20 \text{ kG (superconducting magnets)} \\ \varepsilon/\pi = 0.5\text{x}3\text{x}10^{-5} \text{ m} = 1.5\text{x}10^{-5} \text{ m (beam port radius} \\ \qquad a = 0.3 \text{ m, 10 m away from target)} \end{cases}$$

we get for conditions (II) and (III)

$$\frac{N}{q^2} = 0.118, \qquad \frac{kb}{q} = 102.$$

A reasonable choice is

$$\boxed{q = 4,\ N = 2,\ k = 16,\ b = 25.5}$$

(The fact that $N/q^2 = 0.125$ is slightly larger than 0.118 simply means that we will be able to deliver slightly more energy than W = 1 MJ.) Eqs. (1) through (8) give

$R = 46.25$ m

$I_i = 37.3$ A (37 turns @ 1 A/turn)

$V_{linac} = 6.55$ GV

$B_Q \ell_Q = 27.75$ kGm (quite o.k.)

$\Delta\eta_i/\eta_i = 0.52\text{x}10^{-4}$

$\Delta\eta_b/\eta_f = 1.3\text{x}10^{-3}$ ($<\frac{r}{a} = 3.3\text{x}10^{-3}$ o.k.)

$V_b = 219$ kV (quite o.k.)

$F_f = 3.69$ MHz

Two remarks should be made.

(i) One can reduce k to 8 (thereby saving 8 final transport lines) and increase b to 51, but then $\Delta\eta_b/\eta_f$ will be $2.6\text{x}10^{-3}$ which is too close to the limit of $3.3\text{x}10^{-3}$.

(ii) One can reduce V_{linac}, say, by a factor 2 by increasing q to 8, N to 8, and k to 32. But the increased cost of 6 rings and 16 beam transport lines may offset the reduced cost of the injector linac.

<u>Case B</u> $T_i < T_f$. With $\Delta\nu = \frac{1}{8}$, $B_f = 20$ kG, and $\varepsilon/\pi = 1.5\text{x}10^{-5}$ m conditions (II) and (III) give

$$\frac{N}{q^2} = 0.235\ \frac{\eta_f \gamma_f}{\eta_i \gamma_i}, \qquad \frac{kb}{q} = 204\ \frac{\eta_f \gamma_f}{\eta_i \gamma_i}$$

In this case, although it is possible to operate several synchrotrons in a synchronized fashion, there is a definite preference for N = 1. A reasonable choice is

$$\boxed{\begin{array}{c} q = 1,\ N = 1,\ k = 32,\ b = 27.1 \\[2ex] \dfrac{\eta_i\gamma_i}{\eta_f\gamma_f} = 0.235 \end{array}}$$

The injection energy is given by $\eta_i\gamma_i = 0.235\ \eta_f\gamma_f = 0.1315$ to be

$$T_i = 1.88 \text{ GeV} \qquad \begin{cases} \gamma_i = 1.00847 \\ \eta_i = 0.130 \\ \beta_i = 0.129 \end{cases}$$

Eqs. (1) through (10) give

$R = 185$ m

$I_i = 1.27$ A (10 turns @ 127 mA/turn)

$V_{linac} = 1.88$ GV

$B_Q\ell_Q = 111$ kGm (just o.k. with superconducting quadrupoles)

$\Delta\eta_i/\eta_i = 1.35\text{x}10^{-4}$

$\Delta\eta_b/\eta_f = 0.95\text{x}10^{-3}$ (o.k.)

$V_b = 1.81$ MV (o.k.)

$F_f = 3.69$ MHz, $F_i = 1.07$ MHz

$V_a \sin\phi_s = 3.18$ MV (for $\Delta t = 0.1$ sec) (too high)

<u>Case C</u> $T_i < T_f$. To reduce V_a we can either increase the acceleration time Δt or give up on N = 1 and choose

$$q = 2,\ N = 2,\ k = 32,\ b = 27.1$$

$$\frac{\eta_i \gamma_i}{\eta_f \gamma_f} = 0.471$$

The injection energy is

$$T_i = 7.10 \text{ GeV} \qquad \begin{cases} \gamma_i = 1.0320 \\ \eta_i = 0.255 \\ \beta_i = 0.247 \end{cases}$$

Eqs. (1) through (10) give

$$\begin{aligned}
R &= 92.5 \text{ m} \\
I_i &= 4.86 \text{ A} \quad \text{(20 turns @ 243 mA/turn)} \\
V_{linac} &= 3.55 \text{ GV} \\
B_Q \ell_Q &= 55 \text{ kGm} \quad \text{(o.k.)} \\
\Delta\eta_i/\eta_i &= 0.83\text{x}10^{-4} \\
\Delta\eta_b/\eta_f &= 1.15\text{x}10^{-3} \quad \text{(o.k.)} \\
V_b &= 658 \text{ kV} \quad \text{(o.k.)} \\
F_f &= 3.69 \text{ MHz}, \qquad F_i = 2.11 \text{ MHz} \\
V_a \sin\phi_s &= 527 \text{ kV} \quad (\text{for } \Delta t = 0.1 \text{ sec}) \quad \text{(o.k.)}
\end{aligned}$$

Compared to Case B these parameters are easier to achieve, except now we need a higher voltage injector linac and two synchrotrons.

