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Scale-Independent Inflation and Hierarchy Generation

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We discuss models involving two scalar fields coupled to classical gravity that satisfy the general criteria: (i) the theory has no mass input parameters, (ii) classical scale symmetry is broken only through $-\frac{1}{12}\varsigma\phi^2 R$ couplings where ς departs from the special conformal value of 1; (iii) the Planck mass is dynamically generated by the vacuum expectations values (VEVs) of the scalars (iv) there is a stage of viable inflation associated with slow roll in the two–scalar potential; (v) the final vacuum has a small to vanishing cosmological constant and an hierarchically small ratio of the VEVs and the ratio of the scalar masses to the Planck scale. This assumes the paradigm of classical scale symmetry as a custodial symmetry of large hierarchies.

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The discovery of the weakly interacting Brout-Englert-Higgs (BEH) boson, coupled with the absence of significant evidence for physics beyond the Standard Model, has stimulated a re-evaluation of the possible explanations of the hierarchy problem. In the Standard Model (SM) of the strong and electroweak interactions, which has no fundamental input mass scale other than the BEH mass, an apparent hierarchy problem arises that is due to the additive quadratically divergent radiative corrections to the mass squared of the BEH boson. However, in the pure Standard Model the quadratic divergences are an artifact of the introduction of a mass scale cut-off in momentum space [1]. In the context of field theory, the coefficients of relevant operators have to be renormalised and the theory is defined ultimately by observable renormalised coefficients. In this case neither the quadratically divergent radiative correction to the BEH mass nor the mass counter-term is measurable and only the renormalised mass is physically meaningful. If one maintains scale invariance broken only explicitly by the various trace anomalies and spontaneously to generate the BEH boson mass, then the latter must be viewed as multiplicatively renormalized since no quadratic divergence arises in the trace anomaly. This has further led to the proposal of classically-scale-invariant models that contain the SM, in which the electroweak scale is generated through spontaneous breaking of scale invariance via Coleman-Weinberg mechanism [2, 3].

It has been suggested that scale invariance might even apply at the quantum level through "endogenous" renormalisation which requires that the regulator mass scale, μ, associated with quantum loops in dimensional regularization, is itself generated by a moduli field[1]. Alternatively, one can always introduce an arbitrary cut-off scale Λ, *e.g.*, by way of momentum space cut-off or Pauli-Villars regularization, but then renormalize the theory at a renormalization scale given by a moduli field to remove the Λ dependence[2]. However we will not explore this possibility here, concentrating on whether it is possible to build a viable scale invariant theory broken only spontaneously and via the trace anomaly.

Of course a complete theory must include gravity and, if one is to maintain classical scale invariance, it is necessary to do so in a way that generates the Planck scale through spontaneous breaking of the scale invariance such as occurs in the Brans Dicke theory of gravity [5]. The inclusion of gravity means there are additional additive divergent contributions to the BEH mass but these, too, are unphysical and should be absorbed in the renormalised mass which vanishes if the underlying theory is classically scale invariant.

In this paper we construct a spontaneously broken scale-free model. As such, there is no physical meaning

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[1] For a recent discussion of this see [4]

[2] It is easy to see that if one subtracts at some mass scale M that is specified externally to the defining field theory action, then the trace anomaly arises as the variation of the renormalized action wrt $\ln(M)$. In replacing the subtraction scale M by an actual field χ that is part of the defining action of the theory, there is no residual trace anomaly; the trace anomaly is simply absorbed into the improved stress tensor itself, which then remains traceless.

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to the vacuum expectation value (vev) of a single scalar field and only ratios of vevs are measurable. A mimimal model capable of generating an hierarchy requires the introduction of two scalar fields, ϕ and χ coupled to gravity in the form:

$$S = -\int d^4x\sqrt{-g}[\frac{1}{12}\alpha\phi^2 R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \frac{1}{12}\beta\chi^2 R + \frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi + W(\phi,\chi)] \quad (1)$$

where: $W(\phi,\chi) = \lambda\phi^4 + \xi\chi^4 + \delta\phi^2\chi^2$. This theory has no input mass scales, is conformally invariant if $\alpha = \beta = 1$ and is invariant under independent $\phi \to \pm\phi,\ \ \chi \to \pm\chi$.

