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AN ESTIMATE OF THE BRANCHING RATIOS FOR DALITZ PAIR DECAYS OF THE ω^0 MESON

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The origin of low-mass lepton pairs produced in hadron-hadron collisions is an issue of high current interest. Whether these pairs represent "new physics" or are manifestations of known phenomena is an important open question. Conventional sources for low-mass dilepton pairs include the vector meson decays

$$\rho \to \ell^+\ell^- \text{ (Branching ratio } \sim 5\times10^{-5}),$$

$$\omega \to \ell^+\ell^- \ (\sim 7.6\times10^{-5}),$$

the Dalitz pair decays

$$\pi^0 \to \gamma e^+e^- \ ((1.15\pm.05)\times10^{-2}),$$

$$\eta \to \gamma e^+e^- \ (\sim 5\times10^{-3}),$$

$$\eta \to \gamma \mu^+\mu^- \ (\sim 3\times10^{-4} \text{ theoretical estimate}),$$

$$\omega \to \pi^0 e^+e^-,$$

$$\omega \to \pi^0 \mu^+\mu^- \ (<2\times10^{-3}),$$

and the rare decays

$$\pi^0 \to e^+e^- \ (\sim 6\times10^{-8} \text{ theoretical estimate}),$$

$$\eta \to e^+e^- \ (\sim 2.4\times10^{-9} \text{ theoretical estimate}),$$

$$\eta \to \mu^+\mu^- \ ((2.2\pm0.8)\times10^{-5};\ \text{theory:} \sim 6\times10^{-6}).$$

The branching ratios are from the 1976 Review of Particle Properties, Rev. Mod. Phys. 48, S1 (1976) in the case of

experiment, and from C. Quigg and J.D. Jackson, UCRL-18487 (1968, unpublished) in the case of theory.

The purpose of this note is to fill an apparent gap in the literature by providing estimates for the branching ratio for the Dalitz decay of the ω meson. We assume that the decay $\omega \to \pi^0\gamma$ can be described by the Feynman graph of Fig. 1(a), in the spirit of M. Gell-Mann, D. Sharp, and W.G. Wagner, Phys. Rev. Lett. 8, 261 (1962). Similarly, the Dalitz decay is assumed to proceed by the Feynman diagram in Fig. 1(b). Denote the invariant mass-squared of the lepton pair by s. Then we compute the ratio of partial decay rates

$$\frac{dW(\omega\to\pi^0\ell^+\ell^-)/ds}{W(\omega\to\pi^0\gamma)} = \frac{\alpha}{3\pi}\cdot\frac{(1-4m_\ell^2/s)^{\frac{1}{2}}(1+2m_\ell^2/s)}{s(1-s/m_\rho^2)^2} \times\left[\left(1+\frac{s}{m_\omega^2-m_\pi^2}\right)^2-\frac{4m_\omega^2 s}{(m_\omega^2-m_\pi^2)^2}\right]^{3/2} . \quad (1)$$

[In the limit $m_\pi\to 0$, the right-hand-side becomes ½ × the right-hand-side of eq. (C.5) of Quigg and Jackson, itself a straightforward generalization of eq. (13) of N.M. Kroll and W. Wada, Phys. Rev. 98, 1355 (1955).] Integrating eq. (1) from $s=4m_\ell^2$ to $(m_\omega - m_\pi)^2$, we arrive at the following estimates of branching ratios:

$$\frac{W(\omega\to\pi^0 e^+e^-)}{W(\omega\to\pi^0\gamma)} = 9.1\times 10^{-3} , \quad (2)$$

$$\frac{W(\omega\to\pi^0\mu^+\mu^-)}{W(\omega\to\pi^0\gamma)} = 9.1\times 10^{-4} . \quad (3)$$

The measured branching ratio $W(\omega\to\pi^0\gamma)/W(\omega\to all) = (8.8\pm0.5)\times10^{-2}$ then leads to

$$\frac{W(\omega\to\pi^0 e^+e^-)}{W(\omega\to all)} \simeq 8 \times 10^{-4} \quad , \tag{4}$$

$$\frac{W(\omega\to\pi^0 \mu^+\mu^-)}{W(\omega\to all)} \simeq 8 \times 10^{-5} \quad . \tag{5}$$

To explore the sensitivity of our results to the vector dominance assumption, we remove the ρ-meson propagator by letting $M_\rho^2 \to \infty$ in (1). This has little effect on the rate for electron pairs

$$\left.\frac{W(\omega\to\pi^0 e^+e^-)}{W(\omega\to\pi^0\gamma)}\right|_{\text{non VDM}} = 8.6 \times 10^{-3} \quad , \tag{6}$$

but results in a marked reduction of the rate for muon pairs

$$\left.\frac{W(\omega\to\pi^0 \mu^+\mu^-)}{W(\omega\ \pi^0\gamma)}\right|_{\text{non VDM}} = 5.5 \times 10^{-4} \quad . \tag{7}$$

