

CPT- and Lorentz-Violation Tests with Muon $g - 2$

B. Quinn

*Department of Physics and Astronomy, University of Mississippi,
University, MS 38677, USA*

On behalf of the Muon $g - 2$ Collaboration

The status of Lorentz- and CPT-violation searches using measurements of the anomalous magnetic moment of the muon is reviewed. Results from muon $g - 2$ experiments have set the majority of the most stringent limits on Standard-Model Extension Lorentz and CPT violation in the muon sector. These limits are consistent with calculations of the level of Standard-Model Extension effects required to account for the current 3.7σ experiment–theory discrepancy in the muon’s $g - 2$. The prospects for the new Muon $g - 2$ Experiment at Fermilab to improve upon these searches is presented.

1. The anomalous magnetic moment

The magnetic moment of the muon can be expressed by the relation

$$\vec{\mu} = g \frac{e}{2m} \vec{s} = (1 + a_\mu) \frac{e}{m} \vec{s}, \quad (1)$$

where the first term arises from the leading-order Dirac theory, and the anomaly, $a_\mu = (g - 2)/2$, represents the sum of all higher-order loop diagrams.¹ The anomalous magnetic moment includes Standard-Model (SM) terms from QED, EW, and QCD processes, as well as possible contributions from Beyond the Standard Model (BSM) physics. The muon anomaly was measured to very high precision (540 ppb) by the BNL E821 experiment yielding $a_\mu^{\text{E821}} = 116592089(63) \times 10^{-11}$.² When compared to the most recent SM calculations, the difference between the BNL result and theory is 3.7σ , as shown in Fig. 1.³ This discrepancy may be a sign of new physics. The new Muon $g - 2$ experiment, E989, is currently running at Fermilab and aims to measure a_μ to 140 ppb, a factor of four improvement in precision.

2. CPT- and Lorentz-violating signatures in $g - 2$

In the Muon $g - 2$ experiment, a beam of polarized muons is injected into a storage ring. The anomaly is determined by measuring the ratio

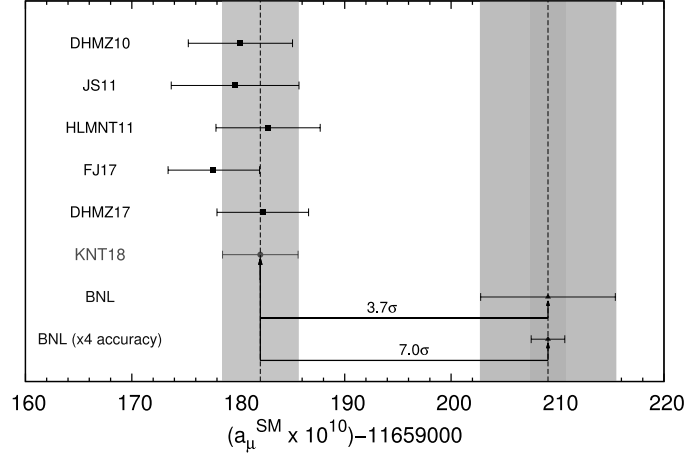


Fig. 1. Comparison of SM evaluations of a_μ with the most recent experimental result and prospect.³

of two frequencies: $\vec{\omega}_a = \vec{\omega}_c - \vec{\omega}_s$, which is the rate at which the muon's spin (ω_s) advances relative to its momentum (ω_c), and the proton Larmor-precession frequency ω_p , which is a measure of the ring magnetic field.⁴ These frequencies are related to the anomaly by

$$a_\mu = \frac{\omega_a}{\omega_p} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}. \quad (2)$$

The Standard-Model Extension (SME) is a general framework that describes CPT- and Lorentz-invariance violation by adding new terms to the SM lagrangian.⁵ A minimal SME expression for the muon sector is

$$\begin{aligned} \mathcal{L} = & -a_\kappa \bar{\psi} \gamma^\kappa \psi - b_\kappa \bar{\psi} \gamma_5 \gamma^\kappa \psi - \frac{1}{2} H_{\kappa\lambda} \bar{\psi} \sigma^{\kappa\lambda} \psi \\ & + \frac{1}{2} i c_{\kappa\lambda} \bar{\psi} \gamma^\kappa \overleftrightarrow{D}^\lambda \psi + \frac{1}{2} i d_{\kappa\lambda} \bar{\psi} \gamma_5 \gamma^\kappa \overleftrightarrow{D}^\lambda \psi. \end{aligned} \quad (3)$$

Equation (3) predicts two Lorentz- and CPT-violating effects: a μ^+/μ^- ω_a difference, $\Delta\omega_a = \langle \omega_a^{\mu^+} \rangle - \langle \omega_a^{\mu^-} \rangle$, and a sidereal ω_a variation.⁶

In terms of the SME coefficients, $\Delta\omega_a = (4b_Z/\gamma) \cos \chi$, where χ is the colatitude of the experiment. Experimentally, it is convenient to perform the analysis on the ratio $\mathcal{R} = \omega_a/\omega_p$ from Eq. (2). The BNL E821 result $\Delta\mathcal{R} = -(3.6 \pm 3.7) \times 10^{-9}$ yields the limit $b_Z = -(1.0 \pm 1.1) \times 10^{-23}$ GeV.⁷ Comparison of $\omega_a^{\mu^+}$ from one experiment with $\omega_a^{\mu^-}$ from a second at a different colatitude affords sensitivity to the d and H coefficients, and doing

so with BNL E821 and an earlier muon $g - 2$ experiment at CERN⁸ gives $(m_\mu d_{Z0} + H_{XY}) = (1.6 \pm 5.6) \times 10^{-23}$ GeV.