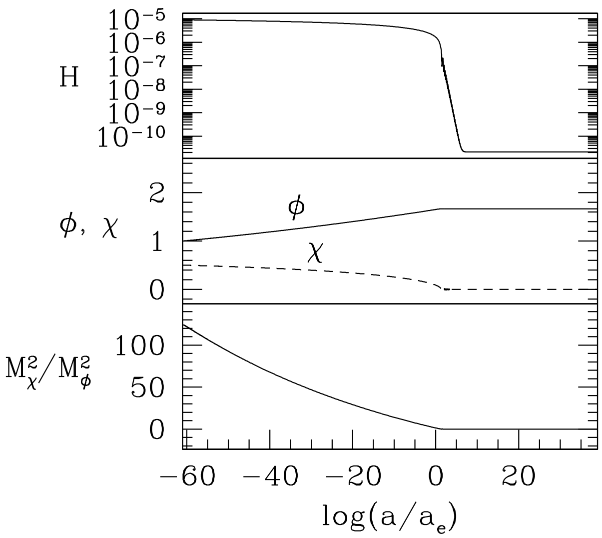

FIG. 1: Plot of the Hubble parameter, H, ϕ, χ and the ratio of the two components of the effective Planck mass, M_ϕ^2 and M_χ^2, as a function of a; we have normalized the x-axis to the scale factor at the end of inflation, a_e.

Such a theory has remarkable properties that we illustrate for one representative choice of parameters (α, β, λ, ξ, δ) in Figure 1. At early times it has a period of inflation during which, as we will show later on, observationally viable spectra of scalar and tensor perturbations can be generated. Furthermore, it has an infra-red (IR) fixed point set by ratios of the coupling constants and which is radiatively stable to quantum corrections and during which the universe undergoes accelerated expansion. The effective planck mass, $M^2 = M_\phi^2 + M_\chi^2$ (where $M_\phi^2 = -\alpha\phi^2/6$ and $M_\chi^2 = -\beta\chi^2/6$) is time varying during the inflationary period (when $M_\phi^2 \ll M_\chi^2$) but constant during the late time accelerated expansion phase (when $M_\phi^2 \gg M_\chi^2$), obeying current constraints on gravitational physics. In the rest of the letter we flesh out this scenario in detail.

The field equations follow from eq.(1):

$$M^2 G_{\alpha\beta} = T^\phi_{\alpha\beta} + T^\chi_{\alpha\beta} - g_{\alpha\beta}W(\phi,\chi) \quad (2)$$

where:

$$T^\phi_{\alpha\beta} = \left(1-\frac{\alpha}{3}\right)\nabla_\alpha\phi\nabla_\beta\phi + \left(\frac{\alpha}{3}-\frac{1}{2}\right)g_{\alpha\beta}\nabla_\mu\phi\nabla^\mu\phi - \frac{\alpha}{3}\phi\nabla_\alpha\nabla_\beta\phi + \frac{\alpha}{3}g_{\alpha\beta}\phi\Box\phi$$

$$T^\chi_{\alpha\beta} = \left(1-\frac{\beta}{3}\right)\nabla_\alpha\chi\nabla_\beta\chi + \left(\frac{\beta}{3}-\frac{1}{2}\right)g_{\alpha\beta}\nabla_\mu\chi\nabla^\mu\chi - \frac{\beta}{3}\chi\nabla_\alpha\nabla_\beta\chi + \frac{\beta}{3}g_{\alpha\beta}\chi\Box\chi \quad (3)$$

and:

$$\Box\phi - \frac{\alpha}{6}\phi R - \frac{\partial W}{\partial\phi} = 0, \quad \Box\chi - \frac{\beta}{6}\chi R - \frac{\partial W}{\partial\chi} = 0. \quad (4)$$

To obtain the normal form of the Einstein equations at late times, M^2 must be positive and therefore at least one of the coefficients α or β must be negative, inconsistent with the conformally invariant choice. However the resultant theory is still scale-independent. Taking the trace of the Einstein field equations we have:

$$-M^2 R = (\alpha-1)\nabla_\mu\phi\nabla^\mu\phi + (\beta-1)\nabla_\mu\chi\nabla^\mu\chi + \alpha\phi\Box\phi + \beta\chi\Box\chi - 4W \quad (5)$$

which determines the Ricci scalar.

