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No fifth force in a scale invariant universe.

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We revisit the possibility that the Planck mass is spontaneously generated in scale invariant scalar-tensor theories of gravity, typically leading to a "dilaton." The fifth force, arising from the dilaton, is severely constrained by astrophysical measurements. We explore the possibility that nature is fundamentally Weyl-scale invariant and argue that, as a consequence, the fifth force effects are dramatically suppressed and such models are viable. We discuss possible obstructions to maintaining scale invariance and how these might be resolved.

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I. INTRODUCTION

The possibility that the gravitational constant, G, or alternatively the Planck mass, $M_{\rm Pl}$ is dynamically generated has been considered for more than half a century. P. Dirac argued that the large number hypothesis indicated the possibility that G obeyed an equation of the form $\Box G \sim \rho$ while C. Brans and R. Dicke proposed the action

$$S_{BD} = -\int d^4x\sqrt{-g}\left[-\frac{\alpha}{12}\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + L_m\right] \quad (1)$$

where $g_{\mu\nu}$ is the metric, R the corresponding Ricci scalar, L_m is the matter Lagrangian, minimally coupled to $g_{\mu\nu}$ and α is a dimensionless constant [1] (we are assuming the mostly minus sign convention). Brans and Dicke's original theory is normally expressed in terms of the dynamical Planck mass $\Phi = -\frac{\alpha}{6}\phi^2$, $V = 0$ and the parameter $\omega_{\rm BD} \sim 1/\alpha$. The Brans-Dicke action has become one of the workhorses of gravitational physics and is used to explore extensions of general relativity that appear in a wide range of fundamental contexts. It has a more modern, complete incarnation - the Horndeski action - which encapsulates all possible Scalar-Tensor theories which have second order equations of motion [2].

If scalar tensor theories are to work, we require a mechanism by which the Planck mass stabilizes at its observed value. This can be achieved through a variety of ways, most notable by picking a potential V such that, during its cosmic history, the scalar field settles at its minimum. The potential can have an explicit mass scale, ϕ_0, in it (e.g. of the form $V \sim (\phi^2 - \phi_0^2)^n$ where n can be positive or negative). The curvature of the effective potential will then set the effective range of the fifth force arising from the scalar field; a judicious choice of curvature can lead to a small enough range that current observational constraints can be avoided.

The non-minimal coupling of ϕ with R can lead to a richer variety of dynamics than those observed for standard scalar fields. In particular it is possible to construct models such that there are no dimensionfull parameters. If we choose $V = \lambda\phi^4$ and observe that, in the absence of matter, the equation of motion for ϕ can be cast as

$$(1-\alpha)\left[\Box\phi + \frac{\nabla_\mu\phi\nabla^\mu\phi}{\phi}\right] + \phi^4\frac{d}{d\phi}\left(\frac{V}{\phi^4}\right) \quad (2)$$

we find that the homogenous solution satisfies

$$a^3\phi\frac{d\phi}{dt} = \text{constant} \quad (3)$$

where a is the scale factor of the Universe. In an expanding Universe, $\dot{\phi} \to 0$ and $\phi \to \phi_0$. The final value will not be set by the minimum of the potential but by the field's initial value. This is a universe of eternal inflation and a spontaneously generated Planck mass, as described in the single scalar model of [3]. It is seen to be equivalent (in the Einstein frame) to a theory with a cosmological constant, fixed Planck scale and a completely decoupled dilaton. In two-field, or more, generalizations we can have inflation, Planck scale generation, and end up in a vacuum with vanishing cosmological constant. The time evolution naturally evolves the system from a Jordan frame to an Einstein frame [3].

Generic Scalar-Tensor theories are very severely constrained by observations. The process of estimating these constraints is well established (and clearly presented in the original paper by Brans and Dicke [1] and then generalized in [4] and [5]). While the original calculation [1] was done in the Jordan frame (i.e. the frame in which ϕ is non-minimally coupled to R) it has now become customary to transform to the Einstein frame where we have the standard Einstein-Hilbert action but where L_m is now coupled to $A(\phi)g_{\mu\nu}$ (where $A(\phi)$ arises from the conformal transformation between frames). The direct coupling between ϕ and matter brings out the interpretation of ϕ as the mediator of a "fifth force" which supplements the ordinary gravitational force.

The presence of ϕ leads to modifications of the usual solutions to the Einstein equations. For example the Schwarzschild-like solutions will have two non-trivial Parametrized Post Newtonian (PPN) parameters, γ and β, which can be constrained using, for example, measurements of the Shapiro time delay, light deflection and the Nordvedt effect. The current, tightest constraints come from an analysis of the Cassini spacecraft placing an upper bound on γ such that $\omega_{\rm BD} > 40,000$ [6]. Comparable constraints (within

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a factor of 2) have been obtained from the analysis of the relativistic pulsar-white dwarf binary, J1738+033 [7].