Thus we have

$$\left.\frac{W(\omega\to\pi^0 e^+e^-)}{W(\omega\to all)}\right|_{\text{non VDM}} \sim 7.6 \times 10^{-4} \quad , \tag{8}$$

$$\left.\frac{W(\omega\to\pi^0 \mu^+\mu^-)}{W(\omega\to all)}\right|_{\text{non VDM}} \sim 5 \times 10^{-5} \quad . \tag{9}$$

The normalized differential Dalitz decay rates are shown in Fig. 2 (for the $\pi^0 e^+e^-$ mode) and in Fig. 3 (for the $\pi^0\mu^+\mu^-$ mode). For each case the spectrum is shown with and without the ρ-meson propagator.

A number of other Dalitz pair decays may also be of eventual interest. Our equation (1) is appropriate, mutatis mutandis, for any decays of the form

$$1^- \text{hadron} \to 0^- \text{hadron} + \ell^+\ell^-$$

or

$$0^- \text{hadron} \to 1^- \text{hadron} + \ell^+\ell^- \quad .$$

For decays of the type

$$0^- \text{hadron} \to \gamma + \ell^+\ell^- \quad ,$$

eqn. (C.5) of Quigg and Jackson applies. Estimates for the Dalitz decay rates of η' and ϕ are presented in Table I.

ACKNOWLEDGEMENT

We thank Eli Rosenberg for drawing our attention to this problem. After we had written this note, Clive Field showed us a new paper on the origins of prompt leptons and photons by N.S. Craigie and D. Schildknecht [CERN preprint TH.2193]. These authors estimate many of the rates we have computed, apparently using an approximation (their eqn. (4.2)) to eqn. (1). For the most part the two sets of estimates agree, but there are a few significant discrepancies. Their estimate of the $\phi \to \eta\mu^+\mu^-$ branching ratio is about 50% larger than ours. We do not know whether this is to be attributed to a

different choice for the off-mass-shell form factor behavior or to their use of approximate kinematics. Their estimate of the $\eta' \to \gamma e^+ e^-$ branching ratio is one-half of ours. This is the factor by which, as noted above, the rates for decays with final-state hadrons differ from those for decays with final-state photons. Our number agrees with the classical formula of R.H. Dalitz, Proc. Phys. Soc. (London) A64, 667 (1951).

TABLE I. Branching Ratios for Dalitz Pair Decays of ϕ and η'.

Mode	$W(A\to B\ell^+\ell^-)/W(A\to B\gamma)$	$W(A\to B\ell^+\ell^-)/W(A\to all)$[a]
$\phi \to \eta\mu^+\mu^-$	3.36×10^{-4}	6.72×10^{-6}
	b) 4.69×10^{-4}	9.38×10^{-6}
$\phi \to \eta e^+e^-$	8.25×10^{-3}	1.65×10^{-4}
	b) 8.51×10^{-3}	1.70×10^{-4}
$\phi \to \pi^0\mu^+\mu^-$	8.96×10^{-4}	1.25×10^{-6}
$\phi \to \pi^0 e^+e^-$	9.00×10^{-3}	1.26×10^{-5}
$\eta' \to \rho^0 e^+e^-$	6.83×10^{-3}	2.08×10^{-3}
$\eta' \to \gamma\mu^+\mu^-$	1.72×10^{-3}	3.44×10^{-5}
$\eta' \to \gamma e^+e^-$	1.79×10^{-2}	3.58×10^{-4}

[a] Based on 1976 Particle Data Group values for $W(A\to B\gamma)/W(A\to all)$.

[b] Including vector meson propagator with ρ-meson mass.

FIGURE CAPTIONS

Fig. 1 (a) Feynman graph for the decay $\omega \to \pi^0 \gamma$, using the vector dominance model assumption of Gell-Mann, Sharp, and Wagner. (b) The corresponding diagram for the Dalitz pair decay $\omega \to \pi^0 \ell^+ \ell^-$.

Fig. 2 Normalized spectrum of Dalitz pair masses for the decay $\omega \to \pi^0 e^+ e^-$ with (solid line) and without (dashed line) the ρ-meson propagator in Fig.1(b). The invariant mass-squared of the dielection pair is s.

Fig. 3: Normalized spectrum of Dalitz pair masses for the decay $\omega \to \pi^0 \mu^+ \mu^-$ with (solid line) and without (dashed line) the ρ-meson propagator in Fig.1(b). The invariant mass-squared of the dimuon pair is s.