We now restrict the analysis to study the cosmological evolution for a Friedmann Robertson Walker (FRW) metric, $g_{\alpha\beta} = (-1, a^2\delta_{ij})$. The FRW equation is given by:

$$H^2 - \frac{D}{3M^2}H - \frac{\rho_T}{3M^2} = 0 \quad (6)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, $D = \alpha\phi\dot{\phi} + \beta\chi\dot{\chi}$ and $\rho_T = \dot{\phi}^2/2 + \dot{\chi}^2/2 + W$. The evolution equations for ϕ and χ can be uncoupled to give:

$$\begin{pmatrix} \Box\phi \\ \Box\chi \end{pmatrix} = \frac{1}{\mathrm{K}}\begin{pmatrix} 1+\frac{\beta^2\chi^2}{6M^2} & -\frac{\alpha\beta\phi\chi}{6M^2} \\ -\frac{\alpha\beta\phi\chi}{6M^2} & 1+\frac{\alpha^2\phi^2}{6M^2} \end{pmatrix}\begin{pmatrix} \mathcal{S}_\phi \\ \mathcal{S}_\chi \end{pmatrix} \quad (7)$$

where $\mathrm{K} = 1 + (\alpha^2\phi^2 + \beta^2\chi^2)/(6M^2)$ and:

$$\mathcal{S}_\phi = \alpha(\alpha-1)\frac{\phi\dot{\phi}^2}{6M^2} + \alpha(\beta-1)\frac{\phi\dot{\chi}^2}{6M^2} + \frac{4\alpha\phi}{6M^2}W + \frac{\partial W}{\partial\phi}$$

$$\mathcal{S}_\chi = \beta(\beta-1)\frac{\chi\dot{\chi}^2}{6M^2} + \beta(\alpha-1)\frac{\chi\dot{\phi}^2}{6M^2} + \frac{4\beta\chi}{6M^2}W + \frac{\partial W}{\partial\chi} \quad (8)$$

As advertised, this theory has an infrared fixed point which can be found by setting $\ddot{\phi} = \dot{\phi} = \ddot{\chi} = \dot{\chi} = 0$ leading to:

$$\overline{\mathcal{S}}_\phi \equiv -4\alpha\frac{\phi}{\alpha\phi^2 + \beta\chi^2}W + \frac{\partial W}{\partial\phi} = 0$$

$$\overline{\mathcal{S}}_\chi \equiv -4\beta\frac{\chi}{\alpha\phi^2 + \beta\chi^2}W + \frac{\partial W}{\partial\chi} = 0 \quad (9)$$

Note that $\phi\overline{\mathcal{S}}_\phi + \chi\overline{\mathcal{S}}_\chi = 0$ is automatically satisfied since our full potential, $W(\phi,\chi)$, is classically scale invariant: $\delta W/\delta\ln\phi + \delta W/\delta\ln\chi = 4W$ This guarantees that nontrivial solutions generally exist in the ratio of the VEV's of ϕ and χ given by:

$$\frac{\langle\chi_0\rangle^2}{\langle\phi_0\rangle^2} = \frac{4\lambda\beta - 2\alpha\delta}{4\alpha\xi - 2\beta\delta} \tag{10}$$

One can readily show that this is an IR stable fixed point so that $\langle\phi_0\rangle$, $\langle\chi_0\rangle$ are the IR vevs of the scalar fields. Note that it is only dimensionless ratios of VEVs that are physical. The absolute value of a VEV, not determined by the static equations, is not measurable.

We are interested in the case that $\langle\phi_0^2\rangle \gg \langle\chi_0^2\rangle$ so that, at late times, a large hierarchy develops. To have an hierarchically light "matter" sector also requires that the χ mass should be small relative to the Planck scale and this in turn requires that the χ mass contribution coming from the $\delta\phi^2\chi^2$ term should be hierarchically small relative to the Planck mass, *i.e.* $\delta \le \langle\chi_0^2\rangle/\langle\phi_0^2\rangle$. Finally if the cosmological constant at late times is small then this requires a fine-tuning of the parameters in W such that it is (or is close to) a perfect square. Furthermore, we need $\lambda \le \langle\chi_0^4\rangle/\langle\phi_0^4\rangle$ which, in the absence of a $\frac{\alpha}{12}\phi^2 R$ term, is natural because ϕ is shift symmetric in the limit the small parameters vanish. Thus the radiative corrections to the small parameters can only be gravitational in origin (we will discuss these corrections later in this letter).

What happens to the scale factor in the IR? For static scalar fields the FRW equation, Eq. 6, implies:

$$3M^2\left(\frac{\dot a}{a}\right)^2 = W = (\lambda + \xi\mu^4 + \delta\mu^2)\phi_0^4 \tag{11}$$

(where $\mu^2 \equiv \langle\chi_0\rangle^2/\langle\phi_0\rangle^2$) and we can define an effective cosmological constant $\Lambda_{\rm eff} = (\lambda + \xi\mu^4 + \delta\mu^2)\phi_0^2/(\alpha + \beta\mu^2)$. With the ordering of the couplings discussed above $\Lambda_{eff} \le \xi\chi_0^4/M^2$. To obtain zero cosmological constant requires fine tuning of the couplings corresponding to the potential having the form of a perfect square.