In this letter we will show that these results can be evaded if L_m is *scale invariant* [1]. That is, when we transform $g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}$, $\phi \to \Omega\phi$ and the matter appropriately (where Ω is a *constant*), we find that $S_M = \int d^4x\sqrt{-g}L_m$ is invariant and then the fifth force is completely non-existent. That the fifth force should be suppressed is simple and in some sense, obvious. In this scenario, scale invariance is a global Weyl symmetry which is broken when the scalar field settles down to its asymptotic value, i.e. when the Planck mass is stabilised. As a result, there is a Goldstone boson - the dilaton - which is the mediator of the fifth force and is, at most, derivatively coupled to the matter sector.

Derivative couplings between the dilaton and the matter sector will lead to a suppression, at large distance in the 5th force. We will show, in detail, that, in fact, in a scale invariant universe, in the symmetry broken phase, the dilaton does not perturbatively couple *at all* to the matter energy momentum tensor. There are several ingredients to this effect. The first one is that, given that the dilaton is derivatively coupled, the relevant terms in the action are of the form $[\delta L/\delta(\nabla^\mu\sigma)]\nabla^\mu\sigma = K_\mu\nabla^\mu\sigma$ where L is the Lagrangian including all terms in σ and K_μ is a four current, the global Weyl symmetry current, which is conserved. In the symmetry broken phase, $K_\mu = 0$ and thus σ decouples from the matter dynamics of the theory. The second ingredient is that the kinetic term for fermions involves a symmetrized derivative, $(\overrightarrow{\nabla} - \overleftarrow{\nabla})$, which is completely blind to a *real* rescaling of the fermion field (as opposed to one involving a complex phase). Thirdly, gauge fields are neutral under Weyl transformations, and the dilaton is automatically decoupled from a classical gauge action. If the Weyl symmetry is valid at the quantum level as well, again the dilaton completely decouples from the gauge field action. We will use this paper to show how all these ingredients come into play and flesh out the proof that scale invariant theories evade fifth constraints by examining a number of specific examples.

We begin, in section Section II by re-deriving the constraints in the standard derivation and then show how they may be evaded for the case of single scalar field coupled to a scale invariant matter action - we should expect, at most, a derivative coupling of the dilaton to the matter source. In the Section III we delve deeper and explored how the Weyl symmetry of a multi-scalar field action actually leads the dilaton to completely decouple from the matter sector. In Section IV we build on our previous results and show how a proxy for the standard model - a fermion that acquires a mass via the Higgs mechanism - will lead to the same result. We also demonstrate the decoupling of the dilaton from gauge bosons. Finally we discuss non-perturbative effects that can lead to the coupling of the dilaton to matter, but in a highly suppressed manner.

In Section V we briefly address the fact that, while quantum corrections would seem to invalidate scale invariance via trace anomalies, this problem can be avoided [3]. It is well known that a quantum theory in $D = 4$ with no input masses and vanishing β-functions to all orders in $\hbar$ is scale symmetric. However, the converse is not necessarily true: a quantum theory in $D = 4$ with no input masses and non-vanishing β-functions is not necessarily non-scale invariant. That is, it can be interpreted as a subsector of a fully scale invariant theory. In these theories, ratios of observables to fixed mass scales, such as ϕ_c/M, where ϕ_c is a classical field VEV (or an external momentum scale in a scattering amplitude) and M is a fixed mass scale, do not occur as arguments of logs. Rather renormalization group running occurs in Weyl invariant ratios, e.g., ϕ_c/χ_c which respects overall scale symmetry [3]. In short, in these theories there is no absolute mass scale in nature, but rather just dimensionless ratios of VEV's. In Section VI we summarize our findings.

II. EVADING FIFTH FORCE CONSTRAINTS WITH THE DILATON.

We take as our starting point the action presented in Equation 1. The modified Einstein field equations are

$$-M^2 G_{\alpha\beta} = \left(1-\frac{\alpha}{3}\right)\partial_\alpha\phi\partial_\beta\phi - \left(\frac{1}{2}-\frac{\alpha}{3}\right)\partial_\mu\phi\partial^\mu\phi g_{\alpha\beta} + \frac{\alpha}{3}\left(\phi g_{\alpha\beta} - \nabla_\alpha\nabla_\beta\phi\right) + V g_{\alpha\beta} - T_{m\alpha\beta} \quad (4)$$

where we have defined $M^2 = -\alpha\phi^2/6$. The modified Klein-Gordon equation is

$$\Box\phi + \frac{\alpha}{6}R\phi + V_\phi = 0 \quad (5)$$

where $V_\phi = dV/d\phi$.

We are interested in studying these equations in two limits. First we will expand around Minkowski space, $\eta_{\alpha\beta}$ and will assume that ϕ has stablized around a minimum value, ϕ_0. Hence we are interested in linear fluctuations around the scalar field minimum, $\phi = \phi_0 + \varphi$ and the Minkowski metric, $g_{\alpha\beta} = \eta_{\alpha\beta} + \mathrm{diag}(\Phi, \Psi\delta_{ij})$. Second, we are interested in the Newtonian, or quasi-static regime where we can discard all time derivatives of the metric and scalar field. Taking the trace of Equation 4 to eliminate the Ricci scalar in Equation 5 and the taking the two approximations described above we end up with

$$\nabla^2\varphi = -\frac{\alpha}{6(1-\alpha)}\frac{\phi_0}{M_{\rm Pl}^2}T_m \quad (6)$$

where we have defined $M_{\rm Pl}^2 \equiv -\alpha\phi_0^2/6$ and we have assumed that contributions from $d^2V/d\phi^2$ are negligible.

