ULTIMATE GRADIENT LIMITATION IN NIOBIUM SUPERCONDUCTING ACCELERATING CAVITIES*

M. Checchin†, M. Martinello
Fermilab, Batavia, IL 60510, USA / Illinois Institute of Technology, Chicago, IL 60616, USA
A. Grassellino, A. Romanenko, S. Posen,
Fermilab, Batavia, IL 60510, USA
and J. F. Zasadzinski
Illinois Institute of Technology, Chicago, IL 60616, USA

Abstract

The present study is addressed to the theoretical description of the ultimate gradient limitation in SRF cavities. Our intent is to exploit experimental data to confirm models which provide feed-backs on how to improve the current state-of-art. New theoretical insight on the cavities limiting factor can be suitable to improve the quench field of N-doped cavities, and therefore to take advantage of high Q_0 at high gradients.

INTRODUCTION

The ultimate limiting factor of SRF cavities is the penetration of vortexes from the RF surface, causing an abrupt increase of the surface resistance that consumes all the RF power in the resonator, quenching the superconductive state.

The model we present in this study is addressed to the description of vortex nucleation at the surface. We solve the Ginzburg-Landau (GL) equations numerically for constant and variable GL parameter $\kappa(=\lambda/\xi)$ in order to calculate the lower critical field (H_{c1}) and the superheating field (H_{sh}). In addition we calculate the vortex nucleation barrier for both constant κ and sigmoidal profile of κ, which simulates the presence of a dirty layer at the surface.

CRITICAL AND SUPERHEATING FIELDS CALCULATIONS

We calculate H_{c1} and H_{sh} solving numerically the coupled Ginzburg-Landau equations using in both cases a self-consistent multiple shooting method, where the solutions' domain is increased every iteration.

In order to obtain H_{c1} for arbitrary κ we calculate the energy per vortex line, and use it to define the magnetic field at which the Gibbs free energy is equal to zero (i.e. the lower critical field).

The GL equations that describe a vortex line are defined in cylindrical coordinates as in [1]. Those were solved in order to obtain the order parameter $f(r)$, the vector potential $a(r)$ and the magnetic field $h(r)$, all normalized to get adimensional quantities as in [2]. The boundaries conditions are the following:

$$\begin{aligned} f(r_0) = 0 \quad &; \quad f(R) = 1 \\ a(r_0) = 0 \quad &; \quad a(R) = \frac{1}{\kappa R}, \end{aligned} \tag{1}$$

where $r_0 = 10^{-5}$ is a small number used to avoid the singularity at zero, R is the extension of the solutions' domain, usually larger than 10, and $1/(\kappa R)$ is the asymptotic solution for the vector potential.

The energy per single vortex (ϵ) is then defined as [2]:

$$\epsilon = \int_0^\infty \left[h^2(r) + \frac{1}{2}\left(1 - f^4(r)\right)\right] 2\pi r dr. \tag{2}$$

Equating the Gibbs free energy density in adimensional notation for a single non interacting vortex (g) to zero, we find the minimum field at which the vortex is thermodinamically stable in the material - the lower critical field ($h_{c1} = H_{c1}/(\sqrt{2}H_c)$):

$$g = \epsilon - \frac{4\pi}{k}h = 0 \quad \rightarrow \quad h_{c1} = \frac{k\epsilon}{4\pi}. \tag{3}$$

In order to calculate the superheating field the monodimensional GL equations in absence of currents were solved. The same notation considered in [3] was used. The superheating field corresponds to the maximum field for which solutions to the GL equations, or "superconductive" solutions ($f(0) \neq 0$) still exist. The boundary conditions are:

$$\begin{aligned} f'(0) = 0 \quad &; \quad f(Z) = 1 \\ a'(0) = h(0) \quad &; \quad a(Z) = 0, \end{aligned} \tag{4}$$

where $h(0)$ is the field applied parallel to the surface and Z the extension of the solutions' domain.

Both the simulations (h_{c1} and h_{sh}) were performed with the software Mathematica for $0.2 \leq \kappa \leq 3$. Higher κ were not considered since the simulation time would have been substantially longer.