This theory is equivalent to a multi-scalar Jordan-Brans-Dicke theory of gravity with a potential [5–7]. Current constraints on Brans-Dicke theories from Shapiro time delay measurements are particularly stringent and a naive application to this theory leads to $\alpha^2 < 2.5\times10^{-5}$. However, the particular scale invariant form of the potential W implies that $\Box\phi = 0$ at the fixed point, implying that it decouples from the "matter" field, χ, suggesting that the bound may not apply. Clearly this warrants a more detailed study of quasi-static solutions in the presence of matter in a cosmological background, beyond the scope of the current letter.

A remarkable feature of the scale-independent structure, that we see in Fig 1, is that it readily leads to an inflationary era. Non-minimally coupled models of inflation have been looked at before [8–11]. Multifield, non-minimal models have also been looked at in some detail, with a particular focus on models with an explicit Planck mass [12] or perfectly (or almost perfect) conformal invariance (with $\alpha = \beta = 1$) [13].

However this case is characteristically different, with no explicit Planck mass and the slow-roll condition resulting from a cancellation of terms due to the scale invariance of non-gravitational sector. To understand its inflationary regime, it useful to rewrite Eq. 7 in terms of M_ϕ^2 and M_χ^2. In the regime where $W \simeq \xi\chi^4$, Eqs 7 gives us:

$$\frac{d}{dN}\begin{pmatrix} M_\phi^2 \\ M_\chi^2 \end{pmatrix} = \frac{4}{3}\frac{M_\phi^2(M_\phi^2 + M_\chi^2)}{(\alpha-1)M_\phi^2 + (\beta-1)M_\chi^2}\begin{pmatrix} (1-\beta)\alpha \\ (\alpha-1)\beta \end{pmatrix} \tag{12}$$

where $N = \ln a$. Slow-roll results in the $\beta \gg \alpha$ regime where $M_\chi^2 \gg M_\phi^2$ because the scale invariant form of the scalar potential results in a cancellation of the large $\frac{\partial W}{\partial\chi}$ term in eq(8) so that the rhs of eq(12) is proportional to M_ϕ^2. Solving this equation gives the inflationary solution $M_\phi^2 = M_E^2 e^{-\nu N}$ and $M_\chi^2 = M_E^2\left[1 + \gamma\left(1 - e^{-\nu N}\right)\right]$ where $\nu = -4\alpha/3$ and $\gamma = \beta(1-\alpha)/\alpha(1-\beta)$, and we have $N = 0$ at the end of inflation when $M_\phi^2 = M_\chi^2 = M_E^2$. We have checked that this solution is a superb approximation to the numerical solution to Eqs 7.

With our analytical solution in hand, assuming that at the beginning of inflation we have $\phi \sim \chi \sim \Phi_I$, we find that the total number of e-folding during inflation is $N_{\rm tot} = -(1/\nu)\ln[(1+\gamma)/(\beta/\alpha + \gamma)]$. This allows us to determine the value of the effective Planck mass today as a function of $M_I = -\alpha\Phi_I^2$ through $M_E^2 \simeq M_I^2 e^{\nu N_{\rm tot}}$. If $\alpha,\beta \ll 1$ we have that $M_E^2 \simeq M_I^2$ while being possible to have $N_{\rm tot} \to \infty$.

We can also calculate the predictions for the inflationary observables [14]. We have that $H^2(N) \simeq (18\xi/\beta^2)M_\chi^4/M^2$ which we use to determine the slow roll parameters, $\epsilon = -H'$ and $\eta = \epsilon - \epsilon'/2\epsilon$, and then calculate the tensor to scalar ratio, $r = 16\epsilon$ and the scalar spectra index, $n_s = 1 + 2\eta - 4\epsilon$. We then find the expressions:

$$r = \frac{8\nu}{e^{\nu N_e} - 1} \tag{13}$$

$$n_s - 1 = -\nu\frac{e^{\nu N_e} + 1}{e^{\nu N_e} - 1} \tag{14}$$

where N_e is the number of e-foldings before inflation. To obtain (r, n_s) consistent with the Planck measurements [15], i.e. $r \le 0.1$ and $n_s \sim 0.96$, we need $\nu N_e \sim 1$ which means that $\alpha \sim 10^{-2}$ for $N_e \sim 50 - 60$. Future B-mode constraints will further tighten bounds on r, leading to a lower-bound on α. We have ignored the effect of fluctuations in the ϕ which could in principal lead to additional isocurvature fluctuations and non-negligable non-Gaussian effects [16]. A cursory analysis shows that the curvature of the inflationary trajectories in field space is small (i.e. $\dot\phi^2/(\dot\phi^2 + \dot\chi^2) \ll H^2$) and hence we expect deviations from adiabiticity and gaussianity to be small but a more in depth study is required.