[1] We will in fact use Weyl invariance, which is a multiplicative scale transformation of fields incuding the metric that does not affect coordinates, as the key property. Diffeomorphism scale invariance, which transforms the coordinates, $\delta x^\mu = \varepsilon(x)x^\mu$ and is more general, inevitably arises from the fact that we are considering generally covariant theories without mass scales

The Einstein field equations become, in terms of the gravitational potentials,

$$-M_{\rm Pl}^2\nabla^2\Psi = -\frac{1}{2}\frac{3-2\alpha}{3(1-\alpha)}T_{m00}$$
$$-M_{\rm Pl}^2\nabla^2(\Phi-\Psi) = \frac{2\alpha}{3(1-\alpha)}T_{m00} \qquad (7)$$

where we have assumed a non-relativistic source, $T_m \simeq T_{m00}$ and $T_{mij} \simeq 0$. A localized mass gives us $T_{m00} \simeq M\delta^3(\mathbf{r})$ and we can solve for the potentials to find

$$\Psi = -\frac{3-2\alpha}{6(1-\alpha)}\frac{1}{M_{\rm Pl}^2}\frac{M}{r}$$
$$\Phi = \frac{1}{2(1-\alpha)}\frac{1}{M_{\rm Pl}^2}\frac{M}{r} \qquad (8)$$

If we define Netwon's constant via $\Phi = G_0M/r$ we have that the PPN parameter γ defined through

$$\Psi \equiv \gamma\frac{G_0M}{r}$$

is given by

$$\gamma = \frac{2\alpha-3}{4\alpha-3}. \qquad (9)$$

We have recovered the well established expression for γ for scalar-tensor theories.

Crucial, in this derivation, is the fact that φ is sourced by T_m and furthermore, that the energy momentum tensor of φ then sources the gravitational potentials. Because of the non-minimal coupling, φ enters the Einstein field equations in combinations of the form $\phi_0\nabla^2\varphi$, bringing in extra contributions of T_m to the right hand side. We can immediately see that, if the energy momentum tensor of matter fields is traceless there is no extra contribution to the metric potentials.

If the action presented in Equation 1 is scale invariant, the situation changes dramatically. Specifically, assume that $V = \frac{\lambda}{4}\phi^4$ and that, under scale transformations, $\sqrt{-g}L_m$ is invariant. Then consider the following Weyl field redefinitions

$$\phi = \phi_0 e^{\frac{\sigma}{f}}$$
$$g_{\alpha\beta} = \hat{g}_{\alpha\beta}e^{-\frac{2\sigma}{f}} \qquad (10)$$

where ϕ_0 is the stationary solution of the background field equations and σ is a scalar field - the dilaton. Transforming the action, we find

$$S_{BD} \to \int d^4x\sqrt{-\hat{g}}\left[-\frac{\alpha}{12}\phi_0^2\hat{R} + \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi_0) + \hat{L}_m\right] \qquad (11)$$

where we have chosen $f = (1-\alpha)\phi_0^2$ so as to canonically normalize σ. Note that, because of our assumptions about scale invariance, the transformed matter action, $\hat{L}_m$ does not couple directly to the dilaton σ although it may, however, couple to $\partial_\alpha\sigma$. This means that the dilaton equation of motion will be of the form

$$\Box\sigma = \partial_\alpha\sigma S^\alpha \qquad (12)$$

where S^α is constructed from elements of $T_{m\mu\nu}$. In fact, it is likely that $S^\alpha = \partial^\alpha S$ where S is a local function of the matter fields. We then have that σ is non zero inside the source but satisfies $\Box\sigma = 0$ outside, i.e. a damped wave equation. This means that, at late times, any contribution from σ to the energy momentum tensor sourcing the Einstein field equations is severely surpressed (as we shall see in Section IV C) and the standard constraints on Jordan-Brans-Dicke gravity do not apply.

A key aspect to this derivation is the scale invariance of L_m. We have assumed that there will be a derivative coupling to σ as we would expect from Goldstone's theorem. For this coupling to be completely absent, as we saw above, we would naively expect that we would have to restrict ourselves to a conformally matter source and that the result, therefore, follows trivially from our original derivation. In the next section we will dig a bit deeper and consider explicit forms for L_m to see that this is not necessarily the case.

III. THE DILATON IN A MULTI-SCALAR UNIVERSE.

Let us now consider a multi-scalar tensor theory of gravity of the form

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{12}\sum_i^N\alpha_i\phi_i^2R + \frac{1}{2}\sum_i^N\partial_\mu\phi_i\partial^\mu\phi_i - W(\vec{\phi})\right] \qquad (13)$$

where we assume a generalized "$\lambda\phi^2$ potential of the form:

$$W(\vec{\phi}) = \sum_i^N\sum_i^N\phi_i^2W_{ij}\phi_j^2$$

The action in Equation 13 is scale invariant: it is invariant under $g_{\mu\nu} \to \Omega^{-2}g_{\mu\nu}$, $\phi_i \to \Omega\phi_i$ where Ω is a constant. Here, what we call the matter action will be a subset of the scalar field action; for example we can define ϕ_1 to be the ϕ and ϕ_i, with $i = 2,\cdots,N$, to be the matter fields in L_m in the previous section.