The values obtained for $h_{c1}(\kappa)$ and $h_{sh}(\kappa)$ are in agreement with previous calculations [2–6]. The analytic equations that best fit the trends obtained are:

$$\begin{aligned} H_{c1} &= 0.58\sqrt{2}H_c\kappa^{-0.57} \\ H_{sh} &= \sqrt{2}H_c\left(0.72 + 0.18\kappa^{-1} + 0.004\kappa^{-2}\right), \end{aligned} \tag{5}$$

where H_c is the thermodynamic critical field, for niobium $H_c \simeq 190$ mT.

* Work supported by the US Department of Energy, Office of High Energy Physics.
† checchin@fnal.gov

Operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the United States Department of Energy

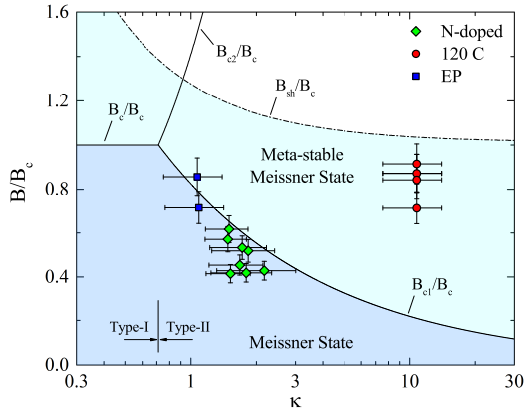

Figure 1: Experimental quench data for cavities treated differently (EP, 120 C baked, N-doped) and simulated $H_{c1}(\kappa)$ and $H_{sh}(\kappa)$ curves. Note that $H_{c2}(\kappa) = \sqrt{2}\kappa H_c$.

VORTEX NUCLEATION AT THE SURFACE

As defined by C.P. Bean and J.D. Livingston [7] the physical explanation of the superheating field is the presence of a vortex nucleation energy barrier at the superconductor surface, that prevents the penetration of vortexes above H_{c1}, dragging the superconductor in a meta-stable Meissner state.

In first approximation the energy barrier is described by the interplay of two forces at the surface: i) the attractive interaction of the vortex with its image anti-vortex on the opposite side of the surface, and ii) the repulsive interaction between the vortex and the penetrating field from the surface. Considering such forces is then possible to calculate the Gibbs free energy density as composed by three terms: vortex-anti-vortex interaction ($g_{v-v}(x)$), vortex-field interaction (g_{v-f}) and the bulk energy (g_∞) that does not depend on the distance from the surface. In adimensional notation the Gibbs free energy density ($g = 4\pi G(x)/(\lambda H_c)^2$)is:

$$\begin{aligned} g(x) &= g_{v-f}(x) + g_{v-v}(x) + g_\infty \\ &= -\frac{4\pi}{\kappa}\int \frac{\partial h_v(2x)}{\partial x} - \frac{\partial h_f(x)}{\partial x} dx \\ &= \frac{4\pi}{\kappa}\left(h_f(x) - h_v(2x)\right) + c. \end{aligned} \tag{6}$$

When $x \to \infty$, then $g(\infty) = c$, and $c = g_\infty = 4\pi/\kappa\,(h_{c1} - h)$. The Gibbs free energy density is then equal to:

$$g(x) = \frac{4\pi}{\kappa}\left(h_f(x) - h_v(2x) + h_{c1} - h\right), \tag{7}$$

where $h_f(x)$ and $h_v(2x)$ are the the magnetic field penetrating from the surface and the vortex magnetic field respectively ($h = H/(\sqrt{2}H_c)$).

EXPERIMENTAL RESULTS AND SIMULATIONS

Cavities prepared with different recipes were tested at the FNAL vertical test facility. The normalized quench fields measured are reported in Fig. 1 as a function of κ, along with the simulated curves for $H_{c1}(\kappa)$ and $H_{sh}(\kappa)$. The values of κ are calculated using $\lambda_0 = 39$ nm, $\xi_0 = 38$ nm and the mean free path values reported in [8] (the same cavities are used in the two works).

Interesting to notice that EP and N-doped cavities show quench fields lower or equal than H_{c1}, while 120 C baked cavities quench above H_{c1} in the meta-stable Meissner state. The ultimate limiting factor of N-doped cavities seems to be the lower critical field, since no cavities have been seen quenching above H_{c1} yet. On the other hand the same cannot be said for EP cavities since quenching when the high-field-Q-slope is present, so the gradient limitation may be of different nature (e.g. hydrides proximity breakdown).