The generation of a hierarchy requires that the choice of parameters in the tree level Lagrangian is also hierarchical and it is important to check whether this choice is stable against radiative corrections. The choice $\lambda \ll \delta \ll \xi$ is stable against non-gravitational corrections because in the limit that λ and δ vanish there is an enhanced shift symmetry $\phi \to \phi + c$. This implies that non-gravitational corrections to δ are proportional to δ while the corrections to λ are proportional to δ^2 or λ, both being perturbatively small.

Gravitational corrections to scalar masses do not occur if the scalar field is shift symmetric. In the case of the χ field this shift symmetry is broken by its $O(1)$ quartic interaction proportional to ξ so one expects corrections to its mass at two loop order involving both the gravitational coupling and ξ. However this contribution is quadratically divergent and so unmeasurable. The divergence is absorbed in the counter term and the renormalised mass vanishes due to the classical scale invariance. The same is true of the mass generated for the χ field due to its shift breaking coupling to the Ricci scalar.

Turning to gravitational radiative corrections to the ϕ mass and the $\delta\phi^2\chi^2$ coupling we note that any such corrections must be inversely proportional to powers of the effective Planck scale set by the ϕ vev. As a result there are *no* such corrections because any power of ϕ in the numerator will be more than compensated by the powers of ϕ in the denominator.

Thus we find that the classical scale invariance is unbroken by gravitational corrections. Indeed there are no gravitational trace anomalies, trace anomalies being generated only by the non-gravitational couplings which do not spoil the hierarchy. This result follows because the theory in the IR has no heavy states with Planck scale masses and so does not have a "real" hierarchy problem. Indeed, if one employs the "endogenous" renormalisation mentioned in the introduction, the theory has no such trace anomalies as it is scale invariant even at the quantum level.

While the model is very simple, it should be possible to extend it to include the Standard Model states with the χ vev coupled to the BEH scalar to provide the electroweak scale[3]. Of course the SM states should have hierarchically small coupling to the ϕ field but such small couplings will again be radiatively stable due to the enhanced symmetry when the couplings are zero. One problem with the scale independent approach applied to the Standard Model is the presence of the Landau pole associated with the $U(1)$ gauge group factor. This signals that the SM becomes strongly interacting at the scale associated with the Landau pole. It is common to assume that there will be massive bound states associated with this strong interaction that will couple significantly to the BEH boson and create the "real" hierarchy problem[4]. One possible way to evade this is to embed the SM in a theory with no Abelian gauge group factor that does not have a Landau pole [17]. This must be done close to the electroweak scale to avoid introducing the hierarchy problem via new massive states and leads to a profusion of new states that may be visible at the LHC. However the Landau pole in the SM lies above the Planck scale where gravitational effects cannot be neglected and it is far from clear clear what the physics above the Landau pole will be and whether it indeed reintroduces the hierarchy problem. For the same reason we did not insist on the absence of a Landau pole in the model considered here. Similarly it is possible that, when gravity becomes strong, it leads to massive states that generate the real hierarchy problem. However it is not know if this happens and, as with the Landau pole problem, we chose to ignore this possibility here.

We have shown that a simple two-scalar model coupled to gravity can satisfies the general criteria: (i) the theory has no mass input parameters, *i.e.*, is classically scale invariant; (ii) scale symmetry is broken only through the scalar coupling to the Ricci scalar which depart from the special conformal value of $-1/6$; (iii) the Planck mass is dynamically generated by the scalar VEV's (iv) there is a viable stage of inflation associated with slow roll in the two–scalar potential; (v) the final vacuum has a small to vanishing cosmological constant and an hierarchical ratio between the Planck scale and the scalar mass scale. Our analysis assumes the paradigm of classical scale symmetry as a custodial symmetry of large hierarchies and we have argued that the hierarchies are preserved even when quantum corrections are included. We will present generalizations of this scheme to multi-scalar theories as well as the inclusion of SM states and expand the formal implications elsewhere [18].

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