As shown in [8, 9], this system has a conserved current which is tied to the underlying Weyl symmetry of the theory. The evolution equations for the scalar fields are

$$\Box\phi_i - \frac{\alpha_i}{6}\phi_iR - W_{\phi_i} = 0 \qquad (14)$$

where $W_{\phi_i} = \partial W/\partial\phi_i$ and R is the Ricci scalar which, in this case, is given by

$$-\frac{1}{6}\sum_{i=1}^N\alpha_i\phi_i^2R = \sum_{i=1}^N\left[(\alpha_i-1)\nabla_\mu\phi_i\nabla^\mu\phi_i + \alpha_i\phi_i\Box\phi_i\right] + 4W \qquad (15)$$

Multiplying each of the field equations 14 by ϕ_i and adding all of them together, one finds a conservation law of the form $\nabla_\mu K^\mu = 0$ where $K^\mu = \nabla^\mu K$ and

$$K = \frac{1}{2}\sum_{i=1}^{N}(1-\alpha_i)\phi_i^2 \tag{16}$$

We can easily understand the dynamics of this theory at the level of the background. If we take the ϕ_i to be functions of time t only, we have that the conservation equation give us

$$\ddot{K} + 3\frac{\dot{a}}{a}\dot{K} = 0 \tag{17}$$

and can be solved to give

$$K = c_1 + c_2\int\frac{dt}{a^3(t)}. \tag{18}$$

Therefore we find that, under general conditions, K will evolve to a constant value, $K \to K_0$. In other words, the scalar fields will evolve such that their values will be constrained to lie on the ellipse given by 16. Furthermore, one can show that, if W_{ij} is non-singular, that there will be a fixed point on this ellipse where the ratios between all possible ϕ_i^2 are determined by the coupling constants. We then have that the effective Planck mass, $M_{\rm Pl}$ is determined by the initial conditions of the scalar fields and the coupling constants in the theory. This behaviour is a generalization of the simple scalar field model presented in the introduction.

The phenomenology of the two scalar model is rich and has been extensively explored before. In particular, [8, 10–12] suggested that one of the fields could be a non-minimally coupled standard model Higgs and have extensively studied the phenomenology of what they have dubbed "Higgs-Dilaton cosmology. We have explored the fixed point structure and the inflationary regime in [3, 9] arguing that a scale-invariant, two field model can unify the IR and UV accelerated regimes into a viable cosmological model. A number of authors have explored various phenonomenological aspects of this theory in [13–16].

As before, we want to focus on what happens once the Planck mass has stabilised. In effect, the global scale invariance of the theory will have been broken and, as one would expect, a massless Goldstone mode, the dilaton will emerge. We will show, in this case, that the dilaton is uncoupled from the matter sector. In other words, there is no fifth force. To see how this happens in practice, we change variables to

$$\begin{aligned}\phi_i &= e^{-\frac{\sigma}{f}}\hat{\phi}_i \\ g_{\mu\nu} &= e^{2\frac{\sigma}{f}}\hat{g}_{\mu\nu}\end{aligned} \tag{19}$$

where $\hat{\phi}_i$ are constrained to lie on the ellipse given by

$$\bar{K} = \frac{1}{2}\sum_{i=1}^{N}(1-\alpha_i)\hat{\phi}_i^2 = f^2 \tag{20}$$

where f^2 is a constant.

Transforming the full action we find

$$\begin{aligned} S &= \int d^4x\sqrt{-\hat{g}}\Bigg[-\frac{1}{12}\sum_i^N\alpha_i\hat{\phi}_i^2\left(\hat{R}-\frac{6}{f^2}\partial_\mu\sigma\partial^\mu\sigma-\frac{6}{f^2}\Box\sigma\right) \\ &\quad +\frac{1}{2}\sum_i^N\partial_\mu\hat{\phi}_i\partial^\mu\hat{\phi}_i+\frac{1}{2f^2}\sum_i^N\hat{\phi}_i^2\partial_\mu\sigma\partial^\mu\sigma+\frac{1}{f}\partial_\mu\sigma\sum_i^N\hat{\phi}_i\partial^\mu\sigma_i \\ &\quad -W(\vec{\phi})\Bigg] \end{aligned} \tag{21}$$

which can be integrated by parts and rewritten as

$$\begin{aligned} S &= \int d^4x\sqrt{-\hat{g}}\Bigg[-\frac{1}{12}\sum_i^N\alpha_i\hat{\phi}_i^2\hat{R}+\frac{1}{2}\sum_i^N\partial_\mu\hat{\phi}_i\partial^\mu\hat{\phi}_i \\ &\quad +\frac{1}{f^2}\bar{K}\partial_\mu\sigma\partial^\mu\sigma+\frac{1}{f}\partial_\mu\sigma\partial^\mu\bar{K}-W(\vec{\phi})\Bigg] \end{aligned} \tag{22}$$

Given that $\bar{K} = f^2$ is a constant we have there are no cross-terms between σ and $\hat{\phi}_i$ and thus the dilaton is completely decoupled from everything else; in particular there are no derivative couplings between the dilaton and the remaining fields. The dilaton is canonically normalized and satisfies $\Box\sigma = 0$ so that in can be set to zero in the symmetry broken phase.