We calculate the Bean-Livingston barrier for increasing values of κ and applied field equal to the lower critical field for that particular κ (shown in Fig. 2). The simulations in Fig. 2 underlines that the energy barrier height decreases for increasing κ, which, in first approximation, implies that the efficiency to exclude thermodynamically stable vortexes from the material's bulk decreases as κ increases. Such phenomenon should in principle prevent 120 C baked cavities to quench above H_{c1}, since possessing the highest κ among the cavities studied, but this is not the case as shown in Fig. 1.

So, why do 120 C baked cavities quench in a meta-stable Meissner state while N-doped cavities do not?

We know from LE-μSR measurements [9] that 120 C baked cavities near surface presents a strong variation of mean free path (l) throughout the penetration depth. Such strong l variation is instead not present in N-doped cavities which possess a relatively flat mean free path profile within the penetration depth.

In light of such knowledge we simulate the Bean-Livingston barrier for various κ profiles varying as a function of depth. To do so we introduce an analytic sigmoid form for $\kappa(x)$ with a variable GL parameter at the surface (κ_s), a constant value in the bulk ($\kappa_\infty = 1.04$), and saddle point chosen around 20 nm from the surface (the κ profile considered are showed in the inset of Fig. 3c).

While the calculation of the penetrating field was carried out by simply considering $\kappa(x)$ instead of a constant value, the vortex field was calculated for every $g(x)$ point, since

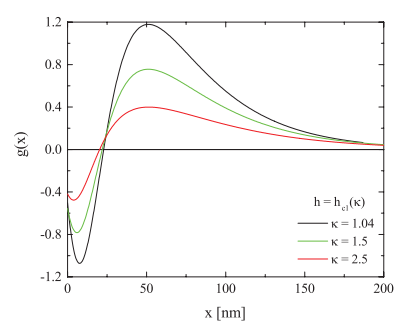

Figure 2: Gibbs free energy density for various constant κ for $h = h_{c1}(\kappa)$.

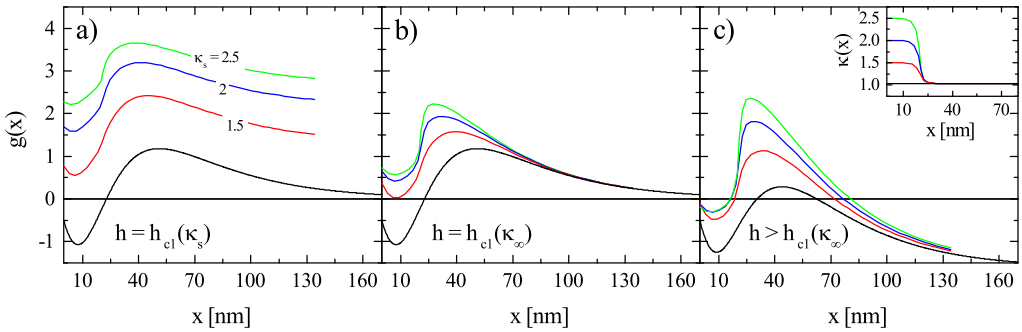

Figure 3: Gibbs free energy density calculated for different κ profiles - shown in the inset - for applied field h equal to (a) $h_{c1}(\kappa_s)$, (b) $h_{c1}(\kappa_\infty)$ and (c) in the range $h_{c1}(\kappa_\infty) < h < h_{sh}(\kappa_s)$.

the $\kappa(x)$ profile seen by the vortex is different for every x coordinate. In particular, we approximate that the vortex order parameter and vector potential depend on a κ profile that is cylindrical symmetric. In other words $\kappa(x-r)$ varies from its value at the position x - the vortex center - with increasing concentric constant values for every r till its value is equal to κ_s. Such approximation permits us to calculate $h_v(2x)$ with the correct κ profile along x, without solving non-cylindrically symmetric GL equations.