E. Example 2 - Heavy ion demonstration experiment (HIDE)

$$\begin{cases} P = 5 \text{ TW} \\ W = 100 \text{ kJ} \\ E = 20 \text{ MJ/g} \\ r = 1 \text{ mm} \end{cases}$$

Condition (I) gives

$$\lambda = 0.159 \text{ g/cm}^2$$

and the range/energy curve gives for U_{238}

$$T_f = 7.4 \text{ GeV} = 1.18\times10^{-9}\text{J} \qquad \begin{cases} \gamma_f = 1.0333 \\ \eta_f = 0.260 \\ \beta_f = 0.252 \end{cases}$$

<u>Case A</u> $T_i = T_f$. Taking

$$\begin{cases} \Delta\nu = \frac{1}{4} \\ B_f = 10 \text{ kG} \quad \text{(conventional magnets)} \\ \varepsilon/\pi = 0.260\times5\times10^{-5}\text{ m} = 1.3\times10^{-5}\text{ m} \quad \text{(beam port} \\ \qquad \text{radius a = 0.25 m, 5 m away from target)} \end{cases}$$

we get for conditions (II) and (III)

$$\frac{N}{q^2} = 0.100, \qquad \frac{kb}{q} = 79.9 \ .$$

A reasonable choice is

$$\boxed{q = 3,\ N = 1,\ k = 12,\ b = 20}$$

Eqs. (1) through (8) give

R = 64.2 m

I_i = 8.46 A (20 turns @ 423 mA/turn)

V_{linac} = 2.47 GV

$B_Q \ell_Q$ = 32.1 kGm (just o.k. for conventional quadrupoles)

$\Delta\eta_i/\eta_i$ = 1.0×10^{-4}

$\Delta\eta_b/\eta_f$ = 2.0×10^{-3} ($< \frac{r}{a} = 4\times10^{-3}$ o.k.)

V_b = 337 kV (o.k.)

F_f = 2.25 MHz

<u>Case B</u> $T_i<T_f$. With $\Delta\nu = \frac{1}{8}$, B_f = 10 kG, and $\varepsilon/\pi = 1.3\mathrm{x}10^{-5}$ m conditions (II) and (III) give

$$\frac{N}{q^2} = 0.200\ \frac{\eta_f\gamma_f}{\eta_i\gamma_i}, \qquad \frac{kb}{q} = 160\ \frac{\eta_f\gamma_f}{\eta_i\gamma_i}\ .$$

A reasonable choice is

$$\boxed{\begin{array}{c} q = 1,\ N = 1,\ k = 32,\ b = 25 \\ \dfrac{\eta_i\gamma_i}{\eta_f\gamma_f} = 0.200 \end{array}}$$

The injection energy is given by $\eta_i\gamma_i = 0.2\ \eta_f\gamma_f = 0.0538$ to be

$$T_i = 320\ \mathrm{MeV} \qquad \left\{\begin{array}{l} \gamma_i = 1.00144 \\ \eta_i = 0.0537 \\ \beta_i = 0.0536 \end{array}\right.$$

Eqs. (1) through (10) give

R = 192.6 m

I_i = 0.180 A (10 turns @ 18 mA/turn)

V_{linac} = 320 MV

$B_Q\ell_Q$ = 96.2 kGm (o.k. only with superconducting quadrupoles)

$\Delta\eta_i/\eta_i = 4.08\mathrm{x}10^{-4}$

$\Delta\eta_b/\eta_f = 2.10\mathrm{x}10^{-3}$ ($<4\mathrm{x}10^{-3}$ o.k.)

V_b = 3.03 MV (rather high)

F_f = 2.00 MHz, F_i = 0.426 MHz

$V_a \sin\phi_s$ = 1.85 MV (for Δt = 0.1 sec)(rather high)

<u>Case C</u> $T_i<T_f$. To reduce V_a and V_b we can give up on N = 1 and choose

$$q = 2,\ N = 2,\ k = 32,\ b = 25$$

$$\boxed{\frac{\eta_i \gamma_i}{\eta_f \gamma_f} = 0.400}$$

The injection energy is

$$T_i = 1.267 \text{ GeV} \qquad \begin{cases} \gamma_i = 1.00571 \\ \eta_i = 0.107 \\ \beta_i = 0.106 \end{cases}$$

Eqs. (1) through (10) give

$R = 96.3$ m

$I_i = 0.713$ A (20 turns @ 36 mA/turns)

$V_{linac} = 633$ MV

$B_Q \ell_Q = 48.1$ kG m (o.k. with superconducting quadrupoles)

$\Delta\eta_i/\eta_i = 2.44\text{x}10^{-4}$

$\Delta\eta_b/\eta_f = 2.50\text{x}10^{-3}$ ($<4\text{x}10^{-3}$ just o.k.)