It is interesting to rephrase the result in terms of $\hat{\phi}_1$ (with non-minimal coupling $\alpha \equiv \alpha_1$) and the matter action, $\hat{L}_m$ constructed from the remaining $N-1$ fields. For simplicity we restrict ourselves to $N = 2$ and minimal coupling for the second field, $\chi \equiv \phi_2$). The background equations of motion fix $\hat{\phi}_1 = \phi_0$ and $\chi_0 = 0$. We then have $f^2 = \frac{1}{2}(1-\alpha)\phi_0^2$ and, as in the previous section, we can define an effective Planck mass, $M_{\rm Pl} \equiv -\frac{1}{6}\alpha\phi_0^2$. The matter action is simply

$$\hat{L}_m = \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + W(\phi_0,\hat{\chi}) \tag{23}$$

where scale invariance is now explicitly broken by the expectation value of ϕ_1. Again, note that there is no coupling at all to the dilaton, as advertised.

IV. ADDING MATTER FIELDS.

A. Complex Scalar and Fermions

We could construct a more realistic model of the matter sector which includes fermions, gauge fields and a Higgs sector. It turns out that it is sufficient to consider a fermion, ψ, coupled to a complex scalar field, H; gauge fields are conformally invariant and automatically decouple from the dilaton. The gravitational part of the action is

$$S_{BD} = \int d^4x\sqrt{-g}\left[-\frac{\alpha}{12}\phi^2R+\frac{1}{2}\partial_\mu\phi\partial_\nu\phi-V(\phi)\right] \tag{24}$$

where $V = \frac{\lambda}{4}\phi^4$.

We have that fermions, ψ will transform as $\psi \to \Omega^{3/2}\psi$ and therefore, the fermion action with a scale invariant mass term must take the form

$$S_\psi = \int d^4x\sqrt{-g}\left[\frac{i}{2}\bar{\psi}(\overrightarrow{\nabla\!\!\!/} - \overleftarrow{\nabla\!\!\!/})\psi - g\bar{\psi}\psi_R H - g\bar{\psi}\psi_L H^*\right] \tag{25}$$

where $\psi_L = \frac{1-\gamma^5}{2}\psi$, $\psi_R = \frac{1+\gamma^5}{2}\psi$ and γ^5 is a Dirac matrix. We have defined the covariant Dirac operator, $\overrightarrow{\nabla\!\!\!/} = E^{a\mu}\gamma_a\partial_\mu$, where $E^{a\mu}$ is the vierbein such that $g^{\mu\nu} = \eta_{ab}E^{a\mu}E^{b\nu}$ and η_{ab} is the Minkowski metric.

The action for the complex scalar field will take the form

$$S_H = \int d^4x\sqrt{-g}\left[\nabla_\mu H\nabla^\mu H^* + U(\phi,H)\right] \tag{26}$$

where the potential takes the form

$$U(\phi,H) = \xi(H^*H)^2 + \delta\phi^2 H^*H \tag{27}$$

The full action is then given by $S = S_\phi + S_\psi + S_H$. We can easily introduce Yang-Mills gauging by suitably correcting the covariant derivative and adding in the gauge kinetic term.

Ignoring the phase of the Higgs field, we can define $H = h/\sqrt{2}$ to end up with a two scalar theory

$$\begin{aligned} S \;=\; \int d^4x\sqrt{-g}\Big[&\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}g^{\mu\nu}\partial_\mu h\partial_\nu h \\ &+\frac{i}{2}\bar{\psi}(\overrightarrow{\nabla\!\!\!/} - \overleftarrow{\nabla\!\!\!/})\psi - g'\bar{\psi}\psi h - W(\phi,h) - \frac{1}{2}\alpha\phi^2 R\Big] \end{aligned} \tag{28}$$

where $g' = g/\sqrt{2}$ and $W = V + U$.

As before, we now want to extract the dilaton by transforming the fields as follows:

$$\begin{aligned} \phi &= \hat{\phi}e^{-\frac{\sigma}{f}} \\ h &= \hat{h}e^{-\frac{\sigma}{f}} \\ g_{\mu\nu} &= e^{2\frac{\sigma}{f}}\hat{g}_{\mu\nu} \\ E^{a\mu} &= e^{\frac{\sigma}{f}}\hat{E}^{a\mu} \\ \psi &= e^{\frac{3\sigma}{2f}}\psi' \end{aligned} \tag{29}$$

Applying this transformation, integrating by parts and defining the kernel, $\bar{K} = \frac{1}{2}(1-\alpha)\hat{\phi}^2 + \frac{1}{2}\hat{h}^2$, we find

$$\begin{aligned} S \;=\; \int d^4x\sqrt{-\hat{g}}\Big[&\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} + \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\hat{h}\partial_\nu\hat{h} + \frac{1}{f}\partial_\mu\partial^\mu\bar{K} \\ &+\frac{1}{2f^2}\bar{K}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{12}\alpha\hat{\phi}^2\hat{R} - W + \frac{i}{2}\bar{\psi}'(\overrightarrow{\nabla\!\!\!/} - \overleftarrow{\nabla\!\!\!/})\psi' \\ &-g'\bar{\phi'}\phi'\hat{h}\Big] \end{aligned} \tag{30}$$

As in the multi-scalar case, we have a conserved Weyl current; canonically normalizing the dilaton we have $f^2 = \bar{K}$ and which decouples the dilaton kinetic term from the remaining scalar fields.