Because of the introduction of the κ profile, the Gibbs free energy definition in Eq. 6 considers $\kappa(x)$ instead of a constant value, while g_∞ depends only κ_∞ since defined in the material bulk. The new Gibbs free energy density definition then is:

$$g(x) = \frac{4\pi}{\kappa(x)}\left(h_f(x) - h_v(2x)\right) + \frac{4\pi}{\kappa_\infty}\left(h_{c1}(\kappa_\infty) - h\right). \quad (8)$$

Figure 3a shows the simulated variation of the Gibbs free energy for different κ profiles - showed in the inset - for fixed applied reduced field ($h = h_{c1}(\kappa_s)$). As κ_s increases, the Gibbs free energy in the whole material assumes values larger than zero. In this situation vortexes are not energetically favorable to nucleate in the superconductor bulk nor at its surface. No quench should therefore happen when $h = h_{c1}(\kappa_s)$.

If the field is increased till $h = h_{c1}(\kappa_\infty)$, then vortexes become thermodinamically stable in the superconductor bulk ($g_\infty \to 0$), but at the surface there still be the energy barrier to prevent their penetration. In Fig. 3b is shown the energy barrier at the surface for the κ profiles plotted in the inset of Fig. 3c. The presence of a dirty layer in the surface affects positively the vortex nucleation barrier at the surface, increasing its height as κ_s increases. The meta-stable Meissner state is therefore stabilized by the presence of the dirty layer.

Once the field is further increased between $h_{c1}(\kappa_\infty)$ and $h_{sh}(\kappa_s)$ when the dirty layer is present, the barrier is less affected by the presence of higher field than when κ is constant. Such important effect is clearly shown comparing Fig. 3b to Fig. 3c. This implies that the meta-stable Meissner state is stabilized and the cavity might reach the superheating field.

Reconsidering Fig. 2 for a constant GL parameter, we have shown that the energy barrier decreases as κ increases. On the other hand if the superficial κ_s is increased while the bulk κ_∞ is maintained constant, the trend is the opposite and the barrier increases in height and width with κ. Therefore, 120 C baked cavities are quenching in the meta-stable state probably because of the presence of a non-constant κ profile inside the penetration depth. N-doped cavities on the contrary show constant κ within distances longer than the penetration depth (κ can be considered constant), and their quench at H_{c1} might be explained by the suppression of their energy barrier driven by both the "constant" κ and superficial defects.

CONCLUSIONS

In this paper we addressed theoretically the ultimate limiting factor of the high gradient quench of 120 C baked cavities and N-doped cavities. The presence of the dirty layer is beneficial in order to stabilize the meta-stable Meissner state, and push 120 C baked cavities up to the superheating field. The lower quench field of N-doped cavities at H_{c1} can instead be explained by the suppression of the vortex nucleation barrier because of a constant κ and by the presence of defects. Sill, no clear conclusion can be done for EP cavities since quenching in a different regime where high surface losses are present.

The model presented is able to give insights for future developments on high quality factors at high gradients. Indeed, a superficial dirty layer may improve the gradient, and, at the same time, may improve the Q-factor, since small mean free paths ($\sim$ 20 nm) minimize the Mattis-Bardeen surface resistance. Therefore, by a smart surface engineering it may be possible to achieve high Q-factors up to high gradients, allowing future high gradient accelerators to minimize their cryogenic losses.

REFERENCES

[1] P. Rentop, Computing 16, 61 (1976)

[2] P. Tholfsen and H. Meissner, Phys. Rev. 169, 413 (1968)

[3] A. J. Dolgert, S. J. Di Bartolo and A. T. Dorsey, Phys. Rev. B 53, 5650 (1996)

[4] J. L. Harden and V. Arp, Cryogenics 3, 105 (1963)

[5] J. Matricon and D. Saint-James, Phys. Lett. 24A, 241 (1967)

[6] M. K. Transtrum, G. Caletani and J. P. Sethna, Phys. Rev. B 83, 094505 (2011)

[7] C. P. Bean and J. D. Livingston, Phys. Rev. Lett. 12, 14 (1964)

[8] M. Martinello, A. Grassellino, M. Checchin, S. Posen, O. Melnychuk and D. A. Sergatskov, in Proceedings of the 7th International Particle Accelerator Conference WEPMR003, Busan, South Korea (2016)

[9] A. Romanenko A. Grassellino, F. Barkov, A. Suter, Z. Salman and T. Prokscha, Appl. Phys. Lett. 104, 072601 (2014)