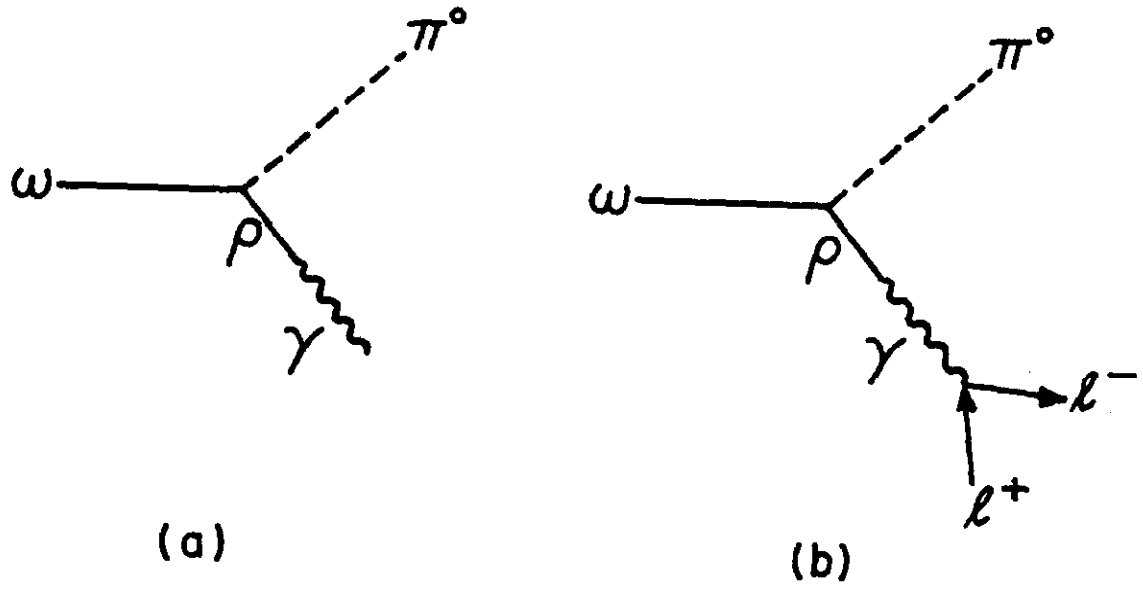

Fig. 1

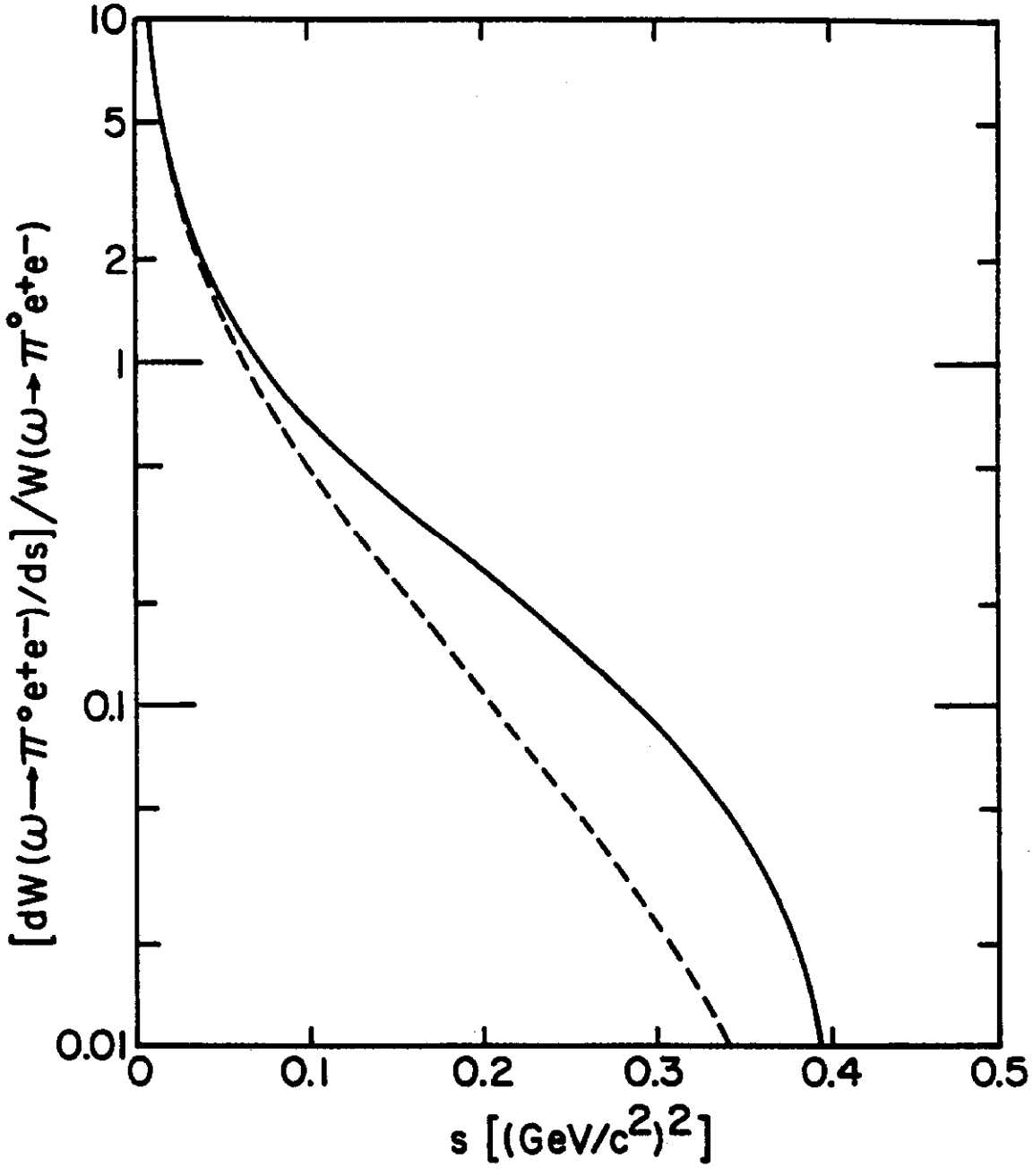

Fig. 2

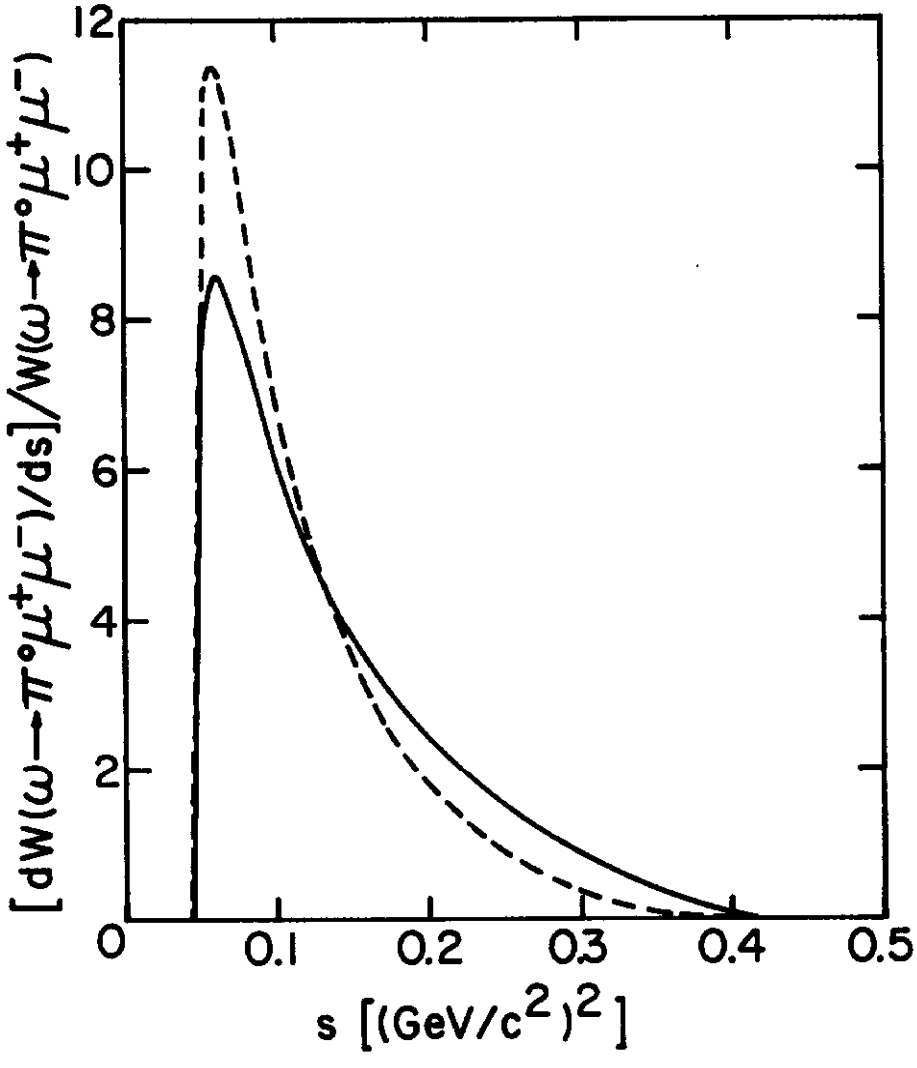
12
10
8
6
4
2
0
0.1
0.2
0.3
0.4
0.5
$[dW(\omega \to \pi^0 \mu^+ \mu^-)/ds]/W(\omega \to \pi^0 \mu^+ \mu^-)$
$s\ [(GeV/c^2)^2]$

Fig. 3