Focusing on the symmetry broken phase where we have $\hat{\phi} = \phi_0 + \tilde{\phi}$ and $\phi_0 \gg \hat{h}$ we can solve for $\partial\tilde{\phi}$ to get

$$\partial\tilde{\phi} = -\frac{1}{1+\alpha}\frac{\hat{h}}{\phi_0}\partial\hat{h} \tag{31}$$

which means that $\partial\tilde{\phi} \ll \partial\hat{h}$. Furthermore, we have that $\bar{K} \simeq \frac{1}{2}(1-\alpha)\phi_0^2$ and so $\tilde{\phi} \simeq -\hat{h}^2/2(1-\alpha)$ and the leading order terms in the potential are

$$W(\hat{\phi},\hat{h}) \simeq \frac{\lambda}{4}\phi_0^4 + \frac{\delta'}{2}\phi_0^2\hat{h}^2 + \frac{\xi'}{4}\hat{h}^4 \tag{32}$$

where δ' and ξ' can be expressed in terms of λ, δ, ξ and α. The resulting action (with $M_{\rm Pl}^2 = -\frac{1}{6}\phi_0^2$) is Einstein gravity:

$$S \;=\; \int d^4x\sqrt{-\hat{g}}[\frac{1}{2}M_{\rm Pl}^2\hat{R} + L_m] \tag{33}$$

with

$$L_m = \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\hat{h}\partial_\nu\hat{h} - W + \frac{i}{2}\bar{\psi}'(\overrightarrow{\nabla\!\!\!/} - \overleftarrow{\nabla\!\!\!/})\psi' - g'\bar{\phi'}\phi'\hat{h} \tag{34}$$

As in the previous cases, we have found that there is no coupling between the dilaton and the matter sector and thus, such a scale invariant theory won't be subject to fifth force constraints.

B. Gauge Bosons

Covariant (lower index) vector bosons are neutral under the Weyl transformations laid out in Equation 29 . This is related to how the notion of length is contained in the covariant metric, and not in the coordinates under Weyl symmetry. This means that there is a big difference between contravariant and covariant and one has to be careful: $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$ has dimensions of L^2 (where $L \sim$ length), but $g^{\mu\nu} \to \Omega^{-2}g^{\mu\nu}$ has dimensions of L^{-2}. Hence the contravariant coordinates and their differentials, dx^μ, are dimensionless numbers, and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ has dimensions L^2 via the metric. Covariant coordinates, $dx_\mu = g_{\mu\nu}dx^\nu$ thus carry L^2.

Therefore, derivatives $\partial_\mu = \partial/\partial x^\mu$ are likewise neutral under a Weyl transformation, $\partial_\mu \to \partial_\mu$. When we construct a gauge covariant derivative for electromagnetism or other unitary gauge group based theories, we introduce a vector potential and have $D_\mu = \partial_\mu - ieA_\mu$. Consistency thus dictates that A_μ is also neutral under Weyl transformations, i.e. $A_\mu \to A_\mu$. (Note that Weyl's original gauge field enters as $D_\mu = \partial_\mu - qe'A_\mu$, and gauges scale transformations, where q is the (mass $\sim L^{-1}$)-scale dimension, i.e., $q = 1$ for ϕ and $q = -2$ for $g_{\mu\nu}$).

Hence the electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is also neutral, transforming as $F_{\mu\nu} \to F_{\mu\nu}$, but $F^{\mu\nu} = g^{\mu\rho}g^{\nu\lambda}F_{\rho\lambda} \to \Omega^{-4}F^{\mu\nu}$ has dimensions of L^{-4}, as an energy density. Since Maxwellian electromagnetic fields $\vec{E}$ and $\vec{B}$ have mass dimension L^{-2}, we see that they must be identified with $\vec{E} \sim F_0^i$ and $\vec{B} \sim F_i^j$.

The canonical kinetic term for gauge theories is therefore

$$\mathscr{L} = -\frac{1}{4}g^{\mu\rho}g^{\nu\lambda}F_{\mu\nu}F_{\rho\lambda} \tag{35}$$

and we see that that $\mathscr{L} \to \Omega^{-4}\mathscr{L}$ is an energy density. Since, $\sqrt{-g} \to \Omega^4\sqrt{-g}$, the action, $S_A = \int\sqrt{-g}\mathscr{L}$, is invariant. Since the Dilaton follows by replacing $\ln\Omega \to \sigma/f$, we see that it decouples from the classical vector potential action.