Sidereal variation in ω_a is investigated using a Lomb–Scargle test⁹ for a significant amplitude of ω_a oscillation at the sidereal frequency. The Lomb–Scargle method is optimized for data unequally spaced in time, as is the case for E821. The limits on such an amplitude in the BNL data are $A^{\mu^-} < 4.2$ ppm and $A^{\mu^+} < 2.2$ ppm, which by the relationship $A^\mu = 2\check{b}_\perp^\mu \sin \chi$ is equivalent to $\check{b}_\perp^{\mu^-} \leq 2.6 \times 10^{-24}$ GeV and $\check{b}_\perp^{\mu^+} \leq 1.4 \times 10^{-24}$ GeV.⁷ These BNL E821 limits as well as other on both minimal ($d = 4$) and nonminimal ($d \geq 5$) SME coefficients¹⁰ are the most stringent in the muon sector.

3. Prospects for E989

The goal for Muon $g - 2$ at Fermilab is to reduce the BNL a_μ uncertainty by a factor of four from 540 ppb to 140 ppb. This will be achieved by utilizing Fermilab's much higher intensity muon beam to collect 21 times the BNL μ^+ statistics, and by reducing the overall systematic uncertainty by a factor of 2.5 through detector upgrades and improved analysis techniques. Reaching that goal would increase the significance of the BNL discrepancy from 3.7σ to $\sim 7\sigma$ given the same central value for a_μ .

With regard to sidereal-variation Lorentz and CPT tests, sensitivity roughly scales with ω_a uncertainty. Thus, E989 should be able to reach limits of $\sim 5 \times 10^{-25}$ GeV and could do even better due to the possibility to search for the oscillation with a Fourier-transform method since the E989 data will be time-stamped allowing binning in equally-spaced time periods. Also, the full three-year run for Muon $g - 2$ will include data for most of the calendar year. Contrary to BNL E821, which ran the same three months in each year of operation, this permits a search for annual variation in a_μ .

Obviously, to measure $\Delta\omega_a$, Muon $g - 2$ needs μ^+ and μ^- data. The Fermilab Muon $g - 2$ schedule features μ^+ runs extending through early 2021. The Collaboration is exploring the technical requirements to carry out a μ^- run, as was done in E821. Items to be addressed include issues related to the lower initial muon flux and the need to improve the storage-ring vacuum. The optimal time for a switchover to μ^- depends on the results of the current, approved μ^+ runs.

A μ^- run does not simply represent one more test. The μ^- data gives access to many additional SME coefficients. Furthermore, JPARC is preparing E34, a muon $g - 2$ experiment with an ultra-cold muon beam.¹¹ E34 proposes to measure a_{μ^+} to 450 ppb. This would make possible a sub-

stantial improvement of the $(m_\mu d_{Z0} + H_{XY})$ limit for two reasons. First, the BNL/CERN 540 ppm/7000 ppm precisions would be replaced with Fermilab/JPARC 140 ppm/450 ppm. Second, this limit is proportional to $(\cos \chi_1 - \cos \chi_2)$, and there is much greater difference between Fermilab's and JPARC's colatitudes than between BNL and CERN. Because E34 utilizes muonium, and thus cannot measure a_{μ^-} , the only possibility for realizing this potential improvement is with a Fermilab Muon $g - 2 \mu^-$ run.

Finally, it has been shown that a nonminimal SME coefficient $\check{H}_{230}^{(5)} \simeq 3 \times 10^{-25} \text{ GeV}^{-1}$ can account for the 3.7σ discrepancy in muon $g - 2$.¹² The result from BNL E821 of $\check{H}_{230}^{(5)} = (2.9 \pm 3.0) \times 10^{-24} \text{ GeV}^{-1}$ is compatible with this level. With an E989 μ^- run and the promise of Fermilab and JPARC sensitivity goals, it may be possible to not only establish a significant discrepancy, but also make a statement concerning whether or not Lorentz and CPT violation is the physics responsible for it.

Acknowledgments

This work was supported in part by the US DOE and Fermilab under contract no. DE-SC0012391. The author is grateful to Alan Kostelecký for the CPT'19 invitation and conversations.

References

1. M. Passera, *J. Phys. G* **31**, R75 (2005).
2. Muon $g - 2$ Collaboration, G.W. Bennett *et al.*, *Phys. Rev. Lett.* **92**, 161802 (2004).
3. A. Keshavarzi, D. Nomura, and T. Teubner, *Phys. Rev. D* **97**, 114025 (2018).
4. Muon $g - 2$ Collaboration, J. Grange *et al.*, arXiv:1501.06858.
5. D. Colladay and V.A. Kostelecký, *Phys. Rev. D* **58**, 116002 (1998).
6. R. Bluhm, V.A. Kostelecký, and C.D. Lane, *Phys. Rev. Lett.* **84**, 1098 (2000).
7. Muon $g - 2$ Collaboration, G.W. Bennett *et al.*, *Phys. Rev. Lett.* **100**, 091602 (2008).
8. CERN–Mainz–Daresbury Collaboration, J. Bailey *et al.*, *Nucl. Phys. B* **150**, 1 (1979).
9. N.R. Lomb, *Astrophys. Space Sci.* **39**, 447 (1976); J.D. Scargle, *Astrophys. J.* **263**, 835 (1982).
10. *Data Tables for Lorentz and CPT Violation*, V.A. Kostelecký and N. Russell, 2019 edition, arXiv:0801.0287v12.
11. E34 Collaboration, M. Otani, *JPS Conf. Proc.* **8**, 025008 (2015).
12. A.H. Gomes, A. Kostelecký, and A.J. Vargas, *Phys. Rev. D* **90**, 076009 (2014).