$V_b = 1.077$ MV (o.k.)

$F_f = 2.00$ MHz, $F_i = 0.844$ MHz

$V_a \sin\phi_s = 343$ kV (for $\Delta t = 0.1$ sec) (o.k.)

These parameters are easier to obtain than those of Case B but now we need a higher voltage linac and two synchrotrons.

For an rf linac the normalized emittance of the beam is approximately[3] $\varepsilon/\pi \cong 10^{-6}$ m. With multiturn injection, stacking in both horizontal and vertical planes the number of injected turns assumed in both Examples above can easily be accommodated within the allowed emittances.

F. We now discuss briefly the possibilities of using other types of accelerator as injectors.

Synchrotron

In Cases B and C discussed above we already have, in essence, the system consisting of a d.c. accumulator/compressor (A/C) ring combined with a synchrotron injector. The only additional feature introduced by separating the A/C ring and the synchrotron is the possibility of filling the A/C ring to its tune-shift limit with several pulses from the synchrotron injector. From Eq. (0) we see that for the same emittance ε/π, the tune-shift limited number of ions in the A/C ring is $2\frac{\eta_f\gamma_f}{\eta_i\gamma_i}$ times that in the synchrotron where the factor 2 represents the ratio between the allowed $\Delta\nu$ values of $\frac{1}{4}$ and $\frac{1}{8}$ for the A/C ring and the synchrotron respectively. For ease of discussion we shall concentrate on Example 2 and take $\frac{\eta_f\gamma_f}{\eta_i\gamma_i} = 2.5$ as in Case C. (This corresponds to an injection energy into the synchrotron of $T_i = 1.267$ GeV. With $q = 3$, that for the optimal A/C ring of Case A, the synchrotron injector-linac will need a voltage of $V_{linac} = 422$ MV.)

With identical R and ε/π, to inject 5 pulses from the synchrotron to fill the same phase-space volume in the A/C ring we have to resort to a non-Liouvillean injection process such as the charge-exchange injection. Such an injection process does not seem to be available for U ions (indeed, perhaps unavailable for anything other than H^- and HI^+).

With more conventional phase-space conserving injection processes we must reduce the phase space volume occupied by the beam in the synchrotron by a factor 5. We can reduce the longitudinal emittance by reducing the circumference of the synchrotron by a factor 5. The 5 pulses can then be injected end-to-end to fill the circumference of the A/C ring. But this requires B_f = 50 kG in the synchrotron which is, again, not available. Indeed, if this field were available it would have been used for the A/C ring.

Finally, we can reduce the transverse emittance ε/π in the synchrotron by a factor 5 (in both planes). The number of ions per pulse is, then, reduced also by a factor 5. But, in principle, we can now inject 5 (horizontal) x 5 (vertical) = 25 pulses into the A/C ring and effectively increase the number of injected ions 5-fold. However, since coherence in transverse oscillation is invariably lost between pulses, contrary to multi-turn injection a practical scheme for multipulse injection without significant dilution of the phase-space density does not exist. In any case, this process will require the synchrotron to have a repetition rate of 250 Hz (25 pulses in 0.1 sec) which is also impractical.

We conclude, therefore, that separating the A/C ring and the synchrotron is not likely to lead to any practical advantage.

For N>1, a simple minded advantage of the separation is the possibility of using the same synchrotron to fill N A/C rings in succession. The repetition rate of the synchrotron must be increased N-fold. On the last pulse the synchrotron can also serve as one of the N A/C rings. Whether the cost savings in making N-1 rings d.c. are off-set by the additional costs incurred in raising the repetition rate of the synchrotron and in the beam transfer lines (at T_f) to the N-1 A/C rings can only be answered by a detailed cost analysis.

Induction Linac

The need of A/C rings implies that the injector can deliver only low current beams. On the other hand, the pulse length is sufficiently long (tens of μsec) to supply the requisit total number of ions. Multiturn injection and longitudinal compression in the A/C ring, then, converts the beam into the desire short (~10 nsec) and high-current (10^2 to 10^3 A) pulses. An induction linac is expected to be capable of delivering just these short and high-current beam pulses. Therefore, with an induction linac we do not anticipate the need for A/C rings at all.

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