What about a renormalization group running gauge coupling, e? The action can be written in the noncanonical normalization as $S_A = \int(1/e^2)\sqrt{-g}\mathscr{L}$. If we use external mass scales to define renormalized running couplings (and for an infinitesimal Weyl transformation, $\Omega \simeq 1+\varepsilon$) we have $(1/e^2) \to (1/e^2) - 2(\beta(e)/e^3)\varepsilon$, (e.g. with $e^2 = e^2(\phi/M)$ and $d\ln(\phi/M) = \varepsilon$ where $\phi \to (1+\varepsilon)\phi$).

With "external" renormalization, i.e., using external input masses M to define renormalized quantities, we have the trace anomaly:

$$\begin{aligned} \frac{1}{\sqrt{-g}}\frac{\delta S_A}{\delta\varepsilon} &= -\frac{2\beta(e)}{e^3}\mathscr{L} \\ &\to \frac{\beta(e)}{2e}F^{\mu\nu}F_{\mu\nu} \quad \text{(canonical normalization).} \end{aligned} \tag{36}$$

However, with "internal" renormalization we use fields in the action in place of M, hence $\ln(\phi/M) \to \ln(\phi/\chi)$. With this Weyl invariant argument of the log, the action is invariant under $\delta I/\delta\varepsilon$ and there is no trace anomaly (i.e., the associated Weyl current is conserved). There is still running of the coupling, $g(\phi/\chi)$, but now in the variable $\ln(\phi/\chi)$. There is still a physical Λ_{QCD}, but now $\Lambda_{QCD}/\chi = \exp\{-8\pi^2/|b_0|e^2_{QCD}(\chi)\}$ and the ratio Λ_{QCD}/χ is Weyl invariant to this order of perturbation theory.

Hence, the dilaton completely decouples from gauge fields in quantum mechanics as well, provided we use "internal renormalization." This is a world in which there are no absolute mass scales, but only dimensionless ratios of field VEV's [3], and may be an underlying symmetry in nature.

C. Higher Dimension Operators

The previous discussion has been restricted to $D \leq 4$ operators. In fact, this provides an easy way to see why dilaton decoupling from spinors occurs: the quantity $\partial_\mu\sigma$ is $C = +$ (i.e. charge conjugation even), while the fermionic current $\bar{\psi}\gamma_\mu\psi$ is $C = -$. However, there will generally occur higher dimension operators, such as those involving the nucleon (i.e. ψ) that arise nonperturbatively in QCD. For example, we might have an operator taking the form:

$$\sqrt{-g}\kappa g^{\mu\nu}\partial_\mu\sigma\bar{\psi}\partial^\mu\psi/f\Lambda_{QCD} \tag{37}$$

We've chosen an operator that is chiral symmetry breaking and hence scales like $\Lambda_{QCD}/\Lambda^2_{QCD}$ However, the fermionic operator now has $C = +$ and the dilaton can couple derivatively to the fermion density.

Now consider a compact source, like a star or planet where the nucleon density can be approximated by a local static function $\overline{\psi}\psi(x) = \rho(\overrightarrow{x})$ and $\psi(\overrightarrow{x}) \to 0$ for $|\overrightarrow{x}| > R$. In this approximation the source $\bar{\psi}\nabla_i\psi \sim (1/2)\nabla_i\rho$, and we have a vanishing surface term:

$$\int d^3x\,\vec{\nabla}^2\rho = 0 \tag{38}$$

If we assume approximate flat space and we can seek a static solution for the σ field around the source. The equation of motion in the static limit is thus:

$$-\nabla^2\widehat{\sigma} + \frac{\kappa}{f\Lambda_{QCD}}\nabla^2\rho(\overrightarrow{x}) = 0 \tag{39}$$

A Green's function solution for the dilaton halo is then:

$$\sigma = -\frac{\kappa}{f\Lambda_{QCD}}\int\frac{1}{4\pi|\overrightarrow{r}-\overrightarrow{x}|}\nabla^2\rho(\overrightarrow{x})d^3x \tag{40}$$

Performing a double integration by parts and using $\nabla^2(4\pi|\overrightarrow{r}-\overrightarrow{x}|)^{-1} = \delta^3(\overrightarrow{r}-\overrightarrow{x})$ yields:

$$\sigma(\overrightarrow{r}) = -\frac{\kappa}{f\Lambda_{QCD}}\rho(\overrightarrow{r}) \tag{41}$$

This is a halo field that simply tracks the source distribution and vanishes outside. Other operators and distributions might produce at most weak $1/r^3$ halos. This is analogous to the fact that pseudoscalar fields, such as axions, couple to $\overline{\Psi}\gamma^5\Psi \sim \Psi^\dagger\overrightarrow{\sigma}\cdot\overrightarrow{\nabla}\Psi$, where we indicate the nonrelativistic limit. This implies that pseudoscalar fields couple to dipole densities $\sim \overrightarrow{S}\cdot\overrightarrow{\nabla}\rho$ (where $\overrightarrow{S}$ is a net spin polarization). It is beyond the scope of the present work to determine if ultra-sensitive experiments could detect such a suppressed short-range halo.

