national accelerator laboratory

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0320

MAGNET END TERMINATION

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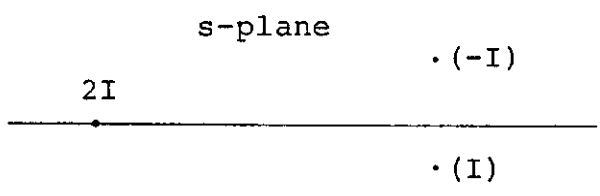
PURPOSE

To calculate the shape of a magnetic surface that matches the field in the gap to a uniform vertical field within the iron. A conformal transformation introduced by Hardt is used to find the above surface in the presence of an array of current elements representing the coil end. Only two dimensional solutions are obtained. Comparison is made with earlier solutions in which the coil end is removed to infinity.

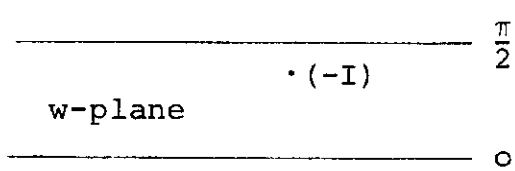
SINGLE CURRENT ELEMENT

The complex potential corresponding to currents shown in the complex plane (s-plane) is



$$G = 4I\,\ell n s - 2I\ell n\,(s-s_o)(s-s_o^*). \quad \text{(emu)} \tag{1}$$

Execute the following zero angle transformation



$$w = \frac{1}{2}\,\ell n s \tag{2}$$

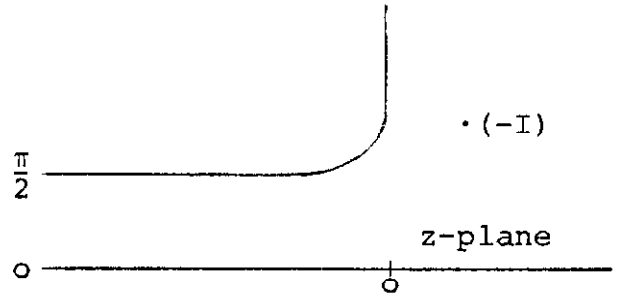
to give

$$G = 8\,Iw - 2I\ell n\,(e^{2w}-e^{2w_o})(e^{2w}-e^{2w_o^*}), \tag{3}$$

where $w_o = \frac{1}{2}\,\ell n s_o$. (4)

Hardt[1] introduces the following transformation,

$$z = \frac{1}{2}\left\{2w - n(e^{w}+e^{w_o})(e^{w}+e^{w_o^*}) + Ae^{w}\right\} \tag{5}$$



which, for the surface, $\mathcal{I}m\ w = \frac{\pi}{2}$, posseses the property

$$\mathrm{Re}\left(\frac{dz}{dG}\right) = \frac{1}{8I} = \text{constant} \tag{6}$$

The property expressed by Eq. (6) may be visualized more readily in terms of the surface magnetic field. Since

$$H^* = i\,\frac{dG}{dz}, \tag{7}$$

then

$$\mathrm{Re}\left(\frac{dz}{dG}\right) = \mathrm{Re}\left(\frac{i}{H^*}\right) = \mathrm{Re}\,\frac{i(H_x+iH_y)}{H_x^2+H_y^2} = \frac{H_y}{H^2} \tag{8}$$

But the flux included in a thin vertical slab of thickness Δx is

$$\Delta\Phi = H\cdot\left(\frac{H_y}{H}\right)\frac{\Delta x}{H_y} = \frac{H^2}{H_y}\,\Delta x \tag{9}$$

Since H^2/H_y is constant, the vertical flux density in the iron is constant.

ARRAY OF CURRENT ELEMENTS

A simple generalization of the previous case was also given by Hardt[1].

$$G = 8\,NI\left\{w - \frac{1}{4N}\,\Sigma_i \ell n(e^{2w} - e^{2w_{oi}})(e^{2w} - e^{2w^*_{oi}})\right\} \tag{10}$$

and

$$z = \frac{1}{2}\left\{2w - \frac{1}{N}\,\Sigma_i \ell n(e^{w} + e^{w_{oi}})(e^{w} + e^{w^*_{oi}}) + Ae^{w}\right\} \tag{11}$$

APPROXIMATION FOR LARGE NEGATIVE Re w

$$z = \frac{1}{2}\left\{2w - \frac{1}{N}\,\Sigma_i\,(w_{oi} + w^*_{oi}) - \frac{e^{w}}{N}\,\Sigma_i\,(e^{-w_{oi}} + e^{-w^*_{oi}}) + Ae^{w}\right\} \tag{12}$$

Notice that this has the form of the solution to the semi-infinite condenser problem.

PROGRAM

Equation (11) and its limiting form, Eq. (12) have been coded to yield the profile ($\mathcal{I}m\ w = \frac{\pi}{2}$). The input to CLIP specifies the desired location of the wire array in the z-plane and an initial guess as to the location of the array in the w-plane. The constant A is arbitrarily chosen to be 2π. Other choices are not needed since they simply shift the array in the w-plane. A simple iterative loop succesively alters the location of the wires in the w-plane until the corresponding transformed position of the array in the z-plane is equal to the desired location. Figure 1 illustrates the profile for the booster F-magnet. Notice that the difference between the exact and the approximate expression is negligible below the center of the coil.

REFERENCE

[1]W. Hardt, private communication, April 1969.

F-MAGNET CASE-F

• Hardt Transformation
x Semi Infinite Condenser