V. OBSTRUCTIONS AND SOLUTIONS.

We have argued that the fifth-force bounds on Brans-Dicke theories are absent if a Weyl scale invariance is only broken spontaneously. In the context of a complete theory of the fundamental forces this requires that the Standard Model (SM) should also be scale invariant with all masses generated spontaneously. Indeed, with the exception of the scalar potential, the SM Lagrangian is scale invariant and the masses of the gauge bosons, the quarks and the charged leptons are generated through spontaneous breaking of the electroweak (EW) symmetry.

However, in the SM the spontaneous breaking of EW symmetry is triggered by the inclusion of a scalar mass term in the Lagrangian that explicitly breaks scale invariance. In the context of the SM this term is at the heart of the naturalness problem that either hampers our understanding of the foundation of the SM or hints at new physics, depending on the eye of the beholder.

A rejuvenated approach has been advocated that builds scale-invariance into the core of the SM [17]. The idea is that the spontaneous EW breaking occurs via dimensional transmutation in which radiative corrections drive the Higgs ruuning quartic scalar coupling negative below the EW scale, leading to the Coleman-Weinberg mechanism [18], and triggering spontaneous EW breaking at that scale. In the original implementations of this idea the classical theory is scale invariant and scale breaking occurs through the trace anomaly arising from the one-loop radiative corrections to the quadratic coupling proportional to $\ln(|H|^2/M^2)$ where M is an explicit mass scale at which the coupling is defined and H is the SM scalar field. However, introducing M as an external input mass leads to explicit breaking of scale symmetry, and such a term will induce non-derivative couplings of the dilaton to the SM states, re-introducing the fifth-force bounds on the Brans-Dicke coupling.

A more ambitious viewpoint argues that any mass scales that might enter via regularization and renormalization should be vacuum expectation values of fields in the action of the theory itself, and thus maintain the Weyl invariance. So, for example, logarithmic corrections to the action of the form $\ln(|H|^2/M^2)$ in a scale broken theory would be replaced by $\ln(|H|^2/\phi^2)$ such that the argument of the logarithm is itself scale invariant. A case for this approach has been made in [3, 19, 20]. This allows for nonzero β-functions and renormalization group running of coupling constants in quantities like $\ln(|H|^2/\phi^2)$, however the Weyl symmetry is now maintained at the quantum level. In this case, scale invariance is only spontaneously broken, so the decoupling of the dilaton persists and there are no fifth-force bounds on the Brans-Dicke coupling.

There remains the question whether neutrino masses explicitly break scale symmetry. In the SM neutrinos are massless due to the absence of right-handed (RH) SM-singlet neutrinos. If they are added to the SM then, after spontaneous EW breaking, neutrinos will acquire Dirac masses upon EW breaking through their Yukawa coupling to the SM scalar. It is possible these couplings are anomalously small and give rise to the observed neutrino masses. As for the quarks and charged leptons they do not lead to explicit scale breaking so the dilaton still decouples. Alternatively the LH neutrinos may acquire Majorana masses via a dimension 5 coupling to two SM scalars through the exchange of a heavy state such a RH neutrino or a heavy scalar state. Provided the RH states also acquire their mass through spontaneous breaking of the scale symmetry the decoupling of the dilaton will be preserved.

VI. DISCUSSION

In this paper we have explicitly shown that perturbatively, in scalar-tensor theories in a scale invariant universe, there is no fifth force. This means that the usual, extremely tight, astrophysical constraints can be completely evaded. We have done so by looking at a representative selection of actions which encapsulate the essential structure of the standard model and beyond. We have discussed how this result may be obstructed in the real world by explicit mass scales but have also described how to evade these obstructions.

Our result is not unexpected. We are considering a global scale symmetry which is spontaneously broken. From Goldstone's theorem we expect the dilaton, which is the mediator of the fifth force, and to be derivatively coupled. In fact, however, the dilaton doesn't couple at all to the energy momentum tensor of the matter fields in the spontaneous symmetry broken phase. The dilaton obeys a damped wave equation and its dynamics are trivial: any residual non-zero fluctuations in the dilaton will dissipate away following the onset of the symmetry broken phase. The fact that the dilaton decouples from the rest of a scale invariant world has been alluded to before. In [21], the author constructed simple scalar field models involving one and two scalar fields and showed there that the massless scalar mode would decouple from any additional static matter sector. Our calculation generalizes the result of [21].

The question remains: does the Universe have an underlying exact scale invariance, that is hidden by spontaneous symmetry breaking? The conventional view, *e.g.*, such as that of string theory where scale symmetry is explicitly broken by the string tension, is that it is not an exact symmetry and, if so, our results do not hold. If there are explicit mass scales built into the fundamental action of the Universe then we would be stuck with the extremely tight constraints on scalar-tensor theories: scalar tensor theories are then disfavored. But the recent resurgence in interest in scale-invariance, driven in part by the discovery of a fundamental scalar particle, the Higgs boson, is leading to a fresh look at some of the impediments and advantages to having a scale-invariant world. It may well be that scale-invariance solves the problems currently facing our understanding of fundamental physics. If, indeed, all explicit mass scales can be dropped from our fundamental action, then scalar-tensor theories will be given a completely new lease on life.

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